

# Banach Space Theory Workshop

## March 4-9, 2012

### MEALS

\*Breakfast (Buffet): 7:00–9:30 am, Sally Borden Building, Monday–Friday

\*Lunch (Buffet): 11:30 am–1:30 pm, Sally Borden Building, Monday–Friday

\*Dinner (Buffet): 5:30–7:30 pm, Sally Borden Building, Sunday–Thursday

Coffee Breaks: As per daily schedule, in the foyer of the TransCanada Pipeline Pavilion (TCPL)

**\*Please remember to scan your meal card at the host/hostess station in the dining room for each meal.**

### MEETING ROOMS

All lectures will be held in the new lecture theater in the TransCanada Pipelines Pavilion (TCPL). LCD projector, overhead projectors and blackboards are available for presentations.

### SCHEDULE

#### Sunday

- 16:00** Check-in begins (Front Desk - Professional Development Centre - open 24 hours)  
Lecture rooms available after 16:00
- 17:30–19:30** Buffet Dinner, Sally Borden Building
- 20:00** Informal gathering in 2nd floor lounge, Corbett Hall  
Beverages and a small assortment of snacks are available on a cash honor system.

#### Monday

- 7:00–8:45** Breakfast
- 8:45–9:00** Introduction and Welcome by BIRS Station Manager, TCPL
- 9:00–9:50** Richard Haydon, *The  $BD$  construction and spaces with very few operators*
- 10:00–10:30** Daniel Freeman, *Embedding into  $BD$  spaces and spaces with very few operators*
- 10:30–10:50** Coffee
- 10:50–11:40** Christian Rosendal, *Isometry groups and maximal symmetry*
- 11:40–13:00** Lunch
- 13:00–14:00** Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall
- 14:00** Group Photo; meet in the foyer of TCPL (it will be outside so bring your jackets)
- 14:15–14:45** Matthew Tarbard, *Banach spaces with few operators*
- 14:45–15:15** Coffee
- 15:15–15:45** Despoina Zisimopoulou, *Bourgain-Delbaen  $\mathcal{L}^\infty$ -sums of Banach spaces*
- 16:00–16:30** Anna Pelczar, *On strictly singular non-compact operators in asymptotic  $\ell_p$  spaces*
- 16:40–17:10** Antonis Manoussakis, *A note on strictly singular non-compact operators*
- 17:30–19:30** Dinner

## Tuesday

- 7:00–9:00 Breakfast  
9:00–9:50 Spiros Argyros, *Using Tsirelson space as the frame for HI constructions*  
10:00–10:30 Pavlos Motakis, *Hereditarily  $\alpha$ -universal Banach spaces*  
10:30–10:50 Coffee  
10:50–11:40 Valentin Ferenczi, *Supports and ranges in Banach spaces*  
11:40–13:30 Lunch  
14:00–14:30 Andras Zsak, *Closed ideals of operators on certain Banach spaces*  
14:40–15:10 Denny Leung, *Ideals of operators on  $(\oplus E_n)_{c_0}$  and  $(\oplus E'_n)_{\ell^1}$*   
15:10–15:30 Coffee  
15:30–16:00 Bentuo Zheng, *Norm closed ideals in the algebra of bounded linear operators on Orlicz sequence spaces*  
16:00–16:30 Niels Laustsen, *Closed operator ideals on the Banach space of continuous functions on the first uncountable ordinal*  
17:30–19:30 Dinner

## Wednesday

- 7:00–9:00 Breakfast  
9:00–9:50 Thomas Schlumprecht, *Shift invariant preduals of  $\ell_1(\mathbb{Z})$*   
10:00–10:30 Kevin Beanland, *Uniformly factoring weakly compact operators*  
10:30–10:50 Coffee  
10:50–11:20 Konstantinos Tyros, *Higher order spreading models*  
11:30–12:00 Ioannis Gasparis, *A new isomorphic  $\ell_1$ -predual which is not isomorphic to a complemented subspace of a  $C(K)$  space*  
12:00–13:30 Lunch  
Free Afternoon  
17:30–19:30 Dinner

## Thursday

- 9:00–9:50 Gideon Schechtman, *A quantitative version of the commutator theorem for zero trace matrices*  
10:00–10:30 Detelin Dosev, *On a class of operators on  $C(K)$*   
10:30–10:50 Coffee  
11:00–11:30 Denka Kutzarova, *Asymptotic geometry of Banach spaces and uniform quotient maps*  
11:30–13:30 Lunch  
13:40–14:30 Nicole Tomczak-Jaegermann, *Geometry determined by random matrices associated to high-dimensional convex bodies*  
14:30–14:50 Coffee  
14:50–15:20 Alejandro Chavez-Dominguez, *"Tensor products" between metric spaces and Banach spaces*  
15:30–16:00 Florent Baudier, *Embedding  $L_p(\mu)$ -spaces for  $0 < p < +\infty$  and their snowflaked versions*  
16:10–16:40 Alexey Popov, *Almost invariant subspaces of operators in Banach spaces*  
16:50–17:20 Haskell Rosenthal, *TBA*  
17:30–19:30 Dinner

## Friday

- 9:00–9:50 Jesus Castillo,  *$\mathcal{L}_\infty$ -envelopes of Banach spaces*  
10:00–11:30 Coffee and informal discussions **Check out by 12 noon**  
11:30–13:30 Lunch

\*\* 5-day workshop participants are welcome to use BIRS facilities (BIRS Coffee Lounge, TCPL and Reading Room) until 3 pm on Friday, although participants are still required to checkout of the guest rooms by 12 noon. \*\*

# Banach Space Theory Workshop

## March 5-9, 2012

### ABSTRACTS

(in alphabetic order by speaker surname)

**Spiros Argyros** (National Technical University of Athens)

Title: *Using Tsirelson space as the frame for HI constructions*

Abstract: (joint work with P. Motakis) We shall present a new Hereditarily Indecomposable reflexive space, denoted as  $\mathfrak{X}_{\text{ISP}}$ , built on Tsirelson space. This is possible with the use of saturation under constraints. The most significant property of  $\mathfrak{X}_{\text{ISP}}$  is that it satisfies the hereditary Invariant Subspace Property (ISP). We will explain the definition of the space and we will outline the basic properties of it.

**Florent Baudier** (Texas A&M University)

Title: *Embedding  $L_p(\mu)$ -spaces for  $0 < p < +\infty$  and their snowflaked versions*

Abstract: In this talk we will show that the bi-Lipschitz embedding theory between  $L_p(\mu)$ -spaces for the range  $0 < p < +\infty$  is completely understood. Then we will discuss what kind of embedding can be constructed when a bi-Lipschitz embedding is ruled out in the previous classification. In particular we will study the embeddability of snowflaked versions of  $L_p(\mu)$ -spaces. And if time permits we will present embeddings results regarding snowflakings of general metric spaces.

This is a joint work with Fernando Albiac.

**Kevin Beanland** (Virginia Commonwealth University)

Title: *Uniformly factoring weakly compact operators*

Abstract: In this talk we discuss uniform versions of the Davis-Figiel-Johnson-Pelczynski weakly compact factorization theorem. In particular we show that if  $X$  and  $Y$  are separable Banach spaces and  $Y$  has a shrinking basis, a Borel (in the strong operator topology) collection of weakly compact operators factor through the same reflexive space. This result uses descriptive set theoretic results of Argyros and Dodos and has several consequences for particular choices of spaces  $X$  and  $Y$ . This work is joint with Dan Freeman.

**Jesus Castillo** (Universidad de Extremadura)

Title:  *$\mathcal{L}_\infty$ -envelopes of Banach spaces*

Abstract: Let  $\mathcal{A}$  be a class of Banach spaces. An  $\mathcal{A}$ -envelope of a Banach space  $X$  is a couple  $(A(X), \delta)$  formed by a Banach space  $A(X) \in \mathcal{A}$  and an isometric embedding  $\delta : X \rightarrow A(X)$  with the property that for every  $A \in \mathcal{A}$  every operator  $\tau : X \rightarrow A$  can be extended through  $\delta$  to an operator  $T : A(X) \rightarrow A$  with  $\|T\| = \|\tau\|$ .

In this talk we will show how to construct envelopes for the following classes of  $\mathcal{L}_\infty$ -spaces:  $C(K)$ -spaces, Lindenstrauss spaces,  $\lambda$ -separably injective spaces, universally  $\lambda$ -separably injective spaces,  $\lambda$ -Lindenstrauss-Pelczyński spaces,  $\mathcal{L}_{\infty, \lambda}$ -spaces. To this end we will develop a rather flexible device to construct such envelopes which unifies several constructions presented or outlined in the literature ranging from the Bourgain-Pisier spaces to Gurarii spaces.

## References

- [1] A. Avilés, F. Cabello, J.M.F. Castillo, M. González, Y. Moreno, *On Banach spaces of universal disposition*, J. Functional Anal. 261 (2011) 2347-2361.

- [2] J. Bourgain and G. Pisier, *A construction of  $\mathcal{L}_\infty$ -spaces and related Banach spaces*, Bol. Soc. Bras. Mat. 14 (1983) 109–123.
- [3] J. M. F. Castillo and Jesús Suárez, *Extending operators into Lindenstrauss spaces*, Israel J. Math. 169 (2009) 1-27.
- [4] V.I. Gurariĭ, *Spaces of universal placement, isotropic spaces and a problem of Mazur on rotations of Banach spaces (Russian)*. Sibirsk. Mat. Ž. 7 (1966), 1002–1013.
- [5] W. Kubis, *Fraïssé sequences - a category-theoretic approach to universal homogeneous structures*, arXiv:0711.1683v1, 2007.
- [6] G. Pisier, *Counterexample to a conjecture of Grothendieck*, Acta. Math. 151 (1983) 181-208.
- [7] M. Zippin, *Extension of bounded linear operators*, in Handbook of the Geometry of Banach spaces vol 2 (W.B. Johnson and J. Lindenstrauss eds.), Elsevier, 2003; pp. 1703-1741.

**Alejandro Chavez-Dominguez** (Texas A&M University)

Title: *"Tensor products" between metric spaces and Banach spaces*

Abstract: The relationships between tensor products and ideals of operators have long been exploited in Banach space theory. One noteworthy example is identifying the dual norms of operator ideals of linear mappings: in several cases, this dual norm can be naturally recognized as a norm on a tensor product. When these operator ideals of linear mappings between Banach spaces are generalized to Lipschitz mappings between a metric space and a Banach space, one would still like to identify the dual spaces but the tensor product arguments do not immediately make sense. In this talk we introduce a concept that plays the role of a tensor product between a metric space and a Banach space (inspired by work of Arens and Eells in the 50's), which allows us to transfer the duality arguments to this nonlinear situation.

**Detelin Dosev** (Weizmann Institute)

Title: *On a class of operators on  $C(K)$*

Abstract: In this talk we will discuss some recent development on the problem of classifying the commutators on  $C(K)$ . We will make an overview of some known results and will discuss a class of operators on  $C(K)$  which satisfy conditions that are sufficient for an operator on  $C(K)$  to be a commutator. The results obtained rely on a theorem of Kalton about decomposition of Borel measures on an infinite compact metric space.

**Valentin Ferenczi** (Universidade de So Paulo)

Title: *Supports and ranges in Banach spaces*

Abstract: I will focus on some aspects of Banach's hyperplane problem and on Gowers' program of classification of Banach spaces "up to subspaces", based on dichotomies. Gowers' two dichotomies, as well as some other dichotomies obtained in 2007 in collaboration with Christian Rosendal, will be mentioned. Finally, a property of some Gowers and Maurey space, proved recently in collaboration with Thomas Schlumprecht, will be presented, as well as the relation of this result to Gowers' program.

**Daniel Freeman** (University of Texas at Austin)

Title: *Embedding into  $BD$  spaces and spaces with very few operators*

Abstract: We will discuss how the generalised Bourgain-Delbaen construction can be used to show that every Banach space with separable dual may be embedded into a Banach space whose dual is isomorphic to  $\ell_1$ . We will then discuss how to use this embedding theorem to prove that every separable reflexive Banach space embeds into a Banach space with the scalar-plus-compact property. The talk will cover joint work with S.A. Argyros, R. Haydon, E. Odell, Th. Raikoftsalis, Th. Schlumprecht, and D. Zisimopoulou.

**Ioannis Gasparis** (Aristotle University of Thessaloniki)

Title: *A new isomorphic  $\ell_1$ -predual which is not isomorphic to a complemented subspace of a  $C(K)$  space*

Abstract: We shall present an example of a subspace  $X$  of  $C(\omega^\omega)$  with  $X^*$  isomorphic to  $\ell_1$  and such that neither  $X$  is isomorphic to  $c_0$ , nor  $C(\omega^\omega)$  is isomorphic to a subspace of  $X$ . It follows that  $X$  is not isomorphic to a complemented subspace of a  $C(K)$  space.

Speaker: **Richard Haydon** (University of Oxford)

Title: *The BD construction and spaces with very few operators*

Abstract: I shall present the Bourgain-Delbaen construction in a fairly general form and give an introduction to some of its recent recent applications. It is intended that this part of the talk will provide a common foundation for other speakers as the workshop progresses. I shall also give a detailed account of some recent joint work with Argyros and Raikoftsalis in which a Banach space is constructed that contains  $\ell_1$  and has the scalar-plus-compact property.

**Denka Kutzarova** (University of Illinois at Urbana-Champaign)

Title: *Asymptotic geometry of Banach spaces and uniform quotient maps*

Abstract: (joint with S.J. Dilworth, G. Lancien and N.L. Randrianarivony) Recently, Lima and Randrianarivony pointed out the role of the property (beta) of Rolewicz in nonlinear quotient problems, and answered a ten-year-old question of Bates, Johnson, Lindenstrauss, Preiss and Schechtman. We prove that the modulus of asymptotic uniform smoothness of the range space of a uniform quotient map can be compared with the modulus of (beta) of the domain space. We also provide conditions under which this comparison can be improved.

**Niels Laustsen** (Lancaster University)

Title: *Closed operator ideals on the Banach space of continuous functions on the first uncountable ordinal*

Abstract: Let  $\omega_1$  be the first uncountable ordinal. By a result of Rudin, bounded operators on the Banach space  $C[0, \omega_1]$  have a natural representation as  $[0, \omega_1] \times [0, \omega_1]$ -matrices. Loy and Willis observed that the set of operators whose final column is continuous when viewed as a scalar-valued function on  $[0, \omega_1]$  defines a maximal ideal of codimension one in the Banach algebra  $\mathcal{B}(C[0, \omega_1])$  of bounded operators on  $C[0, \omega_1]$ . Our main result gives a coordinate-free characterization of this ideal, and implies that  $\mathcal{B}(C[0, \omega_1])$  contains no other maximal ideals. We also obtain a list of equivalent conditions describing the strictly smaller ideal of operators with separable range, and finally, if time permits, I shall explain some further results regarding the structure of the lattice of all closed ideals of  $\mathcal{B}(C[0, \omega_1])$ .

This is joint work with Tomasz Kania.

**Denny Leung** (National University of Singapore)

Title: *Ideals of operators on  $(\oplus E_n)_{c_0}$  and  $(\oplus E'_n)_{\ell^1}$*

Abstract: The unique maximal ideals of operators on  $(\oplus \ell^2(n))_{c_0}$  and  $(\oplus \ell^1(n))_{c_0}$  have been identified in [1, 3]. Building upon the proofs of these results, I will present some additional observations that allow us to identify the unique maximal ideals of operators on the dual spaces  $(\oplus \ell^2(n))_{\ell^1}$  and  $(\oplus \ell^\infty(n))_{\ell^1}$ . For the space  $(\oplus \ell^2(n))_{\ell^1}$ , the result was obtained in [2].

## References

- [1] N.J. LAUSTSEN, R.J. LOY AND C.J. READ, The lattice of ideals in the Banach algebra of operators on certain Banach spaces, *J. Funct. Anal.* **214**(2004), 106-131.
- [2] N.J. LAUSTSEN, TH. SCHLUMPRECHT AND A. ZSÁK, The lattice of closed ideals in the Banach algebra of operators on a certain dual Banach space, *J. Operator Th.* **56:2**(2006), 391-402.
- [3] N.J. LAUSTSEN, E. ODELL, TH. SCHLUMPRECHT AND A. ZSÁK, Dichotomy theorems for random matrices and closed ideals of operators on  $(\oplus_{n=1}^\infty \ell_1^n)_{c_0}$ , Preprint.

**Antonis Manoussakis** (Technical University of Crete)

Title: *A Note on Strictly Singular non-Compact Operators*

Abstract: We present a method for constructing bounded strictly singular non-compact operators on mixed Tsirelson spaces defined either by the families  $\mathcal{A}_n$  or  $\mathcal{S}_n$  of a certain class, as well as on spaces built on them, including HI spaces.

This is joint work with A. Pelczar-Barwacz.

**Pavlos Motakis** (National Technical University of Athens)

Title: *Hereditarily  $\alpha$ -Universal Banach spaces*

Abstract: (joint work with S. A. Argyros) Let  $(e_n)_n$  be a Schauder basic sequence. We say that  $(e_n)_n$   $\alpha$ -embeds into a Banach space  $Y$ , if the rank of the Bourgain embedability tree of  $(e_n)_n$  into  $Y$  is greater than or equal to  $\alpha$ .

For every Schauder basic sequence  $(e_n)_n$  and  $\alpha < \omega_1$  there exists a reflexive HI space  $\mathfrak{X}_\alpha$  such that  $(e_n)_n$   $\alpha$ -embeds into every subspace of  $\mathfrak{X}_\alpha$ .

As consequence we obtain the following.

- (i) For any  $\alpha < \omega_1$ , there exists a reflexive HI space  $\mathfrak{X}_\alpha$  that is Hereditarily  $\alpha$ -Universal, i.e. any basic sequence  $(e_n)_n$   $\alpha$ -embeds into any subspace of  $\mathfrak{X}_\alpha$ .
- (ii) For any  $\alpha < \omega_1$  there exists a reflexive HI space  $\mathfrak{X}_\alpha$ , that is  $\alpha$ -minimal, in the sense of C. Rosendal.

**Anna Pelczar-Barwacz** (Jagiellonian University)

Title: *On strictly singular non-compact operators in asymptotic  $\ell_p$  spaces*

Abstract: We present a method of construction of bounded strictly singular non-compact operator on subspaces in asymptotic  $\ell_p$  spaces, relying on higher order asymptotic behavior of basic sequences. Applications concern convexified mixed Tsirelson spaces and related asymptotic  $\ell_p$  HI spaces. By use of the presented technic we prove also sequential minimality of modified mixed Tsirelson spaces, extending the list of examples studied within the framework of classification program initiated by W.T.Gowers and continued by V.Ferenczi and C.Rosendal. The talk is based on the recent work and joint work with Denka Kutzarova and Antonis Manoussakis.

**Alexey Popov** (University of Waterloo)

Title: *Almost invariant subspaces of operators in Banach spaces.*

Abstract: A subspace  $Y$  of a Banach space  $X$  is almost invariant under an operator  $T$  in  $L(X)$  if  $TY$  is a subspace of  $Y + F$  for some finite-dimensional space  $F$ . We call  $Y$  a half-space if it is of infinite dimension and infinite codimension. Every subspace that is not a half-space is immediately almost invariant under all operators. The question of whether every operator has an almost invariant half-space has been raised in a paper by Androulakis, Popov, Tcaciuc and Troitsky in 2009.

In this work, we obtain almost invariant half-spaces for all polynomially compact operators on reflexive Banach spaces and for some classes of triangularizable operators. We also show that if a norm closed algebra  $A$  of operators on a Banach space has a (complemented) common almost invariant half-space then  $A$  has a common invariant half-space. This is no longer true if we replace “invariant” with “reducing”. Finally, we study operators  $T$  on Hilbert spaces whose almost invariant subspaces include all spaces of the form  $PH$ , where  $P$  is a projection from a fixed masa.

This is a joint work with Laurent Marcoux and Heydar Radjavi.

**Christian Rosendal** (University of Illinois at Chicago)

Title: *Isometry groups and maximal symmetry*

Abstract: I shall present some recent work on the structure of groups acting by isometries on separable reflexive Banach spaces. In particular, this will treat the so called Fredholm group of small perturbations

of scalar multiples of the identity and other small subgroups of the isometry group. This is joint work with V. Ferenczi.

**Gideon Schechtman** (Weizmann Institute)

Title: *A quantitative version of the commutator theorem for zero trace matrices*

Abstract: As is well known, a complex  $m \times m$  matrix  $A$  is a commutator (i.e., there are matrices  $B$  and  $C$  of the same dimensions as  $A$  such that  $A = [B, C] = BC - CB$ ) if and only if  $A$  has zero trace. In such a situation clearly  $\|A\| \leq 2\|B\|\|C\|$  where  $\|D\|$  denotes the norm of  $D$  as an operator from  $\ell_2^m$  to itself.

Does the converse hold? That is, if  $A$  has zero trace are there  $m \times m$  matrices  $B$  and  $C$  such that  $A = [B, C]$  and  $\|B\|\|C\| \leq K\|A\|$  for some absolute constant  $K$ ? If not, what is the behavior of the best  $K$  as a function of  $m$ ?

I'll present some new insight, due to Johnson, Ozawa and myself concerning this question.

**Thomas Schlumprecht** (Texas A&M University)

Title: *Shift invariant preduals of  $\ell_1(\mathbb{Z})$*

Abstract: The Banach space  $\ell_1(\mathbb{Z})$  admits many non-isomorphic preduals, for example,  $C(K)$  for any compact countable space  $K$ , along with many more exotic Banach spaces. In this paper, we impose an extra condition: the predual must make the bilateral shift on  $\ell_1(\mathbb{Z})$  weak\*-continuous. This is equivalent to making the natural convolution multiplication on  $\ell_1(\mathbb{Z})$  separately weak\*-continuous and so turning  $\ell_1(\mathbb{Z})$  into a dual Banach algebra. We call such preduals *shift-invariant*. It is known that the only shift-invariant predual arising from the standard duality between  $C_0(K)$  (for countable locally compact  $K$ ) and  $\ell_1(\mathbb{Z})$  is  $c_0(\mathbb{Z})$ . We provide an explicit construction of an uncountable family of distinct preduals which do make the bilateral shift weak\*-continuous. Using Szlenk index arguments, we show that merely as Banach spaces, these are all isomorphic to  $c_0$ . We then build some theory to study such preduals, showing that they arise from certain semigroup compactifications of  $\mathbb{Z}$ . This allows us to produce a large number of other examples, including non-isometric preduals, and preduals which are not Banach space isomorphic to  $c_0$ .

The talk represents joint work with Matthews Daws, Richard Haydon and Stuart White.

**Matthew Tarbard** (University of Oxford)

Title: *Banach Spaces with Few Operators*

Abstract: In 1993 Gowers and Maurey exhibited a Banach space with few operators, i.e. every operator has the form  $\lambda I + S$  where  $S$  is strictly singular. More recently of course, Argyros and Haydon have improved upon this result, constructing a Banach space with very few operators, i.e. every operator has the form  $\lambda I + K$  where  $K$  is compact. Their space has a number of interesting properties, in particular it is a hereditarily indecomposable  $\ell_1$  predual.

In this talk, we address the following question; must a HI  $\ell_1$  predual with few operators necessarily have very few operators? We will introduce new examples of Banach spaces which provide a negative solution to this question. Our spaces also exhibit a number of other remarkable features. In particular, we will see that we can construct HI  $\ell_1$  preduals with Calkin algebra having any finite dimension. Motivated by work of Daws, Haydon, Schlumprecht and White on shift invariant  $\ell_1$  preduals, we also obtain a space with  $\ell_1$  Calkin algebra.

**Nicole Tomczak-Jaegermann** (University of Alberta)

Title: *Geometry determined by random matrices associated to high-dimensional convex bodies*

Abstract: The talk is based on a series of joint papers by Radoslaw Adamczak, Olivier Guedon, Rafal Latała, Alexander Litvak, Alain Pajor, and the speaker.

We study random matrices with independent columns or independent rows, which are log-concave isotropic random vectors in  $\mathbb{R}^N$ . Fundamental examples of such random vectors are given by vectors uniformly distributed on isotropic convex bodies of volume 1 in  $\mathbb{R}^N$ . The behaviour of fundamental spectral invariants of matrices such as extremal singular numbers is proved to be very close to the case of

matrices with independent identically distributed entries; this in turn is analogous to classical results from the random matrix theory.

We will also describe applications to the Restricted Isometry Property, s geometry of random polytopes and a rate of convergence of the approximation of empirical covariance matrices to the covariance matrix.

**Konstantinos Tyros** (University of Toronto)

Title: *Higher order spreading models*

Abstract: The notion of the spreading model of a Banach space has been introduced by A. Brunel and L. Sucheston in 1974. In this talk a generalization of this notion, called higher order spreading model, will be presented. In particular, for every Banach space  $X$  and every countable ordinal  $\xi$ ,  $\xi$ -order spreading models of  $X$  are defined. The new definition is based on the  $\mathcal{F}$ -sequences and the plegma families. The structural behavior of the higher order spreading models is mainly determined by the combinatorial properties of the plegma families. We will mention several points of the corresponding theory like generalizations of classical results concerning the spreading models, examples establishing the new definition and illustrating the boundaries of the theory.

**Bentuo Zheng** (University of Memphis)

Title: *Norm closed ideals in the algebra of bounded linear operators on Orlicz sequence spaces*

Abstract: For each  $1 < p < \infty$ , we consider a class of  $p$ -regular Orlicz sequence spaces  $\ell_M$  that are “close” to  $\ell_p$ , and study the structure of the norm closed ideals in the algebra of bounded linear operators on such spaces. We show that the maximal proper ideal in  $L(\ell_M)$  is the set of all  $\ell_M$  strictly singular operators and the immediate successor of the ideal of compact operators in  $L(\ell_M)$  is the closed ideal generated by the formal identity from  $\ell_M$  into  $\ell_p$ .

**Despoina Zisimopoulou** (National Technical University of Athens)

Title: *Bourgain-Delbaen  $\mathcal{L}^\infty$ -sums of Banach spaces*

Abstract: Motivated by a problem stated by S. A. Argyros and Th. Raikoftsalis, we introduce a new class of Banach spaces. Namely, for a sequence of separable Banach spaces  $(X_n, \|\cdot\|_n)_{n \in \mathbb{N}}$ , we define the Bourgain-Delbaen  $\mathcal{L}^\infty$ -sum of the sequence  $(X_n, \|\cdot\|_n)_{n \in \mathbb{N}}$  which is a Banach space  $Z$  that is constructed under the Bourgain-Delbaen method of construction. In particular, for every  $1 \leq p < \infty$ , taking  $X_n = \ell_p$  for every  $n \in \mathbb{N}$  (or  $X_n = c_0$  for every  $n \in \mathbb{N}$ ) the aforementioned space  $Z_p$  (respectively  $Z_0$ ) is strictly quasi prime and admits  $\ell_p$  (respectively  $c_0$ ) as a complemented subspace. We study the operators acting on  $Z_p$  and we prove that for every  $n \in \mathbb{N}$ , the space  $Z_p^n = \sum_{i=1}^n \oplus Z_p$  admits exactly  $n + 1$ , pairwise not isomorphic, complemented subspaces. The same holds for  $Z_0$ .

**Andras Zsak** (Peterhouse, University of Cambridge)

Title: *Closed ideals of operators on certain Banach spaces*

Abstract: In recent years several people studied the problem of classifying the closed ideals of operators on a Banach space for certain specific examples of spaces. After a quick survey, we shall describe what we know about closed ideals of operators on  $(\bigoplus_{n=1}^{\infty} \ell_1^n)_{c_0}$ . Here one is lead to consider sequences of operators into  $L_1[0, 1]$ . We present a very natural conjecture concerning operators with approximate lattice bounds, and a dichotomy theorem for random matrices. If time permits, we shall conclude with a characterization of operators on the space  $C(\omega^\omega)$  that factor through  $c_0$ .