Combinatorial game Theory
Jan. 9–Jan. 14

MEALS

*Breakfast (Buffet): 7:00–9:30 am, Sally Borden Building, Monday–Friday
*Lunch (Buffet): 11:30 am–1:30 pm, Sally Borden Building, Monday–Friday
*Dinner (Buffet): 5:30–7:30 pm, Sally Borden Building, Sunday–Thursday
Coffee Breaks: As per daily schedule, 2nd floor lounge, Corbett Hall
***Please remember to scan your meal card at the host/hostess station in the dining room for each meal.**

MEETING ROOMS

All lectures will be held in Max Bell 159 (Max Bell Building accessible by walkway on 2nd floor of Corbett Hall). LCD projector, overhead projectors and blackboards are available for presentations. Note that the meeting space designated for BIRS is the lower level of Max Bell, Rooms 155–159. Please respect that all other space has been contracted to other Banff Centre guests, including any Food and Beverage in those areas.

SCHEDULE

Sunday

16:00	Check-in begins (Front Desk - Professional Development Centre - open 24 hours)
	Lecture rooms available after 16:00 (if desired)
17:30–19:30	Buffet Dinner, Sally Borden Building
20:00	Informal gathering in 2nd floor lounge, Corbett Hall (if desired)
	Beverages and a small assortment of snacks are available on a cash honor system.

Monday

7:00–8:45	Breakfast
8:45–9:00	Introduction and Welcome by BIRS Station Manager, Max Bell 159
9:00-9:10	Overview of the week's activities
9:10-10:00	Olivier Teytaud, *Monte Carlo Tree Search Methods.*
10:00-10:30	Coffee
10:30-11:00	Angela Siegel, *Distributive and other lattices in CGT.*
11:00-11:50	Elwyn Berlekamp, *Report on the latest Coupon Go tournament*
11:50–13:00	Lunch
13:00–14:00	Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall
14:00	Group Photo; meet on the front steps of Corbett Hall
14:30-15:00	Coffee Break, 2nd floor lounge, Corbett Hall
15:00-15:30	Takenobu Takizawa, *A Combination of Basic Go Positions.*
15:30-17:00	Group Sessions (A) Impartial, (B) Go/Monte Carlo methods
17:30–19:30	Dinner
20:00–	NoGo Tournament

Tuesday

7:00–9:00	Breakfast
9:00-10:00	Alan Guo & Mike Weimerskirch, *Lattice point methods in misere games*
10:00-10:30	Kyle Burke,*Neighboring Nim: a PSPACE-complete NimG variant*
10:30-11:00	Coffee Break, 2nd floor lounge, Corbett Hall
11:00-11:30	Tristan Cazenave, *Developments on the Monte-Carlo approximation of temperature*
11:30-12:00	Martin Müller, *Combining Monte-Carlo tree search and combinatorial game theory*
12:00-12:30	Peter Hegarty, *On m-covering families of Beatty Sequences with Irrational Moduli*
12:30–13:30	Lunch
14:00-17:00	(A) Impartial (B) GO/Monte Carlo methods (C) Loopy Games
15:00-15:30	Coffee Break, 2nd floor lounge, Corbett Hall.
17:30–19:30	Dinner
20:00–	(1) Simultaneous Go session and (2) NoGo tournament

Wednesday

7:00–9:00	Breakfast
9:00-9:45	Erik Demaine, *Geometric Puzzles.*
9:45-10:10	Teigo Nakamura, *A Method for Analyzing Complex Go Capturing Races*
10:10-10:30	Carlos Santos, *A Non-trivial Surjective Map onto the Short Conway's Group*
10:30-11:00	Coffee Break, 2nd floor lounge, Corbett Hall
11:00-11:30	Bill Spight, *Influence of Go Stones in the Laboratory*
11:30-12:00	Neil McKay, *Ordinal Sums with base* $*$
12:00-12:30	Thane Plambeck, *Impartial Tic-Tac-Toe*
12:30–13:30	Lunch
	Free Afternoon
17:30–19:30	Dinner
20:00–	NoGo tournament continues

Thursday

7:00–9:00	Breakfast
9:00-9:30	JP Grossman, *Optimizing the search for periodicity in* $.\boldsymbol{6}$
9:30-10:00	Vladimir Oudalov, *A characterization of complete seki*
10:00-10:30	Ryan Hayward, *Recent results on Hex and Rex*
10:30-11:00	Coffee Break, 2nd floor lounge, Corbett Hall
11:00-11:30	Aviezri Fraenkel, *Learn How To Beat Your Fractional Beatty Game Opponent*
11:30-12:00	Urban Larsson, *Blocking games and invariant games*
12:00-12;20	Richard Guy, *Introduction of Cumulative Subtraction Games*
12:30–13:30	Lunch
14:00-17:00	(A) Impartial Games; (B) Go/Monte Carlo Methods; (C) NoGo.
15:00-15:30	Coffee Break, 2nd floor lounge, Corbett Hall
17:30–19:30	Dinner
20:00–	NoGo finals and Problem Session

Friday

7:00–9:00	Breakfast
9:00-10:00	Informal Discussions
10:00-10:30	Coffee Break, 2nd floor lounge, Corbett Hall
10:30-11:30	Informal Discussions
11:30–13:30	Lunch
Checkout by	**12 noon.**

* 5-day workshops are welcome to use BIRS facilities (2nd Floor Lounge, Max Bell Meeting Rooms, Reading Room) until 3 pm on Friday, although participants must checkout of the guest rooms by 12 noon. *

Combinatorial game Theory
Jan 9–Jan 14, 2011

ABSTRACTS

Speaker: **Elwyn Berlekamp** (University of California, Berkeley)
Title: *Report on the latest Coupon Go tournament*
Abstract: Late in 2010 a Coupon Go tournament was held in Korea. This talk will give an overview of the tournament and the results.

Speaker: **Kyle Burke** (Wittenberg University)
Title: *Neighboring Nim: a PSPACE-complete NimG variant*
Abstract: Neighboring Nim is a version of Nim where heaps are embedded onto vertices of a graph. A turn consists of traversing an edge (adjacent to the last play) then removing sticks from the resulting vertex. Even with small heap sizes, the game is PSPACE-hard.

Speaker: **Tristan Cazenave** (Paris-Dauphine University)
Title: *Developments on the Monte-Carlo approximation of temperature*
Abstract:

Speaker: **Erik Demaine** (MIT)
Title: *Geometric Puzzles*
Abstract:

Speaker: **Aviezri Fraenkel** (Weizmann Institute of Science)
Title: *Learn How To Beat Your Fractional Beatty Game Opponent*
Abstract: The P-positions of impartial take-away games on two piles usually split the positive integers into two nonintersecting sequences. Here we consider the case where the P-positions are given a priori as two sequences whose intersection has infinite cardinality. The challenge is to find appropriate succinct game rules for a game having the given P-positions. We present a solution in terms of two exotic numeration systems, for a seemingly first such problem.

Speaker: **JP Grossman** (D. E. Shaw)
Title: *Searching for Periodicity in .**6***
Abstract: **.6** is the only unsolved single-digit octal game. In this take-and-break game, a move consists of removing a bean from a heap and leaving the remaining beans from that heap in exactly 1 or 2 non-empty heaps. It is conjectured that the nim-values for this game are eventually periodic; finding the period (if it exists) requires fast computation of the nim-values. We review the "rare values" algorithm that effectively reduces the computation time for the first N nim-values from $\mathrm{O}(N^2)$ to $\mathrm{O}(N)$. We present several low-level optimizations and show how to parallelize the computation, resulting in significant additional speedups.

Speaker: **Alan Guo & Mike Weimerskirch** (Duke University; Macalester College)
Title: *Lattice point methods in misere games*
Abstract: Positions in normal play heap games with bounded heap size d can be thought of as elements of the lattice $C = \mathbb{N}^d$. We imbed C in $\mathbb{Z}^d$, with $\mathbb{Z}^d \backslash C$ declared to be **defeated** positions. Misère play is treated similarly with gameboard $C = \mathbb{N}^d \setminus \{(0, 0, \ldots, 0)\}$. This can be generalized to arbitrary gameboards. A description of the optimal strategy of such games using Hilbert Series provides an alternative to Plambeck's Quotient Monoid approach.

Speaker: **Ryan Hayward** (University of Alberta)
Title: *Recent results on Hex and Rex*
Abstract: I will describe an explicit Hex handicap strategy: the first player can win the nxn game if allowed to color (n+1)/6 cells on the first move. This is joint with Philip Henderson, and will appear in GONC4.

For Rex, a.k.a. Misere Hex, I will describe some new proofs — and strengthened versions — of Lagarias and Sleator's theorem (for $n \times n$ boards, each player can prolong the game until the board is full, so the 1st(2nd) player wins if n is even(odd)), and Evan's theorem (for even n, opening in the acute corner wins). Also, for n even and at least 4, we find another 1st-player winning opening (adjacent to the acute corner, on the 1st player's side), and for n odd and at least 3 and for each 1st-player opening, we find 2nd-player winning replies. Finally, in response to comments by Martin Gardner, for each n up to 5 we give a simple winning strategy for the $n \times n$ board. This is joint work with Bjarne Toft and Philip Henderson.

Speaker: **Peter Hegarty** (Chalmers University)
Title: *On m-covering families of Beatty sequences with irrational moduli.*
Abstract: Beatty's famous theorem states that the sequences $([n\alpha])_{n=1}^{\infty}$ and $([n\beta])_{n=1}^{\infty}$ are complementary (i.e.: their multiset union forms an exact cover of $\mathbb{N}$) if and only if α and β are both positive irrationals and satisfy $1/\alpha + 1/\beta = 1$. The famous game of Wythoff Nim has as its non-zero P-positions the pairs $\{[n\phi], [n\phi^2]\}_{n=1}^{\infty}$. Thus, not surprisingly, Beatty sequences are interesting for both number theorists and combinatorial game theorists. My talk here will concentrate on presenting a certain class of extensions of Beatty's classical result, which include the still unresolved Tiling Conjecture of Fraenkel. The latter concerns Beatty sequences with rational moduli, which seem harder to deal with. For irrational moduli, I will describe my recent generalisation of Uspensky's theorem classifying eventual exact (e.e.) covers of the positive integers, to e.e. m-covers, for any $m \in \mathbb{N}$, by homogeneous Beatty sequences with irrational moduli. The classification follows from a pleasing property of *integer* arithmetic progressions. The method also applies to inhomogeneous Beatty sequences, still with irrational moduli, but the arithmetical description is more complicated. I will also speculate on how one might make sense of the notion of an exact m-cover when m is not an integer, and present a 'fractional version' of Beatty's theorem. My hope is that some of what I present will inspire members of the current audience to find new applications of Beatty sequences to combinatorial games.

Speaker: **Urban Larsson** (Chalmers University)
Title: *The $\star$-operator and Invariant Subtraction Games*
Abstract: We study a class of 2-player impartial games, so-called *in- variant subtraction games*. The games are played on $\mathcal{B} = \mathbb{N}_0 \times \mathbb{N}_0$ (or $\mathbb{N}_0^k$, k a positive integer), which we call the game board, and where $\mathbb{N}_0 = \{0, 1, \ldots\}$. The players take turns in moving one single piece from a given position in $\mathcal{B}$ towards the position 0. The last player to move is the winner. Here (translation) invariance means that each allowed move is available inside the whole board, and subtraction means that no move is allowed that increases the x- or y-coordinate of the position. Hence the set of allowed moves may be considered as a subset $\mathcal{M} \subset \mathcal{B} \setminus \{0\}$. Then a new game, $\star$ of the old game, is defined by taking the P-positions, except 0, as moves in the new game. Some important classes of games satisfy $G = (G^{\star})^{\star}$. In fact (in a joint result with Hegarty and Fraenkel 2010) we proved that such a 'duality'-relation provides a solution to a nice conjecture of Duchêne and Rigo (2009) on pairs of complementary Beatty sequences and invariant subtraction games. In this talk I focus on a few subsequent results on the $\star$-operator of which the most basic is that the sequence (G^{2n}) *converges* and where G^n denotes the resulting game after n repeated applications of $\star$ on the game G. The notion of a *permutation game* (the case $k = 2$) is introduced. This is an invariant subtraction game, where each row and column of $\mathcal{B}$ contain precisely one move, but, where both row 0 and column 0 are void of moves. Let G denote a permutation game. Then G^2 is also, so that the family of permutation games is *closed* under double application of $\star$. We round off with some observations on the P-positions of (Wythoff Nim)*, the moves of so-called *Ornament games* (the $\star$ of games whose moves are defined via pairs of complementary Beatty sequences with rational moduli), and then finally we give a very short proof of the 'duality' of k-pile Nim.

Speaker: **Neil McKay** (Dalhousie University)
Title: *Ordinal Sums with base* $*$
Abstract: For a partizan game G, we consider how to play the modified game in which from any position of G both players have an additional option to 0. As position in such a game is either $\{\cdot|\cdot\}$ or both players have at least one option, these games are *all-small*. Moreover, the canonical forms of these games are ordinal sums with base $*$. We also consider sums of such games and discuss games, including YELLOW-BROWN HACKENBUSH where such games appear.

Speaker: **Martin Müller** (University of Alberta)
Title: *Looking for Common Ground in Monte-Carlo Tree Search and Combinatorial Game Theory*
Abstract: In analyzing and playing the game of Go, both combinatorial game theory (CGT) and Monte-Carlo tree search (MCTS) have had notable successes. On the surface, these approaches seem very far apart: CGT shines in the endgame, when a board can be split into a disjunctive sum of local games. CGT uses local search and evaluation in order to compute exact answers to very hard endgame puzzles. In contrast, MCTS uses a single global tree search driven by statistics from a large number of "playouts", randomized continuations of games. While the results are typically not exact, MCTS methods have improved the state of the art in computer players for Go and many other games.

This talk briefly surveys relevant previous and ongoing work towards the goal of a combined CGT+MCTS approach, and proposes some directions for future research.

Speaker: **Teigo Nakamura** (Kyushu Institure of Technology
Title: *A Method for Analyzing Complex Go Capturing Races*
Abstract: We developed a new genre of application of CGT to the game of Go, that is, to count liberties in capturing races several years ago. Capturing race is a particular kind of life and death problem in which two adjacent opposing groups are fighting to capture the opponent's group each other. A position of capturing races can be decomposed into independent subpositions and we can apply CGT to analyze the outcome. We extend our methodology to be applied to more complex capturing races in which three or more groups are involved.

Speaker: **Vladimir Oudalov** (Rutgers University)
Title: *A Characterization of complete seki*
Abstract:

Speaker: **Thane Plambeck** (Counterwave)
Title: *Impartial Tic-Tac-Toe*
Abstract: Answering a question of Tim Chow, we show how to calculate and describe the best play of misere disjunctive sums in Impartial Tic-Tac-Toe (both players play "X", and whoever completes the final three-in-a-row configuration on the final board loses).

Speaker: **Carlos Santos** (FCU Lisbon)
Title: *A Non-trivial Surjective Map onto the Short Conway's Group*
Abstract: Berlekamp asked the question "What is the habitat of $*2$?". It was possible generalize the question: "for a game G, what is the largest n such that $*n$ is a position of G?". This led to the concept of nim dimension analyzed in some CGT papers. This paper analyzes an even more general question: "What is the habitat of the short Conway's group?". The goal was to prove that all the short combinatorial games are positions of a particular well-known game. Fortunately, a game like this exists in CGT literature and it is implemented in the Siegel's CGSuite. We show a proof that generalized KONANE has this property

Speaker: **Angela Siegel** (Dalhousie University)
Title: *Distributive and other Lattices in CGT.*
Abstract: Early last year, Albert & Nowakowski generalized the 'Games born on Day n form a distributive

lattice' result. We show that their results also apply to some loopy games. We also also show that the day n option-closed games form a non-distributive but planar lattice.

Speaker: **Bill Spight** (WLS)
Title: *Influence of Go Stones in the Laboratory*
Abstract:

Speaker: **Takenobu Takizawa** (Waseda University)
Title: *A Combination of Basic Go Positions.* Abstract: This talk is about last stage of the end-games of mathematical go. First, analyzing one-point yose, small ko and small hidden ko positions where the conditions are: (1) Black is a ko-monster, (2) Black is a ko-master, (3) Neutral thread environment, (4) White is a ko-master, and (5) White is a ko-monster. under (A) Japanese, and (B) Chinese rules Second, analyzing combinations of one-point yose position, plus small ko and hidden-ko position, where the conditions and rules are same as the first case.

Speaker: **Olivier Teytaud** (Universite Paris-Sud)
Title: *Monte-Carlo Tree Search Methods*
Abstract: This talk will give an overview of the Monte-Carlo Tree Search Methods which have been a revolution in games and planning since 2006.