OPEN DYNAMICAL SYSTEMS: ERGODIC THEORY, PROBABILISTIC METHODS AND APPLICATIONS
12W5050
APR 8 - APR 13, 2012

1. Participants

Wael Bahsoun, Loughborough University, UK
Viviane Baladi, Ecole Normale Superieure, France
Sanjeeva Balasuriya Connecticut College, USA
Oscar Bandtlow, Queen Mary University of London, UK
Arno Berger, University of Alberta, Canada
Erik Bollt, Clarkson University, USA
Chris Bose, University of Victoria, Canada
Henk Bruin, University of Surrey, UK
Leonid Bunimovich, Georgia Institute of Technology, USA
Michael Dellnitz, University of Paderborn, Germany
Mark Demers, Fairfield University, USA
Carl Dettmann, The University of Bristol, UK
Peyman Eslami, Concordia University, Canada
Andrew Ferguson, University of Bristol, UK
Gary Froyland, University of New South Wales, Australia
Cecilia González Tokman, University of Victoria, Canada
Paweł Góra, Concordia University, Canada
Nicolai Haydn, Southern California, USA
Shafiqul Islam, University of Prince Edward Island, Canada
Oliver Junge, Technische Universität München, Germany
Rainer Klages, Queen Mary University of London, UK
Kevin Lin, University of Arizona, USA
Carlangelo Liverani, University of Rome Tor Vergata, Italy
Simon Lloyd, University of Sao Paulo, Brazil
Daniel Mansfield, UNSW, Australia
James Meiss, University of Colorado, Boulder, USA
Ian Melbourne, University of Surrey, UK
Rua Murray, University of Canterbury, New Zealand
Matthew Nicol, University of Houston, USA
William Ott, University of Houston, USA
Kathrin Padberg-Gehle, TU Dresden, Germany
Tomas Persson, Lund University, Sweden
Mark Pollicott, University of Warwick, UK
Anthony Quas, University of Victoria, Canada
Shane Ross, Virginia Tech, USA
Naratip Santitissadeekorn, Clarkson University, USA
Yuzuru Sato, Hokkaido University, Japan
Robyn Stuart, UNSW, Australia
Dalia Terhesiu, University of Rome Tor Vergata, Italy
Serge Troubetzkoy, Marseille, France
Paul Tupper, Simon Fraser University, Canada
Polina Vytnova, University of Warwick, UK
Tom Watson, UNSW, Australia
James Yorke, University of Maryland, USA
Roland Zweimüller, University of Vienna, Austria

Date: April 3, 2012.

2. Schedule

	Monday	Tuesday	Wednesday	Thursday	Friday
8:30	Demers (1)	Baladi (1)	Dellnitz (1)	Zweimüller (1)	Meiss (1)
9:30	Haydn (1)	Liverani (1)	Pollicott (1)	Terhesiu (1)	Balasuriya (1/2) Lin (1/2)
10:30	TEA BREAK	TEA BREAK	TEA BREAK	TEA BREAK	TEA BREAK
11:00	Dettmann (1)	Bollt (1/2) Santitissadeekorn (1/2)	Padberg-Gehle (1/2) Ross (1/2)	Bandtlow (1)	Bruin (1)
12:00	Klages (1/2)	Ott (1/2)	GROUP PHOTO	Junge (1/2)	CHECK OUT
12:30	LUNCH	LUNCH	LUNCH	LUNCH	LUNCH
14:00	Lloyd (1/2) González-Tokman (1)	Melbourne (1)	HIKE (13:30)	Góra (1/2) Murray (1/2)	
15:00		Nicol (1/2)		Eslami (1/2)	
15:30	TEA BREAK	TEA BREAK		TEA BREAK	
16:00	Bunimovich (1)	Yorke (1)		Ferguson (1/2) Stuart (1/2) Sato(1/2)	
17:00	Tupper (1/2)	Watson (1/2)			
17:30	DINNER	DINNER	DINNER	DINNER	

Note: (1) is used for 50 min talks and (1/2) for 25 min talks.

3. Titles and abstracts

Viviane Baladi, Ecole Normale Superieure

Title: On the Whitney-Holder differentiability of the SRB measure in the quadratic family

Abstract: For a smooth one-parameter family of smooth hyperbolic discrete-time dynamics (i.e. Anosov systems, which are structurally stable), the SRB measure depends differentiably on the parameter, say t, and the derivative is given by an explicit "linear response" formula (Ruelle, 1997). When structural stability does not hold, linear response may break down. This was first observed for piecewise expanding interval maps, where linear response holds for tangential families, but where the modulus of continuity can be $t \log t$ for transversal families (Baladi-Smania, 2008). The case of smooth unimodal maps is much more delicate. Ruelle (Misiurewicz case) and Baladi-Smania (slow recurrence case) recently obtained linear response for fully tangential families (remaining in a topological class). We now study the transversal case (e.g. the quadratic family), where we obtain Hölder upper and lower bounds (in the sense of Whitney, along suitable classes of parameters). (Joint work with M. Benedicks and D. Schnellmann).

Sanjeeva Balasuriya, Connecticut College

Title: Flow barriers and flux in non-autonomous flows

Abstract: When a non-autonomous flow exhibits non-periodic time-dependence, the well-known methods of characterising fluid transport via Poincaré maps and turnstile lobe dynamics fail. Nevertheless, "flow barriers" can be thought of as time-varying stable and unstable manifold structures, whose mutual intersection may occur in various ways in each time-slice. These manifolds are not genuine flow barriers since fluid transport may occur across them in various ways depending on their locations, but can still be thought of as important flow regulators, or boundaries of Lagrangian Coherent Structures. Explicit methods of both locating such manifolds, and quantifying flux across them, are examined in the time-aperiodically perturbed instance. A time-dependent flux definition which has an exact relationship to a Melnikov function is established, and the results are illustrated via an example: a time-aperiodic version of a double-gyre.

Oscar Bandtlow, Queen Mary University of London

Title: Lagrange-Chebyshev approximation of transfer operators

Abstract: The talk will be concerned with the problem of how to approximate spectral data of transfer operators arising from open dynamical systems. I will discuss an easily implementable method that adapts to the smoothness of the underlying system. More precisely, I will show that its speed of convergence is exponential if the system is analytic and algebraic if the system is smooth.

Erik Bollt, Clarkson University

Title: Basis Markov Systems, Estimated Transfer Operators of Open Systems, and Absorbing Markov Chains

Abstarct: A dynamical system that is open relative to a given partition element, or (window set of interest) produces a Markov chain that is characterized as having an absorbing set, or escape hole when approximated by a Galerkin-Ulams method. On the other hand, a Markov partition of a dynamical system can produce a presentation of the action of the dynamical system that is as a finite graph. We have previously [1] taken this characteristic to define Basis Markov to extend the notion of Markov partitions to stochastically perturbed dynamical systems. For a chosen class of noise profiles, the FrobeniusPerron operator associated to the noisy system is exactly represented by a stochastic transition matrix of finite size. Here we will reprise this discussion to connect notions of approximating a dynamical system by a Markov dynamical system and using its corresponding Markov or Basis Markov partition. Furthermore, the connection allows discussion of leaking measure and escape rates to discussion of quality of the Markov Basis estimator. Connections will be drawn to compact operators and finite estimates. Connections will also be drawn between leaking measure, escape and the description of holes and absorbing sets from Markov chain theory.

[1] Erik Bollt, Pawel Gora, Andrzej Ostruszka, and Karol Zyczkowski, *Basis Markov Partitions and Transition Matrices for Stochastic Systems*. SIAM J. APPLIED DYNAMICAL SYSTEMS, Vol. 7, No. 2, pp. 341360, 2008.

Henk Bruin, University of Surrey

Title: Renormalization and Thermodynamic Formalism in Subshifts

Abstract: A well-known fact, going back to work of Hofbauer in the 1970s, is that the Manneville-Pomeau map ($f(x) = x/(1-x)$ on the interval $[0, 1/2]$ and $f(x) = 2x - 1$ on $(\frac{1}{2}, 1]$) has a phase-transition for the geometric potential $V = -t \log |f'|$, at $t = 1$, where continuous equilibrium states are supplanted by an atomic measure at $x = 0$. A discovery, probably due to Artur Lopes, is that V is the fixed point of a natural renormalisation process on the level of potentials.This leads to the question whether there is some underlying structure explaining the connection between renormalisation and phase transitions.

In order to explore this, we investigate a similar renormalisation operator for prime examples of renormalisable dynamics: the Thue-Morse substitution in symbolic dynamics and the Feigenbaum map in one-dimensional dynamics; these systems are closely related to each other. We identify a variety of renormalisation-invariant potentials, and investigate their thermodynamics properties. (Joint work with Renaud Leplaideur).

Leonid Bunimovich, Georgia Institute of Technology

Title: Finite time dynamics

Abstract: Traditionally we expect that rigorous results could be obtained only for asymptotic in time statements. For instance all basic notions, like Lyapunov exponents, escape rates, etc, deal with the limit when time goes to infinity. Alternatively the size of a hole must tend to zero. It occurred that meaningful finite time results for dynamics and for finite size holes could be obtained as well. Moreover, even in such classical (and presumed dead) area as finite Markov chains some new (finite time) results were obtained.

Michael Dellnitz, University of Paderborn

Title: The Computation of Invariant Sets via Newton's Method

Abstract: In connection with the understanding of complex dynamics it is crucial to have reliable numerical tools at hand, which allow for the robust computation of *invariant sets* or *invariant manifolds*. In this talk we will give an overview about related so-called *set oriented* methods, which have been developed over the last years. In the main part we will particularly focus on the presentation of a novel numerical approach that enables the computation of general invariant sets by Newtons method. By construction, this technique is applicable even to (unstable) invariant sets related to non-stationary or aperiodic behavior. Additionally it will be shown how to utilize these set oriented methods in the context of global (multiobjective) optimization.

Mark Demers, Fairfield University

Title: Dispersing billiards with holes

Abstract: In 1979, Pianigiani and Yorke posed the problem of characterizing escape rates and limiting distributions for a chaotic billiard table with small holes. Despite the intuitive appeal of this problem, the technical difficulties associated with billiards have prevented a complete answer from being given. In this talk, we present a functional analytic framework for dispersing two-dimensional billiards, including tables with corner points, in which we can prove that the transfer operator of the associated billiard map has a spectral gap. We then show how this framework can be applied to billiard tables with a variety of different types of holes. (Joint work with Hongkun Zhang).

Carl Dettmann, University of Bristol

Title: Escape and diffusion through small holes

Abstract: A dynamical system may be "opened" by allowing trajectories to leak out through one or more holes, subsets of phase space. Given a distribution of initial conditions, We can then pose questions about the probability of surviving within

the system, as a function of time, the size and position of the hole(s). Open billiard dynamics can be related to a number of physical experiments and applications involving escape of particles from a cavity. In several geometries the leading coefficient of the survival probability can be determined, including connections with the Riemann Hypothesis and phenomena such as asymmetric transport. A chain of systems linked by their holes can also model deterministic diffusion. Very recent results for escape and diffusion in one-dimensional maps will be discussed, including an expansion for the escape rate beyond linear order in hole size, and an exact additivity formula for diffusion coefficients.

Peyman Eslami, Concordia University

Title: On the acim-stability of piecewise expanding dynamical systems

Abstract: In the context of piecewise expanding maps of an interval it is known that the presence of periodic turning points is an obstacle to acim-stability. I will talk about a stability result for such maps which involves an improved Lasota-Yorke inequality; furthermore, I will discuss a possible connection between stability of a closed dynamical system and escape rates of associated open dynamical systems.

Andrew Ferguson, University of Bristol

Title: The dimension of some sets generated by systems with holes

Abstract Let (X, d) denote a compact metric space and $T : X \to X$ an open and expanding map. For an non-empty open set $U \subset X$ we define the survivor set X_U to be the set of points which under forward iteration do not enter U. In this talk I will discuss the dimension theory of these sets for a family of non-conformal systems. (Joint work with Thomas Jordan and Michal Rams).

Cecilia González Tokman, University of Victoria

Title: A semi-invertible operator Oseledets theorem

Abstract: Semi-invertible multiplicative ergodic theorems establish the existence of an Oseledets splitting– a decomposition into generalized Jordan blocks– for cocycles of non-invertible linear operators (such as transfer operators) over an invertible base. We present a semi-invertible multiplicative ergodic theorem that for the first time can be applied to the study of transfer operators associated to the composition of piecewise expanding maps randomly chosen from a set of cardinality of the continuum. (Joint work with Anthony Quas).

Paweł Góra, Concordia University

Title: Random map model of metastable system

Abstract: A metastable system is a dynamical system with at least two almost invariant subsystems. The system spends long time in one of the subsystems, then jumps to another, stays there for a long time, then jumps to another, etc. The main question is: what portion of the time the system spends in each of the subsystems. We introduce a random map model of this situation, where the jumps from subsystem J_i to subsytem J_j occur with probabilities $p_{i,j}$. The main result, in special case of piecewise expanding systems with two subsystems is that as $p_{i,j}^{(n)}$ converge to zero we have:

$$\frac{\alpha_1}{\alpha_2} = \frac{\mu_2(H_{2,1})}{\mu_1(H_{1,2})} \lim_{n\to\infty} \frac{p_{2,1}^{(n)}}{p_{1,2}^{(n)}} ,$$

where α_i is average time spent in subsystem J_i, $H_{i,j}$ is the "gate" from i-th to j-th subsystem and μ_i are the invariant measures on the subsystems in the limit case of closed gates and zero jump probabilities. The results are both one and higher dimensional. There are generalizations for more then two subsystems. (Joint work with Abraham Boyarsky and Peyman Eslami)

Nicolai Haydn, Southern California

Title: The almost sure invariant principle for uniformly strong mixing measures

Abstract: Invariant measures on shift spaces over finite or infinite alphabets are uniformly strong mixing if they satisfy a weak kind of mixing condition. We show that the information function satisfies the almost sure invariance principle. Previously for such measures the CLT and the weak invariance principle had been proven.

Oliver Junge, TU München

Title: Estimating long term behavior of flows without trajectory integration: An infinitesimal generator approach

Abstract: The long-term distributions of trajectories of a flow are described by invariant densities, i.e. fixed points of an associated transfer operator. In addition, global slowly mixing structures, such as almost-invariant sets, which partition phase space into regions that are almost dynamically disconnected, can also be identified by certain eigenfunctions of this operator. Indeed, these structures are often hard to obtain by brute-force trajectory-based analyses. In a wide variety of applications, transfer operators have proven to be very efficient tools for an analysis of the global behavior of a dynamical system.

The computationally most expensive step in the construction of an approximate transfer operator is the numerical integration of many short term trajectories. In this paper, we propose to directly work with the infinitesimal generator instead of the operator, completely avoiding trajectory integration. We propose two different discretization schemes; a cell based discretization and a spectral collocation approach. Convergence can be shown in certain circumstances. We demonstrate numerically that our approach is much more efficient than the operator approach, sometimes by several orders of magnitude.

Rainer Klages, Queen Mary University of London

Title: Where to place a hole to achieve a maximal diffusion coefficient

Abstract A particle moving deterministically in a chaotic spatially extended environment can exhibit normal diffusion, with its mean square displacement growing proportional to the time. Here we consider the dependence of the diffusion coefficient on the size and the position of dynamical channels ('holes') linking spatial regions. The system properties can be obtained analytically via a Taylor-Green-Kubo formula in terms of a functional recursion relation, leading to a diffusion coefficient varying with the hole positions and non-monotonically on their size. We derive analytic formulas for small holes in terms of periodic orbits covered by the holes. The asymptotic regimes that we observe show deviations from a simple random walk approximation, a phenomenon that should be ubiquitous in dynamical systems and might be observed experimentally. The escape rate of the corresponding open system is also calculated. The resulting parameter dependencies are compared with the ones for the diffusion coefficient and explained in terms of periodic orbits. (Joint work with G.Knight, O.Georgiou and C.Dettmann).

Kevin Lin, University of Arizona

Title: Reliability of Driven Oscillator Networks

Abstract: A network of dynamical systems driven by a fluctuating time-dependent signal is said to be reliable if, upon repeated presentations of the same signal, it gives essentially the same response each time; reliability is of interest in a number of applications where the degree to which a system is reliable constrains its ability to encode and transmit information. In this talk, I will discuss how the question of reliability can be formulated within the framework of random dynamical systems. Then, focusing on a class of pulse-coupled phase oscillator networks, I will present a general result that explains why such networks are typically reliable in the absence of feedback loops, and discuss a specific example where the presence of feedback leads to unreliable response.

Carlangelo Liverani, University of Rome Tor Vergata

Title: Partially hyperbolic systems close to trivial extensions

Abstract: we study the statistical properties of small perturbation of smooth expanding maps of the circle times identity. (Joint work with D.Dolgopyat, J. De Simoi)

Simon Lloyd, University of Sao Paulo

Title: Slowly decaying modes for skew-product systems

Abstract: The transfer operator describes the action of a dynamical system on densities, and by analysing its spectrum we determine the relaxation rates of perturbations to an invariant density. Perturbations with slow relaxation rates have been used to identify partitions of the phase space into regions with weak interaction. Using a 'semi-invertible' version of Oseledets' Multiplicative Ergodic Theorem allows transfer operator techniques to be applied in the random/skew-product setting. To illustrate, I will look at the composition of piecewise-expanding interval maps according to a subshift.

James Meiss, Colorado Boulder

Title: Transport in Transitory Systems: Mixing in Droplets

Abstract: A dynamical systems is transitory if it is nonautonomous only on a compact interval of time. Such dynamics is appropriate to the study of, for example, fluid mixing in open pipe flows. In this talk I will discuss a simple model for the flow within a droplet in a sinuous pipe of finite length. The incompressible vector field is also globally Liouville, and thus has a Lagrangian-like generating form. We use this form to reduce the computation of volumes to integration along the orbits of curves of intersection of past and future (partially) invariant surfaces. This gives an efficient method for numerical computation of lobe volumes. A goal is to build the most efficient laminar mixer by tuning the pipe shape.

Ian Melbourne, University of Surrey

Title: Convergence and asymptotics of moments for billiards and Lorentz gases

Abstract: I will describe some results on convergence of moments for a range of maps and flows, based on work with Andrew Torok. For Axiom A diffeomorphisms and flows, the central limit theorem holds for Holder observables and we show that all moments converge to the corresponding Gaussian moment. This result also holds for nonuniformly hyperbolic systems with exponentially decaying return tails, eg the finite horizon Lorentz gas). When the return tails decay more slowly, as is the case for the infinite horizon Lorentz gas, we obtain sharp results for which moments converge (agreeing with the Physics literature) and asymptotics for the moments that do not converge (correcting results in the Physics literature).

Rua Murray, University of Canterbury

Title: Numerical approximation of conditionally invariant measures

Abstract: It is well known that open dynamical systems can admit an uncountable number of (absolutely continuous) conditionally invariant measures for each prescribed escape rate in (0,1). We propose (and illustrate) a convex optimisation based selection scheme (essentially maximum entropy) for gaining numerical access to some of these measures. The work is similar in spirit to methods reported in Banff in 2006 for calculating absolutely continuous invariant measures of non-singular dynamical systems, but contains some interesting new twists, including: (i) the natural escape rate is not known in advance, which can destroy convex structure in the problem, leading to additional technical challenges compared to the invariant measure case; (ii) exploitation of convex duality to solve each approximation step induces important (but dynamically relevant and not at first apparant) localisation of support; (iii) significant potential for non-rigorous application to the approximation of other dynamically interesting objects (or example, invariant manifolds). (Joint work with Chris Bose).

Matt Nicol, University of Huston

Title: Erdös Rényi limit laws for dynamical systems

Abstract: We establish Erdös-Rényi limit laws for Lipschitz observations on a class of non-uniformly expanding dynamical systems, including logistic-like maps. These limit laws give the maximal average of a time series over a time window of logarithmic length. We also give results on maximal averages of a time series arising from Hölder observations on intermittent-type maps over a time window of polynomial length. We consider the rate of convergence in the limit law for subshifts of finite type and establish a one-sided rate bound for Gibbs-Markov maps. (Joint work with Manfred Denker).

William Ott, University of Houston

Title: Memory loss for time-dependent dynamical systems

Abstract: We discuss recent results on memory loss for time-dependent dynamical systems. We then discuss this point of view in the context of open systems.

Kathrin Padberg-Gehle, TU Dresden

Title: Finite-time entropy: a probabilistic approach for measuring nonlinear stretching

Abstract: Transport and mixing processes in dynamical systems are often difficult to study analytically and therefore a variety of numerical methods have been developed. Finite-time Lyapunov exponents (FTLEs) or related stretching indicators are frequently used as a means to estimate transport barriers. Alternatively, eigenvectors, singular vectors, or Oseledets vectors of numerical transfer operators find almost-invariant sets, finite-time coherent sets, or time asymptotic coherent sets, respectively, which are minimally dispersed under the dynamics. While these families of approaches (geometric FTLEs and the probabilistic transfer operator) often give compatible results, a formal link is still missing.

We propose a new entropy-based methodology for estimating finite-time expansive behavior along trajectories in autonomous and nonautonomous dynamical systems. We introduce the finite-time entropy (FTE) field as a simple and flexible way to capture nonlinear stretching directly from the entropy growth experienced by a small localized density evolved by the transfer operator. The FTE construction elucidates in a straightforward way the connection between the evolution of probability densities and the local stretching experienced.

We develop an extremely simple and numerically efficient method of constructing an estimate of the FTE field within a set-oriented framework. The FTE field is instantaneously calculable from a numerical transfer operator – a transition matrix of conditional probabilities that describes a discretized version of the dynamical system. Our novel approach is illustrated by several examples. (Joint work with Gary Froyland).

Mark Pollicott, University of Warwick

Title: Asymptotics of geodesic flows

Abstract: Geodesic flows on negatively curved manifolds provide interesting examples of settings where dynamical properties have an interesting geometrical interpretation. This is illustrated by the connection between geodesic excursions into cusps and dynamical theory of extremes. We will survey recent results and discuss new problems.

Shane Ross, Virginia Tech

Title: Geometric and probabilistic descriptions of chaotic phase space transport

Abstract: Several geometric and probabilistic methods for studying chaotic phase space transport have been developed and fruitfully applied to diverse areas from orbital mechanics to fluid mechanics and beyond. For example, concepts such as invariant sets and their stable and unstable manifolds are commonly used to characterize the behavior of autonomous and periodic systems, but these concepts may be extended to analyze realistic dynamical systems which are aperiodic and/or data-based. We will discuss this development, and the connection with concepts such as symbolic dynamics, chaos, coherent sets, and optimal control. We will highlight some recent applications to areas such as celestial mechanics, musculoskeletal biomechanics, ship capsize prediction, and atmospheric microbe transport.

Naratip Santitissadeekorn, Clarkson University

Transfer operator approach for finite time coherent sets identification

Abstract: For many application areas, it is becoming important to under transport mechanisms underlying time-dependent dynamical systems. In particular, it is interesting to identify coherent sets, which are minimally dispersive as moving with the flow and typically present important physical structures such as long-lived vortices in oceans or atmosphere. This presentation will concentrate on a transfer operator approach for a finite-time coherent set identification. Few illustrative examples of its application in a geophysical model and atmospheric flow will be presented.

Yuzuru Sato, Hokkaido University, Japan

Title: Noise-induced phenomena and their applications

Abstract: Problems of nonlinear phenomena in noised dynamical systems is investigated based on numerically observed noise-induced phenomena in logistic map, Belousov-Zhabotinsky map (BZ map) and modi ed Lasota-Mackey map. We found that (i) both noise-induced chaos and noise-induced order robustly coexist with the same control parameters of the original dynamical systems, and that (ii) asymptotical periodicity of density varies according to noise amplitude. Application to time-series analysis of rotating fluid is also exhibited.

Robyn Stuart, UNSW

Title: Almost-invariance in Open Dynamical Systems

Abstract: We explore the concept of metastability or almost-invariance in open dynamical systems. In particular, we look at the conditional probability of trajectories remaining in some set, and consider the maximum conditional probability over certain subsets of the state space. For closed systems, this maximum is estimated by the second largest eigenvalue of a discretised Perron-Frobenius operator, and is strongly related to the mixing time of the discretised process. We demonstrate an extension of these bounds to open systems.

Dalia Terhesiu, University of Rome, Tor Vergata

Title: A new technique for sharp mixing rates associated with infinite measure preserving systems

Abstract: We will present a new technique for operator renewal sequences associated with dynamical systems preserving an infinite measure. This technique allows us to improve some results on mixing rates (for infinite measure preserving systems) obtained in previous joint work with Ian Melbourne.

Paul Tupper, Simon Fraser University

Title: Using the Lorentz gas to resolve a paradox of state-dependent diffusion

Abstract: Consider a particle diffusing in a confined volume which is divided into two equal regions. In one region the diffusion coefficient is twice the value in the other region. Will the particle spend equal amounts of time in the two regions in the long

term? Statistical mechanics would suggest yes, since the number of accessible states in each region is the same. However, another line of reasoning suggests that the particle should spend less time in the region with faster diffusion since it will exit it more quickly. I will demonstrate with the Lorentz gas and related models that both answers are consistent with the information given. (Joint work with Xin Yang).

Tom Watson, UNSW

Title: Computing Oseledets subspaces: A short overview

Abstract: Oseledets subspaces are dynamic structures that contain information regarding the direction of local expansion and contraction about arbitrary points of a dynamic system. They are closely related to the Lyapunov exponents and stable/unstable manifolds and as such are of great interest. Up until recently they have been too difficult to approximate by numerical means. I present an overview of three relatively new methods for approximating Oseledets subspaces by Froyland/Lloyd/Quas, Ginelli et. al. and Wolfe/Samelson and discuss their relative merits with a few numerical examples.

James Yorke, University of Maryland

Title: Partial Control

Abstract: We "partially" control systems despite disturbances ξ that we call "disturbance" so that the trajectories stay in some specified region Q. The disturbances can be thought of as either noise or as purposeful, hostile efforts of an opponent. Our goal is to keep trajectories inside some region despite the disturbances. Using the partial control of chaos method, it is possible even in the presence of hostile disturbances to keep trajectories of a dynamical system inside a phase space region from which almost all typically escape.

The main point of this method is that this goal can be achieved even if the applied control is smaller than the applied disturbance. For usual control methods, control must dominate noise. A fundamental step towards partial control is to compute a set we call the "safe set". Until now we have been able to find it only in certain very special situations. We provide a general algorithm to compute the safe sets. It is fast in one- and two-dimensional dynamical systems, though the algorithm is independent of dimension. The algorithm is able to compute the safe sets with the knowledge of the relevant region in phase space, the disturbance value and the desired maximum control. It is called the Sculpting Algorithm, analogous to removing material while sculpting a statue. We sculpt the safe sets. We apply the algorithm to two paradigmatic nonlinear dynamical systems, namely, the Hénon map and the Duffing oscillator. (Joint work with Juan Sambrano, Samuel Zambrano, and Miguel A. F. Sanjuan).

Roland Zweimüeller, University of Vienna

Title: Systems with holes as a tool for recurrent infinite measure systems

Abstract: Recurrent ergodic transformations T with an infinite invariant measure can often be studied most efficiently by identifying a "good" reference set of finite measure, and by reformulating questions about T in terms of the corresponding first-return map S. (Here, "good" often simply means that the new map S is well-behaved, e.g. Gibbs-Markov.) According to Kac' formula, one then has to deal with non-integrable observables and their behaviour under the nice map S. Even for fairly simple maps T there are some very basic probabilistic questions (distributional limit theorems in the case of infinite measures) which have only been answered very recently. I will try to explain a sample question, and then outline why it leads to questions about dynamical systems with holes (conditional limit theorems), and how those can be answered, to finally entail the desired result for T.