09w5006 Interdisciplinary Workshop on *Fixed-Point Algorithms for Inverse Problems in Science and Engineering*

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1 Summary

The objective of this workshop was to bring together researchers with a strong interest in projection and first-order fixed-point algorithms, both from mathematics and from the applied sciences, in order to survey the state-of-the-art of theory and practice, to identify emerging problems driven by applications, and to discuss new approaches for solving these problems.

Various monographs and conference proceedings on projection methods and their applications have been published recently. The participants have not met before and it is very unlikely they will meet again at ordinary optimization conferences. We expect this workshop to be the base for new innovative research and collaborations by its unique mix of experts whose areas of applications are broad, ranging from variational analysis, numerical linear algebra, machine learning, computational physics and crystallography.

2 Overview of the Field and Relationships with the Workshop

In this section, we highlight some of the recent developments and open problems discussed at the workshop. In particular, we focus on recent scientific progress as well as contributions of participants to the workshop. The topics are grouped into four distinct areas, but common themes that arose throughout the conference are the potential of first-order methods for solving large-scale and/or nonconvex problems, and the need for a theoretical foundation to explain their success. A remarkable aspect of the talks was the role that experimental mathematics has played in the development of theoretical intuition. The use of experimental results on benchmark problems has long been standard practice in research on numerical algorithms, however the use of mathematical software to test theoretical hypotheses is a relatively recent phenomenon. The development of this practice has been well-documented in the recent books by Bailey, Borwein and collaborators [5, 4, 6]. The prevalence of computer-aided mathematical discovery in the presentations at this conference indicates that this methodology has matured to an established practice.

2.1 Douglas-Rachford / Difference Map Algorithms

The Douglas-Rachford algorithm [38], which is a linear implicit iterative method, was originally developed in 1956 for solving partial differential equations. In 1979, Lions and Mercier [56] extended the Douglas-Rachford algorithm to an operator splitting method for finding a zero of the sum of two maximal monotone operators (see [31] for an historical account and theoretical details).

The Douglas-Rachford algorithm was discussed in several talks and from different viewpoints [32, 44, 57, 63, 81, 97]. When applied to normal cone operators of two nonempty closed convex sets A and B, with associated projectors P_A and P_B as well as reflectors $R_A = 2P_A - \mathrm{Id}$ and $R_B = 2P_B - \mathrm{Id}$, the governing iteration takes the form

$$x_0 \in X, \qquad (\forall n \in \mathbb{N}) \;\; x_{n+1} = \frac{\mathrm{Id} + R_B R_A}{2} x_n, \tag{1}$$

where Id denotes the identity operator of the Hilbert space X. Under appropriate assumptions, the so-generated sequence $(x_n)_{n\in\mathbb{N}}$ has the remarkable property that $(P_A x_n)_{n\in\mathbb{N}}$ converges to a solution of the underlying feasibility problem, i.e., to a point in $A \cap B$.

As is the case with good algorithms and ideas in science, this method was rediscovered by different people working in different disciplines. Noteworthy is the application of the Douglas-Rachford algorithm in phase retrieval with a support constraint (as opposed to support and nonnegativity), where it is known as the *hybrid input-output (HIO)* algorithm, pioneered by Fienup [47] in 1982. (See also [8] for a view from convex optimization.) A very interesting development originates with Elser [42], who has recently very successfully applied the Douglas-Rachford algorithm to various continuous and discrete, *nonconvex* problems [43, 50]. In the physics community, the algorithm is now known as the *difference map algorithm* and its product space version à la Pierra [77] as *divide and concur*. Novel applications were given in his talk [44], which is available in video format [45]. The constraint sets that arise in the non-convex settings studied by physicists — e.g. spaces of orthogonal or low-rank matrices — often have projections that can be computed efficiently and yet are outside the scope of conventional, linear programming based methods. By including non-convex constraints in the general formalism even NP-complete problems are open to these solution methods. In such applications, where the iterates behave chaotically, the question of convergence shifts to mathematical themes more closely linked to dynamical systems and ergodicity. Macklem [65] illustrated how the software package *Cinderella* [55] is a visual aid in refuting conjectures and building intuition for the Douglas-Rachford algorithm in low dimensions. The flexibility of the projection-based method in crystallographic applications [58, 59, 89] was illustrated with the protein envelope reconstructions reported in the talk by Lo [57]. Finally, Yedidia [97] reported on a recent modification of the belief propagation algorithm based on the difference map algorithm, which led to a new decoder that is currently state-of-the-art.

2.2 Other Projection-type Algorithms

Ben-Israel [18] presented his very recent work [19] on the *inverse* of the classical Newton iteration, which leads to a geometric interpretation of iterations and chaos. Cegielski [28] described general frameworks for projection methods as well as his recent generalization [29] of the classical Opial Theorem, which is of fundamental importance in algorithmic fixed point theory [73]. De Pierro [37] described his recent work on gradient and subgradient methods [53] and provided applications to SPECT (Single Photon Emission Computed Tomography). Ideas of self similarity were presented by Ebrahimi [39], who considered Banach contraction-based techniques [40]. Based on a statistical multiscale criterion, Marnitz [66] proposed an algorithm for solving linear ill-posed equations. His algorithm also employs Dykstra's method for finding the best approximation to the intersection of convex sets. Another new application of projection type methods was reported by Mostafa Nasri [71]. Nasri combines an augmented Lagrangian scheme with projections, for solving equilibrium problems whose feasible sets are defined by convex inequalities. This method finds first an approximate solution of an unconstrained equilibrium problem, and then, either an extragradient-type step or a projection onto a suitable hyperplane is performed.

2.3 Inverse Problems, Convex Analysis, Optimization

In the inverse problems community, a lot of work is currently focused on the development of efficient numerical techniques for solving minimization problems under sparsity-promoting constraints, e.g.

[33, 36, 88, 91, 27], as well as rank reducing constraints, e.g. [46, 75]. Plemmons [78] opened the workshop with a presentation on spectral image analysis. He showed the importance of identifying and quantifying the materials present in the object or scene being imaged. He described a variational fuzzy segmentation model coupled with a denoising/deblurring model based on fast total variation regularized computations, [98]. Beck [15] presented developments in the spirit of his recent work in image recovery [16, 17] that aim at improving acceleration techniques originally proposed by Nesterov [72]. Luke [63] described a dual-space method developed with Jonathan Borwein in which the regularized dual problem is solved via a subgradient descent method with exact line search [20]. The selection of the "best" subgradient is formulated as a best approximation problem, to which a relaxed Douglas Rachford algorithm [64] is applied. The problem of *sensor network localization* was addressed by Henry Wolkowicz [92]. This problem can be modeled as a rank constrained semidefinite programming problem. However, the special structure of the problem allows one to take advantage of the NP-hard rank constraint and solve huge problems (of the order of a million sensors) in reasonable time to machine precision, [54]. A new first order algorithm for a class of smooth constrained minimization problems, called *the moving balls approximation method*, was presented by Marc Teboulle [86], see also [2]. This relies on a simple geometric idea that approximates the constraint set by a sequence of balls, and combines this with a fixed point approach. Another approach to nonsmooth optimization problems arising in signal processing was proposed by Yamada [93], who employed the Moreau envelope to smooth the original problem and used a fixed point model to represent the constraints in the spirit of [94, 95].

Another interesting development in the use of modern optimization tools in signal processing was proposed by Modersitzki [68] in the context of regularized variational image registration (see also [69]).

Convex combinations of resolvents and the underlying potential (the "proximal average") were considered by Wang [90], with particular emphasis on applications in linear algebra, and by Moffat [70] for non-quadratic kernels. (See [9, 11] for underlying theory.) Nonconvex variations were explored by Hare [52]. Bauschke [7] described results on Chebyshev and Klee sets with respect to Bregman distances induced by Legendre functions [13, 14, 10]. Lucet [61] described his implementation of graph-calculus for computational convex analysis, based on a calculus introduced by Goebel [51]. Bot [21] surveyed recent work on the stability of Fenchel duality [22, 23], which provide an answer to a problem posed by Simons. Corvellec [34] presented new results [3, 35] on the *error bound principle*. Yao [96] provided two linear maximal monotone operators that he used to show that the answer to a recent question by B.F. Svaiter [85] is negative [12]. Zinchenko [99] reported on a cost-effective usage of GP-GPUs to accelerate linear-algebraic computations needed to solve large scale optimization problems arising in intensity modulated radiation therapy treatment planning [30]. An application of augmented Lagrangian schemes for nonconvex and nonsmooth problems was presented by Burachik [24]. She described the recently devised Inexact Modified Subgradient algorithm for solving the (convex) dual of a nonconvex optimization problem. Even though the original problem is nonconvex, the method presented enjoys both primal and dual convergence.

In honor of Rudolf Kalman being awarded the US National Medal of Science, Jim Burke presented an interior point algorithm for computing Kalman-Bucy smoothers with constraints [25, 26]. The method obtains the same decomposition that is normally obtained for the unconstrained Kalman-Bucy smoother, hence the resulting number of operations grows linearly with the number of time points.

Scherzer [82] presented some theoretical results establishing the relationship between lower semi-continuity and separate convexity for non-local functionals that have attracted attention in image denoising. Extending his techniques further, Scherzer outlined novel characteristics of Sobolev spaces to derive approximations of the total variation energy regularization and hence to recover existing numerical schemes for total variation minimization in addition to novel numerical schemes.

2.4 Monotone Operator Theory

Algorithm (1) is the iteration of a firmly nonexpansive mapping. Eckstein in his thesis [41] noted that even though the projection operators P_A and P_B are proximal mappings[1], the operator iterated in (1) need itself *not* be a proximal mapping, i.e., it may be the resolvent of a maximal monotone operator that is *not* a subdifferential of a convex function. As such, *monotone operator theory* appears to be critical for a proper understanding of the Douglas-Rachford / difference map algorithm. The talks by Revalski [79] and by Simons [83] go in this direction: Revalski [48, 49, 76, 80] surveyed extended and variational sums of monotone operators, which

[1]Schaad provided a simpler example where A and B are the x-axis and the diagonal in $\mathbb{R}^2$ [81], respectively.

are believed to play a role in the analysis of algorithms featuring resolvents when the sum of the underlying maximal monotone operators is not maximal, and Simons considered generalizations of maximal monotonicity to more abstract settings [84], nowadays called Simons or SSD spaces. López [60] and Martín-Márquez [67] explored monotone operator theory on Hadamard manifolds, including convergence results for a proximal point algorithm. Another important aspect of monotone operators is duality [1, 74]. Combettes [32] examined composite monotone inclusions in duality and proposed primal-dual proximal splitting algorithms to solve them.

3 Outcome of the Meeting

The organizers will edit a Conference Proceedings volume entitled *Fixed-Point Algorithms for Inverse Problems in Science and Engineering*, part of the Springer-Verlag series "Optimization and Its Applications". A good number of the participants will contribute to this volume. In addition, several researchers who were unable to attend the workshop have committed manuscripts as well, including: J. Borwein (Newcastle), Y. Censor (Haifa), G.T. Herman (New York), and S. Reich (Technion).

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