# Banff International Research Station
## for Mathematical Innovation and Discovery

# Complex Analysis and Complex Geometry
# May 31 - June 5, 2009

### MEALS

Breakfast (Buffet)[1]: 7:00–9:30 am, Sally Borden Building, Monday–Friday
Lunch (Buffet)[1]: 11:30 am–1:30 pm, Sally Borden Building, Monday–Friday
Dinner (Buffet)[1]: 5:30–7:30 pm, Sally Borden Building, Sunday–Thursday
Coffee Breaks: As per daily schedule, 2nd floor lounge, Corbett Hall

### MEETING ROOMS

All lectures will be held in Max Bell 159. LCD projector, overhead projectors and blackboards are available for presentations. The meeting space designated for BIRS is the lower level of Max Bell, Rooms 155–159.

### SCHEDULE

### Sunday

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| 16:00 | Check-in begins (Front Desk - Professional Development Centre - open 24 hours). Lecture rooms available after 16:00 |
| 17:30–19:30 | Buffet Dinner, Sally Borden Building |
| 20:00 | Informal gathering in 2nd floor lounge, Corbett Hall |

### Monday

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| 8:15–8:30 | Introduction and Welcome to BIRS by BIRS Station Manager |
| **8:30–9:15** | László Lempert (Purdue) *The uniqueness of geometric quantization* |
| **9:30–10:15** | Sergey Ivashkovich (Lille-1) *Vanishing cycles in holomorphic foliations by Riemann surfaces and foliated shells* |
| 10:20–10:45 | Coffee Break |
| **10:45–11:30** | Zbigniew Błocki (Jagiellonian University) *On geodesics in the space of Kähler metrics* |
| 11:35–13:30 | Lunch |
| 13:00–14:00 | Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall |
| **14:00–14:45** | Sophia Vassiliadou (Georgetown University) *Hartogs extension theorems on complex spaces with singularities* |
| 14:50 | Group Photo; meet on the front steps of Corbett Hall |
| 14:50–15:15 | Coffee Break |
| **15:15–16:00** | Frank Kutzschebauch (Universität Bern) *A solution of Gromov's Vaserstein problem* |
| **16:15–17:00** | Debraj Chakrabarti (Notre Dame) *CR functions on subanalytic hypersurfaces* |

[1]Please remember to scan your meal card at the host/hostess station in the dining room for each meal.

## Tuesday

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| **8:30–9:15** | Egmont Porten (Mid Sweden University) *Higher-order concavity in CR geometry* |
| **9:30–10:15** | Alexander Isaev (ANU) *Infinite-dimensionality of the automorphism groups of homogeneous Stein manifolds* |
| 10:20–10:45 | Coffee Break |
| **10:45–11:30** | Vincent Guedj (Université Aix-Marseille) *Variational approach to complex Monge-Ampère equations* |
| 11:30–13:30 | Lunch |
| **14:00–14:45** | Burglind Juhl-Jöricke (IHES) *Envelopes of holomorphy and holomorphic discs* |
| 14:50–15:15 | Coffee Break |
| **15:15–16:00** | Imre Patyi (Georgia State) *On holomorphic domination* |
| **16:15–17:00** | Laurent Meersseman (PIMS/Universit de Bourgogne) *Uniformization of deformation families of compact complex manifolds* |

## Wednesday

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|---|---|
| **8:30–9:15** | Jörg Winkelmann (Universität Bayreuth) *On Brody curves* |
| **9:30–10:15** | Aaron Zerhusen (Illinois Wesleyan) *Local solvability of the $\bar{\partial}$-equation in certain Banach spaces.* |
| 10:20–10:45 | Coffee Break |
| **10:45–11:30** | Sebastien Boucksom (Institut de mathématique de Jussieu) *Equilibrium measures and equidistribution of Fekete points on complex manifolds* |
| 11:30–13:30 | Lunch |
| | Free Afternoon |

## Thursday

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| **8:30–9:15** | Alexandre Sukhov (Lille-1) *Constructions of pseudoholomorphic discs* |
| **9:30–10:15** | Gordon Heier (UC, Riverside) *On complex projective manifolds of negative holomorphic sectional curvature* |
| 10:20–10:45 | Coffee Break |
| **10:45–11:30** | Joël Merker (Ecole Normale Superieure) *Effective algebraic degeneracy* |
| 11:30–13:30 | Lunch |
| **14:00–14:45** | Xianghong Gong (Wisconsin) *Regularity in the CR embedding problem* |
| 14:50–15:15 | Coffee Break |
| **15:15–16:00** | Vakhid Masagutov (Purdue) *Homomorphisms of Infinitely Generated Analytic Sheaves* |
| **16:15–17:00** | Nikolay Kruzhilin (Steklov Mathematical Institute) *Holomorphic maps of Reinhardt domains* |

## Friday

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| **8:30–9:15** | Jean-Pierre Rosay (Wisconsin) *Pluri-polar sets in almost complex manifolds* |
| **9:30–10:15** | Bruno De Oliveira (University of Miami) *Symmetric differentials, differential operators and the topology of complex surfaces* |
| 10:20–10:45 | Coffee Break |
| **10:45–11:30** | Evgeny Poletsky (Syracuse) *Functions holomorphic along holomorphic vector fields* |
| 11:30–13:30 | Lunch |

**Checkout by 12 noon.**

5-day workshops are welcome to use the BIRS facilities (2nd Floor Lounge, Max Bell Meeting Rooms, Reading Room) until 3 pm on Friday, although participants are still required to checkout of the guest rooms by 12 noon.

# Complex Analysis and Complex Geometry
# May 31 - June 5, 2009

## ABSTRACTS
## (in alphabetic order by speaker surname)

Speaker: **Zbigniew Błocki** (Jagiellonian University)
Title: *On geodesics in the space of Kähler metrics*
Abstract: Our main result is that geodesics in the space of Kähler metrics (as considered by Mabuchi, Donaldson and Semmes) are (fully) $C^{1,1}$, provided that the bisectional curvature is nonnegative. Existence of such geodesics (without curvature assumption) with bounded mixed complex 2nd derivatives was proved by X. X. Chen. It boils down to solving a homogeneous complex Monge-Ampere equation on a compact Kähler manifold with boundary. We also discuss slightly more general equations of this kind.

Speaker: **Sebastien Boucksom** (Institut de mathématique de Jussieu)
Title: *Equilibrium measures and equidistribution of Fekete points on complex manifolds*
Abstract: Fekete points are optimal configurations of points in polynomial interpolation. It is a classical result that Fekete points confined within a given compact set of the complex plane equidistribute towards the potential-theoretic equilibrium measure of the compact set. I will present a joint work with Robert Berman where we extend this result to the higher-dimensional case by a variational principle, working in the more geometric setting of sections of a line bundle over a compact complex manifold

Speaker: **Debraj Chakrabarti** (Notre Dame University)
Title: *CR functions on subanalytic hypersurfaces*
Abstract: We consider the problem of local one-sided holomorphic extension of continuous or smooth CR functions from hypersurfaces with singularities, in particular from the class of subanalytic hypersurfaces, which include the real-analytic ones. We discuss the obstructions to the existence of such extension, which turn out to be different from those in the classical smooth case.

Speaker: **Bruno De Oliveira** (University of Miami)
Title: *Symmetric differentials, differential operators and the topology of complex surfaces*
Abstract: The space of symmetric differentials of order 1, i.e. holomorphic 1-forms, are intimately connected with the topology of a complex surface. On the other hand, the same does not happen for symmetric differentials of higher order. Examples of this difference are: There are families of algebraic surfaces where $h^0(X_t, S^m\Omega^1_{X_t})$ is not locally constant for $m > 1$, a simply connected surface $X$ can have nontrivial symmetric differentials of order $m > 1$ (in fact $\Omega^1_X$ can be ample). To regain the connection with the topology we need to consider a special class of symmetric differentials, we call these differentials closed, they are locally of the form $df_1...df_m$. Opposite to closed differential forms we will show that there is no collection of differential operators characterizing closed symmetric differentials, but as we will see this can be done if we ask to be closed around a general point. A special case of a topological result to be presented is that if a complex surface $X$ has a nontrivial closed symmetric differential of order 2 then $\pi_1(X) \neq 0$.

Speaker: **Xianghong Gong** (University of Wisconsin)
Title: *Regularity in the CR embedding problem*
Abstract: We will prove a new regularity on the local embedding of strongly pseudoconvex CR manifolds of dimension at least 7. This is joint work with Sidney Webster.

Speaker: **Vincent Guedj** (Université Aix-Marseille)
Title: *Variational approach to complex Monge-Ampère equations*
Abstract: I will present a new variational approach to Monge-Ampere equations on compact complex manifolds, which enbles to construct singular solutions to the Dirichlet problem without relying on Yau's fundamental existence result. This is joint work with R.Berman, S.Boucksom and A.Zeriahi.

Speaker: **Gordon Heier** (University of California, Riverside)
Title: *On complex projective manifolds of negative holomorphic sectional curvature*
Abstract: It is a long-standing open problem to show that a complex projective manifold with a Kaehler metric of negative holomorphic sectional curvature has an ample canonical line bundle. In this talk, partial results towards this problem will be presented. This is joint work in progress with Bun Wong.

Speaker: **Alexander Isaev** (Australian National University)
Title: *Infinite-dimensionality of the automorphism groups of homogeneous Stein manifolds*
Abstract: Let $X$ be a Stein manifold of dimension greater than 1 homogeneous with respect to a holomorphic action of a complex Lie group. We show that the Lie algebra generated by complete holomorphic vector fields on $X$ is infinite-dimensional, i.e. it is impossible to introduce the structure of a Lie transformation group on the group of holomorphic automorphisms of $X$. The well-known examples of complex linear space and affine quadric fit into this general situation. The work is joint with Alan Huckleberry.

Speaker: **Sergey Ivashkovich** (Université de Lille-1)
Title: *Vanishing cycles in holomorphic foliations by Riemann surfaces and foliated shells*
Abstract: The purpose of this talk is the study of vanishing cycles of holomorphic foliations by Riemann surfaces on compact complex manifolds. The notion of a *vanishing cycle* was implicitly introduced by S. Novikov in his proof of the existence of compact leaves in smooth foliations by surfaces on the three-dimensional sphere. Later it appeared as an obstruction to the simultaneous uniformizability of the object known as a *skew cylinder*, introduced by Ilyashenko, which is proved to be an extremely useful tool in foliation theory. Our main result consists in showing that a vanishing cycle comes together with a much richer complex geometric object - we call this object a *foliated shell*. A number of related statements will be given and several open questions will be discussed.

Speaker: **Burglind Juhl-Jöricke** (IHES)
Title: *Envelopes of holomorphy and holomorphic discs*
Abstract: The envelope of holomorphy of an arbitrary domain in a Stein manifold is identified with a connected component of the set of equivalence classes of analytic discs immersed into the Stein manifold with boundary in the domain. This has several corollaries, in particular, in case of dimension two for each of its points the envelope of holomorphy contains an embedded (non-singular) Riemann surface passing through this point with boundary contained in the natural embedding of the original domain into its envelope of holomorphy. The method has applications also for the case of projective manifolds.

Speaker: **Nikolay Kruzhilin** (Steklov Mathematical Institute)
Title: *Holomorphic maps of Reinhardt domains*
Abstract: Nonbiholomorphic proper maps from bounded Reinhardt domains are considered. The structure of the boundary of the source domain and the boundary behavior of the map are investigated. The role of the target domain is discussed.

Speaker: **Frank Kutzschebauch** (Universität Bern)
Title: *A solution of Gromov's Vaserstein problem*
Abstract: It is standard material in a Linear Algebra course that the group $\mathrm{SL}_m(\mathbb{C})$ is generated by elementary matrices $E + \alpha e_{ij}$ $i \neq j$, i.e., matrices with 1's on the diagonal and all entries outside the diagonal are zero, except one entry. Equivalently every matrix $A \in \mathrm{SL}_m(\mathbb{C})$ can be written as a finite product of upper and lower diagonal unipotent matrices (in interchanging order). The same question for matrices in $\mathrm{SL}_m(R)$ where $R$ is a commutative ring instead of the field $\mathbb{C}$ is much more delicate. For example if $R$ is the ring of complex valued functions (continuous, smooth, algebraic or holomorphic) from a space $X$ the problem amounts to find for a given map $f : X \to \mathrm{SL}_m(\mathbb{C})$ a factorization as a product of upper and lower diagonal unipotent matrices

$$f(x) = \begin{pmatrix} 1 & 0 \\ G_1(x) & 1 \end{pmatrix} \begin{pmatrix} 1 & G_2(x) \\ 0 & 1 \end{pmatrix} \cdots \begin{pmatrix} 1 & G_N(x) \\ 0 & 1 \end{pmatrix}$$

where the $G_i$ are maps $G_i : X \to \mathbb{C}^{m(m-1)/2}$. Since any product of (upper and lower diagonal) unipotent matrices is homotopic to a constant map (multiplying each entry outside the diagonals by $t \in [0,1]$ we get a homotopy to the identity matrix), one has to assume that the given map $f : X \to \mathrm{SL}_m(\mathbb{C})$ is homotopic to a constant map or as we will say null-homotopic. In particular this assumption holds if the space $X$ is contractible. This very general problem has been studied in the case of polynomials of $n$ variables. For $n = 1$, i.e., $f : X \to \mathrm{SL}_m(\mathbb{C})$ a polynomial map (the ring $R$ equals $\mathbb{C}[z]$) it is an easy consequence of the fact that $\mathbb{C}[z]$ is an Euclidean ring that such $f$ factors through a product of upper and lower diagonal unipotent matrices. For $m = n = 2$ the following counterexample was found by COHN: the matrix

$$\begin{pmatrix} 1 - z_1 z_2 & z_1^2 \\ -z_2^2 & 1 + z_1 z_2 \end{pmatrix} \in \mathrm{SL}_2(\mathbb{C}[z_1, z_2])$$

does not decompose as a finite product of unipotent matrices. For $m \geq 3$ (and any $n$) it is a deep result of SUSLIN that any matrix in $\mathrm{SL}_m(\mathbb{C}[\mathbb{C}^n])$ decomposes as a finite product of unipotent (and equivalently elementary) matrices. In the case of continuous complex valued functions on a topological space $X$ the problem was studied and solved by THURSTON and VASERSTEIN. It is natural to consider the problem for rings of holomorphic functions on Stein spaces, in particular on $\mathbb{C}^n$. Explicitly this problem was posed by GROMOV in his groundbreaking paper where he extends the classical OKA-GRAUERT theorem from bundles with homogeneous fibers to fibrations with elliptic fibers, e.g., fibrations admitting a dominating spray. In spite of the above mentioned result of VASERSTEIN he calls it the **Vaserstein problem:** *Does every holomorphic map $\mathbb{C}^n \to SL_m(\mathbb{C})$ decompose into a finite product of holomorphic maps sending $\mathbb{C}^n$ into unipotent subgroups in $SL_m(\mathbb{C})$?* In the talk we explain a complete solution to GROMOV'S **Vaserstein Problem**. This is a joint work with B. Ivarsson.

Speaker: **László Lempert** (Purdue University)
Title: *The uniqueness of geometric quantization*
Abstract: This is joint work with Szoke, and in progress. In geometric quantization (as in most other schemes of quantization) one associates with a Riemannian manifold a Hilbert space. The manifold represents the classical configurations of a mechanical system, and the Hilbert space is to represent its quantum states. Often the Hilbert space depends on additional choices, and these choices form a smooth or complex manifold $S$. The uniqueness problem asks whether there is a natural isomorphism between the Hilbert spaces $H_s$ and $H_t$ corresponding to different choices $s, t \in S$.

In the 1990s Axelrod, Della Pietra, and Witten suggested to view the $H_s$ as fibers of a Hilbert bundle $H$ over $S$, define a connection on $H$, and use parallel transport to identify its fibers. In the talk I will briefly explain what is unsatisfactory, from the mathematical point of view, in their work. Then I will discuss the mathematical structures to which their idea leads, and properties of these structures. Finally I will tackle the issue of uniqueness when geometric quantization is based on so called adapted Kähler structures.

Speaker: **Vakhid Masagutov** (Purdue University)
Title: *Homomorphisms of Infinitely Generated Analytic Sheaves*
Abstract: We prove that every homomorphism $\mathcal{O}_\zeta^E \to \mathcal{O}_\zeta^F$, with $E$ and $F$ Banach spaces and $\zeta \in \mathbb{C}^m$, is induced by a $\mathrm{Hom}(E, F)$-valued holomorphic germ, provided that $1 \leq m < \infty$. A similar structure theorem is obtained for the homomorphisms of type $\mathcal{O}_\zeta^E \to \mathcal{S}_\zeta$, where $\mathcal{S}_\zeta$ is a stalk of a coherent sheaf of positive depth. We later extend these results to sheaf homomorphisms, obtaining a condition on coherent sheaves which guarantees the sheaf to be equipped with a unique analytic structure in the sense of Lempert-Patyi.

Speaker: **Laurent Meersseman** (PIMS/Universit de Bourgogne)
Title: *Uniformization of deformation families of compact complex manifolds*
Abstract: consider the following uniformization problem. Take two holomorphic (parametrized by the unit disk) or differentiable (parametrized by an interval containing 0) deformation families of compact complex manifolds. Assume they are pointwise isomorphic, that is for each point t of the parameter space, the fiber over t of the first family is biholomorphic to the fiber over t of the second family. Then, under which conditions are the two families locally isomorphic at 0?

After recalling some known results (positive and negative) on this problem, I will give a sufficient condition in the case of holomorphic families. I will then show that, surprisingly, this condition is not sufficient in the case of differentiable families. These results rely on a geometric study of the Kuranishi space of a compact complex manifold.

Speaker: **Joël Merker** (Ecole Normale Superieure)
Title: *Effective algebraic degeneracy*
Abstract: In 1979, Green and Griffiths conjectured that in every projective algebraic variety $X$ of general type, there exists a certain *proper* subvariety Y with the property that every *nonconstant* entire holomorphic curve $f\colon \mathbb{C} \to X$ landing in $X$ must in fact lie inside $Y$. For projective hypersurfaces $X$, Siu showed in 2004 that there is an integer $d_n$ such that every generic hypersurface $X$ in $\mathbb{P}^{n+1}(\mathbb{C})$ of degree $d \geqslant d_n$, such an $Y$ exists. The talk, based on Demailly's bundle of invariant jet differentials and on a new construction of explicit slanted vector fields tangent to the space of vertical jets to the universal hypersurface (realizing an idea of Siu), will present a recent complete detailed proof (joint with Diverio and Rousseau) of such a kind of algebraic degeneracy statement, with the *effective* degree bound :

$$d \geqslant n^{(n+1)^{n+5}}.$$

In the early 1980's, Lang conjectured a deep correspondence between degeneracy of entire holomorphic curves and finiteness/non-denseness of rational points on projective algebraic varieties that the whole subject is, unfortunately, still unable to put in concrete form.

Speaker: **Imre Patyi** (Georgia State University)
Title: *On holomorphic domination*
Abstract: We discuss the question of flexible exhaustions of pseudoconvex open sets in a Banach space by sublevel sets of the norm of holomorphic vector valued functions; this has applications to sheaf and Dolbeault cohomology of complex Banach manifolds. We show that if $X$ is a Banach space with a Schauder basis (e.g., $X = C[0,1]$), $D \subset X$ is pseudoconvex open, $u : D \to (-\infty, \infty)$ is continuous, then there are a Banach space $Z$ and a holomorphic function $h : D \to Z$ such that $u(x) < \|h(x)\|$ for $x \in D$; in this case we say that holomorphic domination is possible in $D$. On a different note we also show that many complex Banach submanifolds of the Banach space $\ell_1$ of summable sequences admit many nowhere critical numerical holomorphic functions.

Speaker: **Evgeny Poletsky** (Syracuse University)
Title: *Functions holomorphic along holomorphic vector fields*
Abstract: We will discuss the following generalization of Forelli's theorem: Suppose $F$ is a holomorphic vector field with singular point at $p$, such that $F$ is linearizable at $p$ and the matrix is diagonalizable with eigenvalues whose ratios are positive reals. Then any function that has an asymptotic Taylor expansion at $p$ and is holomorphic along the complex integral curves of $F$ is holomorphic in a neighborhood of $p$. We also present an example to show that the requirement for ratios of the eigenvalues to be positive reals is necessary.

Speaker: **Egmont Porten** (Mid Sweden University)
Title: *Higher-order concavity in CR geometry*
Abstract: Our original motivation was to understand CR analysis on a class of homogeneous CR manifolds, arising naturally as orbits of group actions on flag manifolds. These examples have a remarkable tendency to be essentially pseudoconcave, a notion of weak pseudoconcavity introduced by Hill and Nacinovich. We will explain how this kind of pseudoconcavity can be used to prove subellipticity of the system of tangential CR equations as well as holomorphic extension to full neighborhoods of the CR manifold. The results were obtained in common research with A. Altomani, D. Hill and M. Nacinovich.

Speaker: **Jean-Pierre Rosay** (University of Wisconsin)
Title: *Pluri-polar sets in almost complex manifolds*
Abstract: The notion of plurisubharmonicity makes sense for functions defined on almost complex manifolds. Pluripolar sets are sets on which a plurisubharmonic function is $-\infty$. I shall discuss the notion of pluripolarity and the important question of logarithmic singularities versus weaker singularities.

The Chirka function with pole at a point has been effciently used for localization of the Kobayashi metric (Gaussier-Sukhov and Ivashkovich-Rosay). I shall discuss more recent results on pluripolarity with applications to uniqueness results (Ivashkovich-Rosay).

Speaker: **Alexandre Sukhov** (Université de Lille-1)
Title: *Constructions of pseudoholomorphic discs*
Abstract: We establish an existence and study the properties of J-complex curves with prescribed boundary conditions in almost complex Stein manifolds.

Speaker: **Sophia Vassiliadou** (Georgetown University)
Title: *Hartogs extension theorems on complex spaces with singularities*
Abstract: I will discuss some generalizations of the classical Hartogs extension theorem to complex spaces with singularities and present an analytic proof using $\overline{\partial}$-techniques (joint work with Nils Ovrelid).

Speaker: **Jörg Winkelmann** (Universität Bayreuth)
Title: *On Brody curves*
Abstract: We discuss a number of properties of Brody curves which underline that the class of Brody curves is rather "delicate", for example, the property of a variety of admitting a non-degenerate Brody curve does not behave well in families.

Speaker: **Aaron Zerhusen** (Illinois Wesleyan University)
Title: *Local solvability of the $\bar{\partial}$-equation in certain Banach spaces.*
Abstract: In a sharp contrast to the situation in finite dimensions, Imre Patyi has shown that the $\bar{\partial}$-equation is not always solvable, even locally, in an infinite dimensional Banach space. On the other hand, László Lempert has shown that in $\ell_1$, the Banach space of 1-summable sequences, the $\bar{\partial}$-equation is solvable for (0,1)-forms on pseudoconvex domains. I will discuss how Lempert's result leads to a proof of local solvability of the $\bar{\partial}$-equation in a large class of Banach spaces which includes any $L_1$ space and the dual space of any $L_\infty$ space.