Dynamics, Probability,

and Conformal Invariance

Saturday, March 12, 2005 to Thursday, March 17, 2005

MEALS

Breakfast (Continental): 7:00 – 9:00 am, 2nd floor lounge, Corbett Hall, Sunday – Thursday
*Lunch (Buffet): 12:00 pm – 1:30 pm, Donald Cameron Hall, Sunday – Thursday
*Dinner (Buffet): 5:30 – 7:30 pm, Donald Cameron Hall, Saturday – Wednesday
Coffee Breaks: As per daily schedule, 2nd floor lounge, Corbett Hall
***Please remember to scan your meal card at the host/hostess station in the dining room for each lunch and dinner.**

MEETING ROOMS

All lectures will be held in Max Bell 159 (Max Bell Building accessible by bridge on 2nd floor of Corbett Hall). Hours: 6 am – 12 midnight. LCD projector, overhead projectors and blackboards are available for presentations. *Please note that the meeting space designated for BIRS is the lower level of Max Bell, Rooms 155-159. Please respect that all other space has been contracted to other Banff Centre guests, including any Food and Beverage in those areas.*

Saturday

16:00 Check-in begins (Front Desk – Professional Development Centre - open 24 hours)
Lecture rooms available after 18:00 (if desired)

17:30-19:30 Buffet Dinner, Donald Cameron Hall

20:00 Informal gathering in 2nd floor lounge, Corbett Hall (if desired)
Beverages and small assortment of snacks available on a cash honour-system basis.

Sunday

7:00-8:45 Breakfast
8:45-9:00 Introduction and Welcome to BIRS by BIRS Station Manager, Max Bell 159

9:00-9:50 Smirnov. Conformally invariant fractals.
10:00-10:50 Duplantier. Conformal fractal geometry and Quantum Gravity
10:50-11:10 Coffee Break, 2nd floor lounge, Corbett Hall
11:10-12:00 Astala

Group Photo; meet on the front steps of Corbett Hall (to take place directly after the last lecture of the morning)

12:00-13:30 Lunch

13:30-14:30 Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall

14:30-17:30 Informal discussions

17:30-19:30 Dinner

19:30-20:00 Burdzy. On the Robin problem in fractal domains.
20:00-20:30 Poltoratski. Toeplitz kernels and uncertainty principle.

Monday

7:00-9:00 Breakfast

9:00-9:50 Minsky. Current events in Kleinian groups, 1
10:00-10:50 Schramm. SLE and conformally invariant random processes, 1
10:50-11:10 Coffee Break, 2nd floor lounge, Corbett Hall
11:10-12:00 Lyubich. Quasi-Additivity Law and local connectivity of unicritical Julia sets.

12:00-13:30 Lunch

13:30-14:20 Cheritat. The case for a Julia set with positive measure.
14:30-15:00 Braverman. Some results on the computability and complexity of Julia sets.
15:00-15:30 Coffee Break, 2nd floor lounge, Corbett Hall
15:30-16:00 Benedicks
16:00-16:30 Zinsmeister. Some remarks about Laplacian growth
16:30-17:00 Marshall. Geodesics and the Zipper algorithm for conformal mapping.
17:00-17:30 Meyer. Quasispheres, Snowballs, and elliptic harmonic measure.

17:30-19:30 Dinner

Tuesday

7:00-9:00 Breakfast

9:00-9:50 Minsky, Current events in Kleinian groups, 2
10:00-10:50 Schramm, SLE and conformally invariant random processes, 2
10:50-11:10 Coffee Break, 2nd floor lounge, Corbett Hall
11:10-12:00 Kenyon. Simple random surfaces

12:00-13:30 Lunch
Free Afternoon

17:30-19:30 Dinner

19:30-20:00 Kennedy. Monte Carlo comparisons of the self-avoiding walk and SLE.

Wednesday

7:00-9:00 Breakfast

9:00-9:50 Minsky. Current events in Kleinian groups, 3
10:00-10:50 Schramm. SLE and conformally invariant random processes, 3
10:50-11:10 Coffee Break, 2nd floor lounge, Corbett Hall
11:10-12:00 Slade. Scaling limits and super-Brownian motion.

12:10-13:30 Lunch

13:30-14:20 Makarov. On quantum Hele-Shaw flow (joint with H. Hedenmalm).
14:30-15:00 Beliaev. Random conformal snowflakes.
15;00-15:30 Coffee Break, 2nd floor lounge, Corbett Hall
15:30-16:00 Kang. Boundary Behavior of SLE
16:00-16:30 Lind
16:30-17:00 Dubedat. Commutation of SLEs.
17:00-17:30 Angel

17:30-19:30 Dinner

Thursday

7:00-9:00 Breakfast

9:00-10:50 Informal discussions
10:50-11:10 Coffee Break, 2nd floor lounge, Corbett Hall
11:10-12:00 Informal discussions

12:00-13:30 Lunch

Checkout by 12 noon.

** 5-day workshops are welcome to use the BIRS facilities (2nd Floor Lounge, Max Bell Meeting Rooms, Reading Room) until 4 pm on Thursday, although participants are still required to checkout of the guest rooms by 12 noon. **

Abstracts

M. Braverman. Intuitively a subset of the complex plane C is computable if we can draw a reliable picture of it with arbitrarily high precision. We present some recent results on the computability and complexity of Julia sets for rational functions.

C. Burdzy. The Robin boundary conditions represent the flow of a substance or heat through a semi-permeable membrane. Let u be a non-negative solution to the heat equation in a bounded domain with Robin boundary conditions. I will address the question of whether the infimum of u over the whole domain is equal to 0.

A. Cheritat. We will decribe a plan by Douady to construct a degree 2 polynomial with a Cremer fixed point and a Julia set with positive measure. We will explain progress made on this approach and what remains to be done.

J. Dubedat. Schramm-Loewner Evolutions (SLEs) have proved a powerful tool to describe the scaling limit of a conformally invariant simple curve. In several instances (percolation, uniform spanning tree ...), one can define in a discrete setting several simple curves. We will discuss questions pertaining to the joint law of these curves in the scaling limit.

Nam-Gyu Kang. We show that the normalized (pre-)Schwarzian derivatives of SLE_κ maps with higher order terms are continuous square integrable martingales with second moment obeying the Duplantier duality. Also we show that they have correlations that decay exponentially in the hyperbolic distance.

The BMO space, or the space of functions of bounded mean oscillation, is the appropriate substitute for L^∞ in many results concerning singular integrals. This notion can be modified in the setting of continuous martingales. The normalized (pre-)Schwarzian derivatives of SLE_κ maps with negligible terms are BMO martingales. As a corollary, they satisfy the John-Nirenberg inequality.

This result may lead to an estimate on the lower bound for the Hausdorff dimension of the boundary of SLE hull. The results we obtain allow us to make a formal argument for the lower bound. The estimate for the upper bound is already established by S. Rohde and O. Schramm. While the Hausdorff dimension of the SLE_κ trace was proved by V. Beffara, it remains an open conjecture for the boundary of the hull when $\kappa>4$.

We reexamine S. Rohde and O. Schramm's derivative expectation to derive the conjectured sharp estimate for the Hölder exponent for $\kappa\neq4$. I. Binder and B. Duplantier derive the same formula from the multifractal spectrum of SLE_κ independently.

T. Kennedy. Monte Carlo simulations of the two dimensional self-avoiding walk (SAW) have given support to the conjecture that the scaling limit of the SAW is SLE with parameter 8/3. These past simulations treated the SAW and SLE as subsets of the plane, i.e., the parameterization of the curves was ignored. In this talk we consider the SAW and SLE as parameterized curves and attempt to compare things that depend on the parameterization.

R. Kenyon. This is work in progress with David Brydges and Jessica Young. We consider a natural model of random immersed surfaces in a (finite or infinite) 2-complex. This is in many ways a natural generalization of the simple random walk. Although little is known about this model, certain expectations can be computed using the Green's function on 1-forms.

M. Lyubich. We prove a Quasi-Additivity Law that compares different moduli of a family of islands in an ocean in a near degenerate situation. We apply it to prove that non-renormalizable unicritical polynomials $z\to z^d+c$ have locally connected Julia sets. (In the quadratic case, $d=2$, it was proved by Yoccoz in about 1990.) It is a joint work with Jeremy Kahn.

D. Marshall. In the early 1980's an elementary algorithm for computing conformal maps was discovered by R. Kühnau and Marshall. The algorithm is fast and accurate, but convergence was not known. Given points $z_0,\ldots,z_n$ in the plane, the algorithm computes an explicit conformal map of the unit disk onto a region bounded by a C^1 curve γ with $z_0,\ldots,z_n\in\gamma$

In this joint work with S. Rohde we prove convergence for Jordan regions in the sense of uniformly close boundaries, and give corresponding uniform estimates on the closed disc for the mapping functions. Improved estimates are obtained if the data points lie on a C^1 curve or a K-quasicircle. The algorithm was discovered as an approximate method for conformal welding, however it can also be viewed as a discretization of the Löwner differential equation.

D. Meyer. A Quasisphere is the image of the sphere under a quasiconformal map (of $\boldsymbol{R}^3$). The largest known class of quasispheres are called snowballs, they are topologically 2-dimensional analogues of the snowflake curve. For those surfaces the qc-embedding can be constructed explicitly. Many questions about the mapping behavior can be answered, at least numerically.

Y. Minsky. The field of Kleinian groups and hyperbolic 3-manifolds has been very active recently,

with the solution of a number of major conjectures, notably the Tameness Conjecture, the Ending Lamination Conjecture, and (presumably) the Geometrization Conjecture. I will survey these conjectures, discuss how they fit together and what their implications are, and try to say something about what comes next.

A. Poltoratski. We generalize the definition of Toeplitz operators to larger spaces of analytic functions. After that we study the problem of injectivity of Toeplitz operators in these spaces. It turns out that many problems of classical analysis, such as distributions of zeros of entire functions (Levinson), completeness of bases of reproducing kernels (Beurling-Malliavin), spectral problems for the Schroedinger and string operators (Krein, Marchenko, ...), naturally become a part of the picture. One can use the Toeplitz approach together with some of the recent advances in complex and harmonic analysis to give shorter proves and further generalizations to the classical results. This talk is based on joint work with N. Makarov.

O. Schramm. Stochastic Loewner evolution (SLE) is a process defined by using one-dimensional Brownian motion as the driving parameter in Loewner's differential equation. There is one free parameter in SLE, which is the speed of the Brownian driving process. The SLE paths are the scaling limits of various natural random processes in the plane, such as the interface of critical percolation, the Ising model or the self avoiding walk. Some such statements have been recently proved by several authors, but others are still conjectural.

I will attempt to make the course useful to the audience regardless of the level of familiarity with SLE. The first talk will present a general overview of the subject, while the latter two will describe central ideas of some proofs and useful techniques. In particular, the third talk will probably be devoted to the level sets of the Gaussian free field and their convergence to SLE(4) (which is joint work with Scott Sheffield).

G. Slade. We explain how critical percolation and related models can be described by super-Brownian motion, in high spatial dimensions. The talk will provide a survey of several results and will not assume previous knowledge of super-Brownian motion.

M. Zinsmeister. (joint work with S.Rohde) We prove some rigorous results linked to some one-parameter family of growth models introduced by the physicists.