Optimal Transportation and Differential Geometry
May 20 – 25, 2012

MEALS

*Breakfast (Buffet): 7:00–9:30 am, Sally Borden Building, Monday–Friday
*Lunch (Buffet): 11:30 am–1:30 pm, Sally Borden Building, Monday–Friday
*Dinner (Buffet): 5:30–7:30 pm, Sally Borden Building, Sunday–Thursday
Coffee Breaks: As per daily schedule, in the foyer of the TransCanada Pipeline Pavilion (TCPL)
***Please remember to scan your meal card at the host/hostess station in the dining room for each meal.**

MEETING ROOMS

All lectures will be held in the new lecture theater in the TransCanada Pipelines Pavilion (TCPL). LCD projector and blackboards are available for presentations.

SCHEDULE

Sunday	
16:00	Check-in begins (Front Desk - Professional Development Centre - open 24 hours)
17:30–19:30	Buffet Dinner, Sally Borden Building
20:00	*Informal* gathering in 2nd floor lounge, Corbett Hall
	Beverages and a small assortment of snacks are available on a cash honor system.
Monday	
7:00–9:00	Breakfast
9:00–9:10	Introduction and Welcome by BIRS Station Manager
9:10-10:00	NEIL TRUDINGER
	On the local theory of prescribed Jacobian equations
10:00-10:20	coffee break
10:20-11:10	NICOLA GIGLI
	Differential structure of metric measure spaces
11:10-11:20	short break
11:20-11:45	SHIN-ICHI OHTA
	Splitting theorems for Finsler manifolds of nonnegative Ricci curvature
11:45–14:00	Free Time and Lunch
14:00-14:50	SIMON BRENDLE
	Minimal tori in S^3 and the Lawson conjecture
14:50-15:10	coffee break
15:10-16:00	JEROME BERTRAND
	Prescribing Gauss curvature using mass transport
16:00-16:10	short break
16:10-17:00	GUOFANG WEI
	Smooth metric measure spaces and Ricci solitons
17:00-19:30	Free Time and Dinner

Tuesday

7:00–9:00	Breakfast
9:00-9:50	GIUSEPPE SAVARE Displacement convexity and evolution variational inequalities of entropy functionals
9:50-10:10	coffee break
10:10-11:00	NASSIF GHOUSSOUB Monge-Kantorovich problems with symmetry
11:00-11:10	short break
11:10-11:35	ALEXANDER KOLESNIKOV Hessian metrics, C(D,K)-spaces, and optimal transportation
11:35–13:00	Lunch
13:00-14:00	**Group photo and guided tour:** The tour of the campus is offered from 1.00-1.50 pm - meet in the 2nd floor lounge of Corbett Hall The group photo is at 1.50 pm - meet in the foyer of TCPL (the photograph is taken outside so participants may need a jacket).
14:00-14:50	JIAKUN LIU Light reflection is nonlinear optimization
14:50-15:10	coffee break
15:10-15:35	TAPIO RAJALA Interpolated measures with bounded density in CD(K,N)-spaces
15:35-15:50	short break
15:50-16:40	KAZUMASA KUWADA Monotonicity of time-dependent transportation costs and coupling by reflection
16:40–19:30	Free Time and Dinner

Wednesday

7:00–9:00	Breakfast
9:00-9:50	STEFANO BIANCHINI The Monge problem for convex costs
9:50-10:10	coffee break
10:10-11:00	JONATHAN KORMAN Optimal transportation with capacity constraints
11:00-11:10	short break
11:10-11:35	BEATRICE ACCIAIO Mass Transport, Robust Pricing and Trajectorial Inequalities
11:35–17:30	Lunch and free afternoon
17:30–19:30	Dinner

Thursday

7:00–9:00	Breakfast
9:00-9:50	LUIGI AMBROSIO
	Equivalence of weak gradients in metric measure spaces
9:50-10:10	coffee break
10:10-11:00	BENOIT KLOECKNER
	The size of Wasserstein spaces
10:00-11:10	short break
11:10-11:35	TOMMASO PACINI
	Differential forms and symplectic geometry on Wasserstein space
11:35–13:30	Lunch
13:30-14:20	WILLIAM WYLIE
	Warped product Einstein metrics and Ricci solitons
14:20-14:30	short break
14:30-14:55	MATTHIAS ERBAR
	Gradient flows of the entropy for jump processes
14:55-15:15	coffee break
15:15-15:40	YI WANG
	The Aleksandrov-Fenchel inequalities of k+1 convex domains
15:40-15:50	short break
15:50-16:15	JUN KITAGAWA
	Regularity for the optimal transport problem on the embedded sphere when standard conditions fail
16:15 –17:30	free discussion
17:30–19:30	Dinner

Friday

7:00–9:00	Breakfast
9:00 - 9:50	BRENDAN PASS
	Optimal transportation with infinitely many marginals
9:50-10:10	coffee break
10:10-11:00	PAUL LEE
	Displacement interpolation from a Hamiltonian point of view
11:00–13:30	Free Time and Lunch
Checkout by 12 noon.	

** 5-day workshop participants are welcome to use BIRS facilities (BIRS Coffee Lounge, TCPL and Reading Room) until 3 pm on Friday, although participants are still required to checkout of the guest rooms by 12 noon. **

Optimal Transportation and Differential Geometry
May 20 – 25, 2012

ABSTRACTS
(in alphabetic order by speaker surname)

Speaker: **BEATRICE ACCIAIO** (University of Perugia)
Title: *Mass Transport, Robust Pricing and Trajectorial Inequalities*
Abstract: Robust pricing basically corresponds to considering extremal pricing rules coming from possible p̈ricing measureswẅhich satisfy marginal constraints. This problem is naturally connected to that of mass transport. Mathematically the crucial difference is that transport plans are required to be martingales in our setting. We will discuss the advantages of relating the robust pricing problem to the theory of mass transportation. In particular, we will see that the duality theorem from optimal transport can be used to establish new robust super-replication results. This dual viewpoint also provides new insights on classical martingale inequalities. For instance, we establish a (new) sharp version of the classical Doob maximal inequality.

Speaker: **LUIGI AMBROSIO** (Scuola Normale Superiore Pisa)
Title: *Equivalence of weak gradients in metric measure spaces.*
Abstract: In a joint work with Gigli and Savaré, we compare several notion of weak (modulus of) gradient in metric measure spaces and prove their equivalence. This equivalence is part of the "calculus program" we developed, largely based on tools from optimal transportation theory. In particular, we prove density in energy of Lipschitz maps in Sobolev spaces independently of doubling and Poincareássumptions on the metric measure space. The case of BV maps, together with some open questions, will also be illustrated (joint work with Di Marino).

Speaker: **JEROME BERTRAND** (IMT)
Title: *Prescribing Gauss curvature using mass transport*
Abstract: I will give a proof of Alexandrov's theorem on the Gauss curvature prescription of Euclidean convex body. The proof is based on mass transport and the classical theory of convex bodies duality. In particular, it doesn't rely on pdes method nor convex polyhedra theory. With this approach, it is also possible to treat the case of equivariant convex bodies in the (Lorentzian) Minkowski space.

Speaker: **STEFANO BIANCHINI** (SISSA)
Title: *The Monge problem for convex costs*
Abstract: We will show that the the optimal transport problem

$$\inf_{\pi} \int c(x-y)\pi(dxdy), \qquad \pi \in \Pi(\mu,\nu)$$

with $c : \mathbb{R}^d \to \mathbb{R}^d$ convex has a solution of the form $\pi = (Id, T)_{\#}\mu$. The proof is based on a decomposition of Sudakov's type.

Speaker: **SIMON BRENDLE** (Stanford University)
Title: *Minimal tori in S^3 and the Lawson conjecture*
Abstract: We show that any embedded minimal torus in S^3 is congruent to the Clifford torus. This answers a question posed by H.B. Lawson, Jr., in 1970.

Speaker: **MATTHIAS ERBAR** (University of Bonn)
Title: *Gradient flows of the entropy for jump processes*

Abstract: We introduce a new transportation distance between probability measures that is built from a Lévy jump kernel. It is defined via a non-local variant of the Benamou-Brenier formula. We study geometric and topological properties of this distance, in particular we prove existence of geodesics. For translation invariant jump kernels we identify the semigroup generated by the associated non-local operator as the gradient flow of the relative entropy w.r.t. the new distance and show that the entropy is convex along geodesics.

Speaker: **NASSIF GHOUSSOUB** (University of British Columbia)
Title: *Monge-Kantorovich problems with symmetry*
Abstract:

Speaker: **NICOLA GIGLI** (University of Nice)
Title: *Differential structure of metric measure spaces*
Abstract:

I'll show that every metric measure space (complete, separable endowed with a Borel locally finite measure) has a natural first order differentiable structure: it is possible to speak about differential and gradients of Sobolev functions. As an application, I'll provide a general definition of distributional Laplacian and show that on spaces with Ricci curvature bounded from below, for the Laplacian of the distance function the standard comparison estimates hold

Speaker: **JUN KITAGAWA** (University of British Columbia)
Title: *Regularity for the optimal transport problem on the embedded sphere when standard conditions fail*
Abstract: We consider regularity for Monge solutions to the optimal transport problem when the initial and target measures are supported on the embedded sphere, and the cost function is the Euclidean distance squared. Gangbo and McCann have shown that when the initial and target measures are supported on boundaries of strictly convex domains in
$mathbbR^n$, there is a unique Kantorovich solution, but it can fail to be a Monge solution. In the case when we are dealing with the sphere with measures absolutely continuous with respect to surface measure, we present two different types of conditions on the densities of the measures to ensure that the solution given by Gangbo and McCann is indeed a Monge solution, and obtain higher regularity as well. This talk is based on joint work with Micah Warren.

Speaker: **BENOIT KLOECKNER** (Universit Grenoble 1)
Title: *The size of Wasserstein spaces*
Abstract: The Hausdorff dimension is one of the most natural ways to measure the size of a metric space, but it cannot distinguish between most Wasserstein spaces since they are often infinite-dimensional. We shall explain how old ideas can be used to generalize Hausdorff dimension in a way that distinguishes many Wasserstein spaces. One of the consequences is that the Wasserstein space of a compact manifold can never be Lipschitz embedded in the Wasserstein space of a compact manifold of lesser dimension.

Speaker: **ALEXANDER KOLESNIKOV** (Higher School of Economics)
Title: *Hessian metrics, C(D,K)-spaces, and optimal transportation*
Abstract: We study the optimal transportation mapping $\nabla\Phi : \mathbb{R}^d \mapsto \mathbb{R}^d$ pushing forward a probability measure $\mu = e^{-V}\ dx$ onto another probability measure $\nu = e^{-W}\ dx$. Following a classical approach of E. Calabi we introduce the Riemannian metric $g = D^2\Phi$ on $\mathbb{R}^d$ and study spectral properties of the metric-measure space $M = (\mathbb{R}^d, g, \mu)$. We prove, in particular, that M admits a non-negative Bakry-Emery tensor provided both V and W are convex. If the target measure ν is the Lebesgue measure on a convex set Ω and μ is log-concave we prove that M is a $CD(K, N)$ space. Applications of these results include some global dimension-free apriori estimates of $\|D^2\Phi\|$. With the help of comparison techniques on Riemannian manifolds we estimate the diameter of M in terms of the dimension and the diameter of Ω.

Speaker: **JONATHAN KORMAN** (University of Toronto)
Title: *Optimal transportation with capacity constraints*
Abstract: The classical problem of optimal transportation can be formulated as a linear optimization problem on a convex domain: among all joint measures with fixed marginals find the optimal one, where optimality is measured against a cost function. Here we consider a variant of this problem by imposing a constraint on the joint measures: among all joint measures with fixed marginals and which are dominated by a given measure find the optimal one. We show uniqueness of the solution, and compute an example. Joint work with Robert McCann.

Speaker: **KAZUMASA KUWADA** (Ochanomizu University)
Title: *Monotonicity of time-dependent transportation costs and coupling by reflection*
Abstract: A new monotonicity in time of a time-dependent transportation cost between distribution of diffusion processes is shown under Bakry-Emery's curvature-dimension condition on a Riemannian manifold. This result is an analog of the L^p-Wasserstein contraction of heat distributions under lower Ricci curvature bound. The cost function comes from the total variation distance between heat distributions on spaceforms. As a corollary, we obtain a comparison theorem for the total variation distance between heat distributions. We also obtain an explicit expression of the cost function. It leads to a time-independent transportation cost which is non-increasing in time for heat distributions even on a negatively curved space. In addition, we show that our monotonicity is stable under the Gromov-Hausdorff convergence of the underlying space under a uniform curvature-dimension and diameter bound. This talk is based on a joint work with Karl-Theodor Sturm.

Speaker: **PAUL LEE** (Chinese University of Hong Kong)
Title: *Displacement interpolation from a Hamiltonian point of view*
Abstract: We discuss displacement interpolations from the point of view of Hamiltonian systems and give a unifying approach to various known results.

Speaker: **JIAKUN LIU** (Princeton University)
Title: *Light reflection is nonlinear optimization*
Abstract: In the special far field case, the light reflection is related to the reflector antenna design problem. Xu-Jia Wang showed that it is an optimal transportation, and so, is a linear optimization problem. In this talk, we study the general case of the light reflection problem, and show that it is related to a nonlinear optimization problem. This problem involves a fully nonlinear PDE of Monge-Ampere type, subject to a nonlinear boundary condition.

Speaker: **SHIN-ICHI OHTA** (Kyoto University)
Title: *Splitting theorems for Finsler manifolds of nonnegative Ricci curvature*
Abstract: I will discuss the notion of Ricci curvature in Finsler geometry and, as its recent application, generalizations of the Cheeger-Gromoll type splitting theorem

Speaker: **TOMMASO PACINI** (SNS, Pisa)
Title: *Differential forms and symplectic geometry on Wasserstein spac*
Abstract: I will give a brief overview of the paper "Differential Forms on Wasserstein Space..." (Memoirs AMS, 2011), joint with Gangbo and Kim. Topics include: differential calculus on Wasserstein space; de Rham cohomology; symplectic structures and Hamiltonian systems. I will try to explain the underlying intuition and the finite-dimensional analogues.

Speaker: **BRENDAN PASS** (University of Alberta)
Title: *Optimal transportation with infinitely many marginals*
Abstract: We formulate and study the problem of aligning a continuum of marginals as efficiently as possible. In our formulation, we look for the stochastic process with prescribed single time marginals

which minimizes the expectation of a certain functional. This problem is a natural extension of a multi-marginal optimal transportation problem studied by Gangbo and Swiech (1998). In this talk, we prove existence, uniqueness and characterization results.

Speaker: **TAPIO RAJALA** (SNS, Pisa)
Title: *Interpolated measures with bounded density in $CD(K,N)$-spaces*
Abstract: I will briefly explain how using only the convexity-inequality for the critical entropy-functional in a $CD(K,N)$-space one can construct geodesics in the Wasserstein space along which all the measures have bounded density. If time permits I will also discuss some applications of these "good" geodesics.

Speaker: **GIUSEPPE SAVARE** (University of Pavia)
Title: *Displacement convexity and evolution variational inequalities of entropy functionals*
Abstract: The talk will focus on the links between the displacement convexity of entropy functionals and the characterizations of their gradient flows in Wasserstein spaces in terms of a family of evolution variational inequalities. In the particular case of the logarithmic entropy the above properties are strictly related to the linearity and the contractivity of the flow. Various properties and applications to metric measure spaces with Ricci curvature bounded from below will be discussed. (In collaboration with L. Ambrosio and N. Gigli)

Speaker: **NEIL TRUDINGER** (Australian National University)
Title: *On the local theory of prescribed Jacobian equations*
Abstract: We develop the fundamentals of a local regularity theory for prescribed Jacobian equa- tions which extend the corresponding results for optimal transportation equations. In this theory the cost function is extended to a generating function through dependence on an additional scalar variable. In particular we recover in this generality the local regularity theory for potentials of Ma,Trudinger and Wang, along with the subsequent development of the underlying convexity theory.

Speaker: **YI WANG** (Stanford University)
Title: *The Aleksandrov-Fenchel inequalities of k+1 convex domains*
Abstract: In this talk, I will report some recent joint work with Sun-Yung Alice Chang in which we generalize Michael-Simon inequality and partially generalize the Aleksandrov-Fenchel inequalities for quermassintegrals from convex domains in the Euclidean space to a class of non-convex domains. I will also talk about optimal constants of the inequality in some special cases as well.

Speaker: **GUOFANG WEI** (UC Santa Barbara)
Title: *Smooth metric measure spaces and Ricci solitons*
Abstract: Joint with W. Wylie, we extended several comparison results (in particular, the Bishop-Gromov volume comparison) for manifolds with lower Ricci curvature bound to smooth metric measure spaces with Bakry-Emery Ricci tensor bounded from below. We will discuss several applications of this. In particular , Peng Wu uses it to show that the infimum of the potential function of a gradient steady Ricci soliton grows linearly. Joint with P. Wu, for a large class of gradient steady Ricci solitons, we obtain optimal growth estimate for the potential function, and show the volume grows at most like polynomial of degree n.

Speaker: **WILLIAM WYLIE** (Syracuse University)
Title: *Warped product Einstein metrics and Ricci solitons*
Abstract: n joint work with Chenxu He and Peter Petersen we show that any algebraic Ricci soliton on a left invariant Lie group can be extended to a homogeneous warped product Einstein space. This extends a result of Lauret for solvable groups. It also provides the existence of many new homogeneous smooth metric measure spaces which are m-Quasi Einstein manifolds. We also give a strong characterization of the geometric structure of such spaces.