

Special Western Canada Linear Algebra Meeting (17w2668)

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1 History of this Conference Series

The Western Canada Linear Algebra meeting, or WCLAM, is a regular series of meetings held roughly once every 2 years since 1993. Professor Peter Lancaster from the University of Calgary initiated the conference series and has been involved as a mentor, an organizer, a speaker and participant in all of the ten or more WCLAM meetings. Two broad goals of the WCLAM series are a) to keep the community abreast of current research in this discipline, and b) to bring together well-established and early-career researchers to talk about their work in a relaxed academic setting.

This particular conference brought together researchers working in a wide-range of mathematical fields including matrix analysis, combinatorial matrix theory, applied and numerical linear algebra, combinatorics, operator theory and operator algebras. It attracted participants from most of the PIMS Universities, as well as Ontario, Quebec, Indiana, Iowa, Minnesota, North Dakota, Virginia, Iran, Japan, the Netherlands and Spain. The highlight of this meeting was the recognition of the outstanding and numerous contributions to the areas of research of Professor Peter Lancaster. Peter's work in mathematics is legendary and in addition his footprint on mathematics in Canada is very significant.

2 Presentation Highlights

The conference began with a presentation from Prof. Chi-Kwong Li surveying the numerical range and connections to dilation. During this lecture the following conjecture was stated: If $A \in M_3$ has no reducing eigenvalues, then there is a $T \in B(H)$ such that the numerical range of T is contained in that of A , and T has no dilation of the form $I \otimes A$. This statement led to interesting discussion among various participants after the lecture. Prof. Ion Zaballa from Spain gave an inspiring lecture on reducing matrix polynomials, which is a topic of interest to our guest of honour, Prof. Peter Lancaster. The morning talks were followed by two insightful lectures given by graduate students, Jane Breen (University of Manitoba) and Xavier Martinez-Rivera (Iowa State University). Both presented recent work, one on clustering in Markov chains, and the other on a signed principal rank characteristic sequence that has connections to the famous principal minor assignment problem. In the afternoon the focus of the workshop switched to matrix theory, especially the spectra of matrices. Prof. David Watkins, a former doctoral student of Prof. Peter Lancaster, discussed fast and stable computation of certain matrices, taking sparsity into account. This talk was based on a paper that had won a SIAM outstanding paper prize. The first day concluded with an interesting lecture on the localization of the spectrum of a matrix. This topic is classical when considering the numerical range or

Gersgorin's disc theorem. In this presentation, Prof. Michael Tsatsomeros, adapted a formulation for drawing the numerical range to constructing a new object called the envelope of A , which does a better job enclosing the spectrum of a matrix. The lecture was highlighted by some fascinating movies constructing the various regions associated with the envelopes of certain matrices.

On Sunday, the first three lectures were concerned with combinatorial matrix theory, with two of them delivering interesting connections to the classical topic of Hadamard matrices. The other, by Prof. Colin Garnett, provided insights on the patterns that allow all possible spectra (SAPs), and proved the $2n$ conjecture for irreducible patterns of order 6, and order 7 over the reals. Participants suggested ways in which the techniques presented might be applicable for reducible patterns. After the break, the next lecture was concerned with Bezout equations of stable matrix functions. This topic has deep roots with the research of Prof. Peter Lancaster and his work was featured prominently throughout the talk.

Our conference concluded with a heart-warming lecture delivered by the distinguished mathematician, friend, and colleague, Prof. Richard Guy. Prof. Guy provided a wonderful perspective on the life and celebrated career of Prof. Peter Lancaster.

Many thanks to BIRS and PIMS for the opportunity and funding for this very successful special WCLAM.

3 Some Open Problems and Questions

We list here a sample of the questions and unresolved problems that led to interesting discussions and constructive thought.

1. (Chi-Kwong Li, College of William and Mary) If $A \in M_3$ has no reducing eigenvalues, then there is a $T \in B(H)$ such that the numerical range of T is contained in that of A , and T has no dilation of the form $I \otimes A$.
2. (Sarah Plosker, Brandon University) What graphs other than chains exhibit PST ? What if we do not require that the state transfer be perfect? What about weighted chains?
3. (Colin Garnett, Black Hills State University) Is it possible to randomly select a pattern that does not satisfy the algebraic condition set forth with large enough n that is in fact spectrally arbitrary? What can we say about the $2n$ conjecture over R versus over C ?
4. (Behruz Tayfeh-Rezaie, Institute for Research in Fundamental Sciences (IPM)) Classify all Hadamard matrices with only two distinct types for 4-tuples of rows. Find another infinite family of Hadamard matrices with only two distinct types for 4-tuples of rows.
5. (Michael Tsatsomeros, Washington State University) Can the envelope of a matrix be refined by considering more eigenvalues and/or eigenvectors of its Hermitian part?

4 Special Occasion Associated with the Meeting

One other important feature of our meeting was a birthday celebration to honour the 88th birthday for Prof. Peter Lancaster. The occasion was held in the BIRS lounge on Saturday evening with all participants being involved. There was a cake brought by Peter's daughter and the event included reading well-wishes from friends and colleagues of Peter's that could not attend the meeting at BIRS.

References

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