Derivative Free and Black Box Optimization

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1 Overview of the Field

Derivative-free optimization (DFO) is the mathematical study of the optimization algorithms that do not use derivatives. While a DFO algorithm was used to test one of the worlds first computers (the MANIAC in 1952), it was not until the 1990s that DFO algorithms were studied mathematically.

Blackbox optimization (BBO) is the study of optimization problems where the objective function is a blackbox. That is, no analytic description of the function is available, but given an arbitrary input the blackbox returns a function value. As BBO naturally arises whenever a computer simulation is involved in an optimization problem, BBO is one of the most rapidly expanding areas of applied optimization. BBO can naturally be approached by DFO.

DFO algorithms have principally fallen into one of two categories: direct search methods and model-based methods. Direct search methods work from an incumbent solution and examine a collection of trial points; if improvement is found, then the incumbent solution is updated, otherwise a search radius parameter is decreased and a new collection of trial points is examined. Model-based methods approximate the objective function with a model function, and use the gradients or even second derivatives of the model function to help guide optimization. (Note that while DFO studies algorithms that do not use derivatives, this does not mean that the objective function is nondifferentiable – for example the objective could be a computer simulation using numerical integration.) It was not until very recently that researchers began mixing direct search and model-based methods to create hybrid methods with improved performance.

2 Workshop Principle Objective

Derivative-free and blackbox optimization have made massive advances over the past decade and, in our opinion, represent one of the most rapidly expanding fields of nonlinear optimization research. We also feel that DFO and BBO represent one of the most important areas in nonlinear optimization for solving future applications in real-world problems. The foundational concepts in DFO and BBO have become sufficiently mature that it is now possible to teach them at a senior undergraduate level. The principle objective of this workshop was to provide Drs. Audet and Hare a week of focused activity to finalize a book titled "Derivative-free and blackbox optimization".

This book is designed to provide a clear grasp of the foundational concepts in DFO and BBO, in order to push these areas into the mainstream of nonlinear optimization.

The book is targeted at two broad audiences, and hope that both will find value. The first audience is individuals interested in entering (or better understanding) the fascinating world of DFO and BBO. The sec-

ond audience is practitioners who have real-world problems to solve that cannot be approached by traditional gradient based methods.

The book does not present the absolute state-of-the-art in modern algorithms and theory. Instead it focuses on the foundational material required to understand and appreciate the state-of-the-art in DFO and BBO. To this end, in addition of studying several optimization methods, this book includes an elementary introduction to parts of nonsmooth analysis that have proved useful in conceiving and analyzing the methods within. The book also presents rigorous convergence theory for the algorithms in a way suitable for students in the mathematical sciences or in engineering.

In the past, practitioners faced with BBO problem have often fallen back on *ad hoc* methods, resulting in a plethora of papers publishing incremental improvements to solution quality. The methods covered in the book have proven convergence results, mathematically supported stopping criterion, and a track-record of practical success. Yet, for all of this, the methods are nonetheless easy to use and elegant in their simplicity. We hope the book can provide better and more consistent starting point for future applications.

3 Workshop Side Objectives

Development of the book mentioned in the principle objective has prompted several new research directions for Drs. Audet and Hare. The side objective of this workshop was to continue discussion and collaboration of these questions.

4 Outcome of the Meeting

A final version of "Derivative-free and blackbox optimization" was submitted to Springer on June 9th, 2017 (the last day of the workshop). The preface of the final version includes the much deserved line

> Special thanks to the Banff International Research Station (BIRS) for hosting us during the final stages of completing this book.

It is therefore clear that the principle objective of the workshop was accomplished.

The side objective of the workshop was addressed through several discussions about current projects and students. Notable progress was made in *how to approximate a subdifferential using numerical directional derivatives* and *analysis of the quality of regression gradient approximation*. Both problems are works in progress, but likely to result in papers in the near future.

References

[1] C. Audet and W. Hare, *Derivative-free and blackbox optimization*, in press, Springer, Springer-Verlag, New York.