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Alberta-Montana Combinatorics and Algorithms Days

Hadi Kharaghani (University of Lethbridge),
Ryan Hayward (University of Alberta),
Mark Kayll (University of Montana),
Robert Woodrow (University of Calgary)

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1 Overview

The Banff International Research Station (BIRS) hosted the second *Alberta-Montana Combinatorics and Algorithms Days* on June 23–25, 2023. The essential purpose of the event was to bring together, for a second time, faculty and students from post-secondary institutions in Alberta and Montana. The participating universities were: University of Lethbridge, University of Calgary, University of

Alberta, and University of Montana.

Combinatorics at its heart is the study of discrete structures. This pithy description, however, belies the inherent complexity of these objects and their deep connections both with each other and with other branches of mathematics such as algebra and number theory. Moreover, combinatorics is seeing ever proliferating applications in fields as diverse as cryptography and algorithm analysis in game theory and elsewhere. All of these exciting and cutting-edge topics and more were touched upon at this year's workshop at BIRS as outlined in the abstracts reproduced below.

Demographics (briefly)

The workshop attracted participants from a broader geographical region than originally envisioned; to wit, there were speakers from Calgary, Edmonton, Lethbridge, Missoula, Vancouver, Burnaby, Hartford, and Bhopal in India. Likewise, the "vertical" representation went even deeper than the organizers had originally hoped. Of the participants, there were six undergraduate students (one of whom spoke), three graduate students (two of whom spoke), one postdoctoral fellow (who spoke), one MITAC summer research student from Bhopal in India, and a freshman at the University of Waterloo. Thus, the total of fifteen talks was rounded out by eleven lectures given by junior and senior faculty.

2 Presentation highlights

Here follows a précis of each of the talks in their original chronological order.

Davoud Abdi (University of Calgary) opened the workshop by presenting on equimorphic objects, that is, objects which are at once embeddable into each other. In the case the objects are infinite, one does not necessarily obtain isomorhpic objects. Speaking on conjectures specific to this phenomenon, Davoud touched upon those of Thommassé and Bonato-Tardif. Thomassé conjectured that a countable relation has either one or many siblings, up to isomorphism. The Bonato-Tardif conjecture is related to this and states that a locally finite tree has one or infinitely many siblings. The talk concluded with counterexamples to these two conjectures.

Following the opening address, Martin Mueller (University of Alberta) spoke on combinatorial games with sum structure, namely, games that are an aggregation of independent subgames. By considering only boolean outcomes;"Did player A win or loose?", using elementary means, he showed how one can improve drastically upon the typical computation of a canonical representative, a known computational bottleneck. A case study for the game of 1-dimensional linear Clobber was presented.

Because of Xinyue Chen's absence, Ryan Hayward (University of Alberta) presented his talk on his behalf. Ryan spoke on the game of Hex, a two-player zero-sum game. Evidently, the first player to move is always guaranteed to win if only they can play a "perfect game." He went on to explain the use of the Benze Hex Solver with the DFPN algorithm in finding the starting positions available for a player to have a "perfect game." In particular, he gave valuable insights into how proof-number search and how depth-first-proof-number search algorithms work and can be used to solve such problems.

Continuing the topic on games, Invited Speaker Svenja Huntemann (Concordia University, Edmonton) presented on the family of so-called placement games. Such games take place on a finite graph where two players take turns placing tokens, without moving or replacing them. Specifically, Svenja outlined the differences between placement games such as Col and Domineering to games with strong placement such as Snort, NoGo, and Hex. She gave a detailed survey about essential combinatorial game theory terminology, and the idea of disjunctive sums for placement games. Svenja went on to introduce game values as a metric for quantifying how much of an advantage the winning player holds and temperature to determine the urgency of making a particular move. She later spoke about enumerating all possible positions of a game to develop the complexity of a complete analysis. The talk concluded by giving an overview of open research areas within this problem context.

Mahya Jamishidian (University of Alberta) gave a review of the related clustering problems Minimum Sum of Radii (MSR), Minimum Sum of Diameters (MSD), and Minimum Sum of Squared Radii (MSSR). In improving upon what was previously known, Mahya presented her findings on developing a 3.389-approximation for MSR and a 6.546-approximation for MSD, improving over respective 3.504 and 7.008. Furthermore, a 11.078-approximation algorithm for MSSR with a similar approach to the MSR and MSD cases known as well. The techniques employed include bi-point solutions, LP, Lagrangian Relaxation, followed by rounding technique and binary search.

Invited Speaker Stephanie Van Willigenburg (University of British Columbia) spoke on the Stanley-Stembridge (3+1) free conjecture and introduced the chromatic symmetric functions, the (3+1)-free conjecture, new cases and tools for resolving it. She continued on an answer to a question of Stanley of whether the (3+1)-free conjecture can be widened, see [3].

Zac Friggstad (University of Alberta) spoke on the difficult subject of the chromatic theory of graphs. Focusing on the minimum sum colouring problem (MSC), Zac recapitulated the facts that linear time algorithms can recognize 2-colourable bipartite graphs and optimally colour cubic graphs. In general, however, the problem is known to be NP-hard. By specializing to the case of chordal graphs, Zac uses linear-programming techniques for minimum latency problems obtaining a 1.796-approximation algorithm for the MSC problem in chordal graphs.

Caleb Van't Land (University of Lethbridge) spoke about the Butson Hadamard matrices and a Bush-type Butson Hadamard matrices. He went over what it means for two Hadamard matrices to be unbiased, and demonstrated a construction for a maximal family of unbiased Bush-type Butson Hadamard matrices, see [5].

Invited Speaker Dave Morris (University of Lethbridge) recapped the conjecture that cycles are the only finite regular graphs that have unique hamiltonian cycle and an eventual proof in 2014 that the conjecture is true in the special case where the graph is vertex-transitive. Dave then spoke on the possible generalization of the vertex-transitive case to infinite graphs.

Van Magnan (University of Montana) presented three directions of generalizations of the Erdős-Ko-Rado theorem and generalized diversity for families of sets. He then talked about the flower base as a set of special transversals which inherits the family's key properties. Every member of the family contains a member of the flower base. Throughout the remainder of his talk, Van went through ways that can work with the flower base to solve small design problems.

Amarpreet Rattan (Simon Frazer University) spoke about how parking functions, trees, and factorizations are connected to one another. He then continued on the statistics of these mathematical objects, and how it can be generalized to k-parking functions, see [4].

Invited Speaker Cory Palmer (University of Montana) began his survey of extremal co-degree problems for hypergraphs by introducing three parameters including Turán number. After a detailed history of the three parameters in hypergraphs he focused on recent developments on the positive co-degree problem, see [1].

Anna Halfpap (University of Montana) gave a detailed review of the general properties of the positive co-degree Turán number. She explained why these properties are desirable, and then talked further about the hypergraph removal lemma towards establishing supersaturation.

For the penultimate address, Invited Speaker Rei Safavi-Naini (University of Calgary) gave an in-depth synopsis of the history of cryptography and fostered an appreciation for the methodology and computational underpinnings behind this technology. Rei spoke about challenges to modern cryptography algorithms such as posed by the realization of quantum computers.

The concluding talk was given by Daniel Johnston (Trinity College, CT) who defined and spoke to the saturation of graphs, and then demonstrated what it means for a graph to be rainbow saturated. Similar to the rainbow Turán number introduced by Keevash, Mubayi, Sudakov, and Verstraëte, Daniel went on to introduce the rainbow saturation number of graphs, see [2].

3 Meeting outcomes

Nestled in Banff, the heart of the Canadian Rockies, BIRS speakers gathered under one roof from: University of Calgary, University of Alberta, University of Montana, University of Lethbridge, Trinity College, University of British Columbia, and Simon Fraser University. Undergraduates, Masters, and PhD students all felt welcome and were encouraged to participate in the student-friendly environment. Participants of this BIRS workshop found themselves to be an equal part of the group which shared a joint curiosity for combinatorics and algorithms with underlying mathematical structures. The organizers facilitated the development of new connections between people from different walks of life that, hopefully, will lead to fruitful collaborations in the near future.

A last minute cancellation due to illness and the timely action by the Program Coordinator provided an opportunity for a MITAC summer intern from India a chance to participate at the workshop. In his words “The BIRS workshop was one of a kind experience for me. I couldn’t think of any place more beautiful and engaging than BIRS. The talks were well organized, perfectly delivered and were very engaging. I learnt many new concepts. And above all, everyone was very friendly and helpful”.

Selected references

References

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[3] Samantha Dahlberg, Andéle Foley, Stephanie van Willigenburg, Resolving Stanley's e-positivity of claw-contractible-free graphs. *J. Eur. Math. Soc. (JEMS)* **22** (2020), no. 8, 2673–2696.

[4] John Irving, Amarpreet Rattan, Trees, parking functions and factorizations of full cycles. *European J. Combin.* **93** (2021), Paper No. 103257, 22 pp.

[5] H. Kharaghani, T. Pender, C. Van't Land and V. Zaitsev, Bush-type Butson Hadamard matrices, Special issue of *Glasnik Matematicki* dedicated to Zvonimir Janko, `https://web.math.pmf.unizg.hr/glasnik/forthcoming.html`