New Directions in Iwasawa Theory (16w5043)

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1   Objectives

A main goal of the BIRS workshop "New Directions in Iwasawa Theory" was to prepare the ground for anticipated significant scientific developments in number theory, and more specifically in Iwasawa theory. Iwasawa theory is undergoing rapid change in several directions, so one of the aims of the workshop was to map out some primary new objectives for Iwasawa theory.

The workshop brought together researchers and advanced Ph.D. students working on Iwasawa theory and related topics in K-theory, commutative algebra and modular forms.

Two specific objectives of the workshop were to

 (i) discuss recent advances in Iwasawa theory, and

(ii) develop deeper ties between Iwasawa theory and algebraic K-theory.

The workshop included researchers with a wide range of expertise in number theory, algebraic K-theory, homological algebra and arithmetic geometry. It also introduced young researchers to previous work, as well as to new directions for research in these areas.

The workshop was timely in view of the recent proofs of a number of Main Conjectures about first Chern classes. These include the non-commutative Main Conjecture over totally real fields in 2010, the final form of the commutative Main Conjecture over imaginary quadratic fields in 2011, and recent progress on Main Conjectures relating to modular forms.

Besides the generous support from the Banff International Research Station, this workshop was also supported by a Focused Research Group award from the National Science Foundation concerning Iwasawa theory and Chern classes. The P.I.'s on this NSF FRG award were the organizers (Bleher, Chinburg, Greenberg, Pappas and Sharifi) and the consultants on the FRG grant were Mahesh Kakde and Martin J. Taylor.

The workshop had 41 participants and eighteen scheduled 50-minute talks.

2   Talks

The 50-minute talks of the workshop gave insights and results related to recent advances in Iwasawa theory. In the following we give a description of each talk, in the order in which the talks were given.

2.1   Ted Chinburg: Iwasawa theory in higher codimension.

In this talk, Chinburg gave a survey of recent work on higher codimension Iwasawa theory, which is joint work with Frauke Bleher, Ralph Greenberg, Mahesh Kakde, George Pappas, Romyar Sharifi and Martin Taylor [2]. This work has to do with relating p-adic L-functions to the behavior of Iwasawa modules that are supported in codimension larger than one as modules for an Iwasawa algebra. One idea, which was discussed in this talk, is that the natural analytic invariants arising from Katz p-adic L-functions pertain to the derived top exterior powers of Iwasawa modules. For first Chern classes, passing to the derived top exterior power makes no difference, but for higher Chern classes it does. This is analogous to the fact that Stark's conjectures pertain to regulators rather than to the individual entries of matrices whose determinants are regulators.
For the video of the talk, see [10].

2.2   Cristian Popescu: Towards module structure in classical Iwasawa theory.

In this talk, Popescu discussed aspects of his recent joint work with Corey Stone on higher Fitting ideals of various Iwasawa modules. Given a commutative Noetherian ring R and a finitely generated R-module M, one looks at a presentation $R^m \xrightarrow{A} R^n \to M \to 0$ and defines the ith Fitting ideal $\text{Fit}_R^i(M)$ to be the ideal generated by the determinants of all $(n-i) \times (n-i)$ minors of A. If R is a PID then the Fitting ideals $\text{Fit}_R^i(M)$, $0 \le i \le n$, determine M up to isomorphism. However, if $R = \Lambda = \mathcal{O}[[t]]$ contains $\mathbb{Z}_p$, then this is not true in general. Popescu discussed a conjecture of Kurihara [21, 22] about higher Fitting ideals in this context.
For the video of the talk, see [26].

2.3   Samit Dasgupta: On the Gross-Stark Conjecture.

In this talk, Dasgupta discussed joint work with Mahesh Kakde and Kevin Ventullo [14]. In the 1980's, Gross conjectured [17] a formula for the expected leading term at $s = 0$ of the Deligne–Ribet p-adic L-function associated to a totally even character ψ of a totally real field F. The conjecture states that after scaling by $L(\psi\omega^{-1}, 0)$, this value is equal to a p-adic regulator of units in the abelian extension of F cut out by $\psi\omega^{-1}$. In this talk, Dasgupta described a proof of Gross's conjecture.
For the video of the talk, see [13].

2.4   Francesc Castella: Λ-adic Gross-Zagier formula for elliptic curves at super-singular primes.

In this talk, Castella discussed joint work with Xin Wan [9]. In 2013, Kobayashi proved an analogue of Perrin-Riou's p-adic Gross-Zagier formula for elliptic curves at supersingular primes [20]. Castella explained an extension of Kobayashi's result to the Λ-adic setting. The main formula he presented is in terms of plus/minus Heegner points up the anticyclotomic tower. He also discussed the proof of this formula, which is via Iwasawa theory, based on the connection between Heegner points, Beilinson-Flach elements and their explicit reciprocity laws, rather than on calculations inspired by the original work of Gross-Zagier.
For the video of the talk, see [8].

2.5 Bharathwaj Palvannan: On Selmer groups and factoring p-adic L-functions.

In this talk, Palvannan first discussed the construction by Haruzo Hida of a 3-variable Rankin Selberg p-adic L-function. Two of the variables of this L-function are "weight" variables and one of its variables is the "cyclotomic" variable. He then pointed out that Samit Dasgupta [12] has factored a certain restriction of this 3-variable p-adic L-function (when the two weight variables are set equal to each other) into a product of a 2-variable p-adic L-function (related to the adjoint representation of a Hida family) and the Kubota-Leopoldt p-adic L-function. Palvannan then discussed his proof of the corresponding result involving Selmer groups that is predicted by the main conjectures [24]. A key technical input in this proof is the study of the (height one) specialization of Selmer groups.
For the video of the talk, see [25].

2.6 Haruzo Hida: Ring theoretic properties of Hecke algebras and cyclicity in Iwasawa theory.

In this talk, Hida showed that one can formulate a certain Gorenstein property of subrings of the universal deformation ring (i.e., the corresponding Hecke algebra) as a condition almost equivalent to the cyclicity of the Iwasawa module over $\mathbb{Z}_p$-extensions of an imaginary quadratic field if the starting residual representation is induced from the imaginary quadratic field. He discussed this connection in some details.
For the video of the talk, see [18].

2.7 Masato Kurihara: Iwasawa theory and Rubin-Stark elements.

In this talk, Kurihara talked about joint work with David Burns and Takamichi Sano [3]. More precisely, he presented an Iwasawa theoretic discussion of Rubin-Stark elements and zeta elements. Moreover, he discussed equivariant main conjectures and their consequences.
For the video of the talk, see [23].

2.8 Malte Witte: On zeta-isomorphisms and main conjectures.

In this talk, Witte explained first the setup of the zeta-isomorphism conjecture of Fukaya and Kato, which is a generalization of the equivariant Tamagawa number conjecture. He then turned to the noncommutative main conjecture for totally real fields and discussed a unicity result for the noncommutative zeta functions constructed by Kakde. Finally, Witte explained how this unicity result can be used to construct zeta-isomorphisms in the sense of Fukaya and Kato. See [34].
For the video of the talk, see [35].

2.9 Kazım Büyükboduk: On the anticyclotomic main conjectures for modular forms.

In this talk, Büyükboduk reported on recent joint work with Antonio Lei [5, 6] on the anticyclotomic Iwasawa theory of the base change of an elliptic modular form to an imaginary quadratic field K in which the prime p splits. Büyükboduk and Lei treated both the definite and indefinite cases in both p-ordinary and non-p-ordinary situations. One of their main results is an equality (up to powers of p) that is predicted by the main conjectures in the definite p-ordinary set up and a Λ-adic Birch and Swinnerton-Dyer formula in the indefinite case.
For the video of the talk, see [7].

2.10 Florian Sprung: The main conjecture for elliptic curves at non-ordinary primes.

In this talk, Sprung explained the proof of the main conjecture for elliptic curves at non-ordinary primes. More precisely, suppose E is an elliptic curve over $\mathbb{Q}$ and that $p > 2$ is a prime of good reduction. In the ordinary main conjecture one assumes that p does not divide a_p, whereas in the non-ordinary main conjecture the assumption is that p divides a_p. Sprung's results generalize work of Wan [33], who worked under the assumption that $a_p = 0$.
For the video of the talk, see [29].

2.11 Karl Rubin: Heuristics for the growth of Mordell-Weil ranks in big extensions of number fields.

In this talk, Rubin reported on joint work with Barry Mazur. He discussed some heuristics for modular symbols, and consequences of those heuristics for Mordell-Weil ranks. For example, these heuristics predict that every elliptic curve over $\mathbb{Q}$ has finite Mordell-Weil rank over the $\hat{\mathbb{Z}}$-extension of $\mathbb{Q}$.
For the video of the talk, see [27].

2.12 Otmar Venjakob: Wach modules, regulator maps and ε-isomorphisms in families.

In this talk, Venjakob discussed joint work with Rebecca Bellovin on the "local ε-isomorphism" conjecture of Fukaya and Kato for (crystalline) families of $G_{\mathbb{Q}_p}$-representations. This can be regarded as a local analogue of the global Iwasawa main conjecture for families, extending earlier work of Kato for rank one modules, of Benois and Berger for crystalline representations with respect to the cyclotomic extension, as well as of Loeffler, Venjakob and Zerbes for crystalline representations with respect to abelian p-adic Lie extensions of $\mathbb{Q}_p$. Nakamura has shown Kato's conjecture for (φ, Γ)-modules over the Robba ring, which means in particular only after inverting p, for rank one and trianguline families. The main ingredient of (the integrality part of) the proof consists of the construction of families of Wach modules generalizing work of Wach and Berger and following Kisin's approach via a corresponding moduli space.
For the video of the talk, see [30].

2.13 Olivier Fouquet: Congruences between motives and congruences between values of L-functions.

In this talk, Fouquet discussed the following question: If two motives are congruent, is it the case that the special values of their respective L-functions are congruent? Or, more precisely: Can the formula predicting special values of motivic L-functions be interpolated in p-adic families of motives? Fouquet explained how the formalism of the Weight-Monodromy filtration for p-adic families of Galois representations sheds light on this question (and suggests a perhaps surprising answer). See [15].
For the video of the talk, see [16].

2.14 Peter Schneider: Rigid character groups, Lubin-Tate theory, and (φ, Γ)-modules.

In this talk, Schneider described joint work with Laurent Berger and Bingyong Xie [1]. Namely, for a finite extension L of $\mathbb{Q}_p$, they formulate a new theory of (φ, Γ)-modules whose coefficient ring is the ring of holomorphic functions on the rigid character variety of the additive group o_L, resp. a "Robba" version of it.
For the video of the talk, see [28].

2.15   Mirela Çiperiani: Local points of supersingular elliptic curves on $\mathbb{Z}_p$-extensions.

In this talk, Çiperiani first discussed work of Kobayashi and Iovita-Pollack, which describes how local points of supersingular elliptic curves on ramified $\mathbb{Z}_p$-extensions of $\mathbb{Q}_p$ split into two strands of even and odd points. She then discussed a generalization of this result to $\mathbb{Z}_p$-extensions that are localizations of anticyclotomic $\mathbb{Z}_p$-extensions over which the elliptic curve has non-trivial CM points. See [11].

2.16   Ming-Lun Hsieh: Hida families and triple product p-adic L-functions.

In this talk, Hsieh presented a construction of the three-variable p-adic L-function attached to the triple product of three Hida families. This p-adic L-function is a three-variable power series with p-integral coefficients interpolating central L-values of triple product L-functions in the balanced case. He gave the explicit interpolation formula at all critical specializations and discussed some problems on this p-adic L-function.
For the video of the talk, see [19].

2.17   Ashay Burungale: On $\mathfrak{p}$-anticyclotomic Iwasawa theory.

In this talk, Burungale presented joint work with Haruzo Hida. Let F be a totally real field, and et p be an odd prime unramified in F and $\mathfrak{p}$ a prime above p. Let K/F be a p-ordinary CM quadratic extension and $K_{\mathfrak{p}}^-$ the maximal p-anticyclotomic extension of K unramified outside $\mathfrak{p}$. Burungale discussed results on the μ-invariant of certain p-adic L-functions over K along the $\mathfrak{p}$-anticyclotomic tower. He also described relevant questions regarding the $\mathfrak{p}$-anticyclotomic Selmer groups.
For the video of the talk, see [4].

2.18   Preston Wake: Ordinary pseudorepresentations, modular forms and Iwasawa theory.

In this talk, Wake discussed joint work with Carl Wang Erickson [31]. Pseudorepresentations appear naturally in the context of modular forms that are congruent to Eisenstein series. Wake talked about the difficulties that arise when defining "ordinary pseudorepresentation", and how to resolve these difficulties. He explained how the deformation theory of pseudorepresentations is related to cyclotomic Iwasawa theory and the geometry of the ordinary eigencurve.
For the video of the talk, see [32].

3   Schedule

The schedule of the workshop allowed for ample discussions among the participants. This was appreciated by everyone. On the last day of the conference, there were two scheduled one-hour discussion sessions, which were used by many participants to work in groups on old and new projects.

Sunday, June 26

16:00	Check-in begins
	(Front Desk - Professional Development Center - open 24 hours)
17:30 - 19:30	Buffet Dinner, Sally Borden Building
19:30	Informal gathering (Corbett Hall, 2nd floor lounge)

Monday, June 27

7:00 - 8:45	Breakfast
8:45 - 9:00	Introduction and welcome by BIRS station manager
9:00 - 9:50	Chinburg: *Iwasawa theory in higher codimension.*
10:00 - 10:30	Coffee break
10:30 - 11:20	Popescu: *Towards module structure in classical Iwasawa theory.*
11:30 - 13:00	Lunch
13:00 - 13:50	Guided Tour of The Banff Centre
	(meet in the 2nd floor lounge, Corbett Hall)
13:50 - 14:00	Group Photo (meet in foyer of TCPL)
14:00 - 14:50	Dasgupta: *On the Gross-Stark Conjecture.*
15:00 - 15:30	Coffee break
15:30 - 16:20	Castella: *Λ-adic Gross-Zagier formula for elliptic curves at supersingular primes.*
16:40 - 17:30	Palvannan: *On Selmer groups and factoring p-adic L-functions.*
17:45 - 19:30	Dinner

Tuesday, June 28

7:00 - 9:00	Breakfast
9:00 - 9:50	Hida: *Ring theoretic properties of Hecke algebras and cyclicity in Iwasawa theory.*
10:00 - 10:30	Coffee break
10:30 - 11:20	Kurihara: *Iwasawa theory and Rubin-Stark elements.*
11:30 - 13:30	Lunch
14:00 - 14:50	Witte: *On zeta-isomorphisms and main conjectures.*
15:00 - 15:30	Coffee break
15:30 - 16:20	Büyükboduk: *On the anticyclotomic main conjectures for modular forms.*
16:40 - 17:30	Sprung: *The main conjecture for elliptic curves at non-ordinary primes.*
17:45 - 19:30	Dinner

Wednesday, June 29

7:00 - 9:00	Breakfast
9:00 - 9:50	Rubin: *Heuristics for the growth of Mordell-Weil ranks in big extensions.*
10:00 - 10:20	Coffee break
10:20 - 11:10	Venjakob: *Wach modules, regulator maps and ε-isomorphisms in families.*
11:30 - 12:20	Fouquet: *Congruences between motives and values of L-functions.*
12:30 - 13:30	Lunch
	Free Afternoon
17:30 - 19:30	Dinner

Thursday, June 30

7:00 - 9:00	Breakfast
9:00 - 9:50	Schneider: *Rigid character groups, Lubin-Tate theory, and (φ, Γ)-modules.*
10:00 - 10:30	Coffee break
10:30 - 11:20	Çiperiani: *Local points of supersingular elliptic curves on $\mathbb{Z}_p$-extensions.*
11:30 - 13:30	Lunch
14:00 - 14:50	Hsieh: *Hida families and triple product p-adic L-functions.*
15:00 - 15:30	Coffee break
15:30 - 16:20	Burungale: *On $\mathfrak{p}$-anticyclotomic Iwasawa theory.*
16:40 - 17:30	Wake: *Ordinary pseudorepresentations, modular forms & Iwasawa theory.*
17:45 - 19:30	Dinner

Friday, July 1

7:00 - 9:00	Breakfast
9:00 - 10:00	Discussions
10:00 - 10:30	Coffee break
10:30 - 11:30	Discussions
11:30 - 12:00	Checkout by Noon (Front Desk - Professional Development Centre)
12:00 - 13:30	Lunch

4 Participants

The participants of the workshop and their affiliations at the time of the workshop were as follows.

Adebisi Agboola	University of California, Santa Barbara
Frauke Bleher	University of Iowa
Athanasios Bouganis	Durham University
Ashay Burungale	Univeristy of Arizona
Kazım Büyükboduk	Koc University of Istanbul
Francesc Castella	University of California, Los Angeles
Ted Chinburg	University of Pennsylvania
Mirela Çiperiani	University of Texas, Austin
Samit Dasgupta	University of California, Santa Cruz
Olivier Fouquet	University Paris-Sud
Ralph Greenberg	University of Washington
Robert Harron	University of Hawaii, Manoa
Haruzo Hida	University of California, Los Angeles
Ming-Lun Hsieh	National Taiwan University
Jennifer Johnson-Leung	University of Idaho
Mahesh Kakde	King's College London
Chan-Ho Kim	Korea Institute for Advanced Study
Manfred Kolster	McMaster University
Masato Kurihara	Keio University
Jack Lamplugh	University College London
Antonio Lei	Laval University
Bharathwaj Palvannan	University of Washington
Georgios Pappas	Michigan State University
Gautier Ponsinet	Laval University
Cristian Popescu	University of California, San Diego
Sujatha Ramdorai	University of British Columbia

Karl Rubin	University of California, Irvine
Takamichi Sano	Osaka University
Peter Schneider	Mathematisches Institut Muenster
Romyar Sharifi	University of Arizona
Florian Sprung	Princeton University
Martin Taylor	Oxford University
Sebastian Tomaskovic-Moore	University of Pennsylvania
Fabien Trihan	Sophia University in Tokyo
Maria Valentino	King's College London
Vinayak Vatsal	University of British Columbia
Otmar Venjakob	Heidelberg University
Preston Wake	University of California, Los Angeles
Jun Wang	University of Arizona
Carl Wang-Erickson	Brandeis University
Malte Witte	Heidelberg University
Yasin Zaehringer	King's College London

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[27] K. Rubin. Heuristics for the growth of Mordell-Weil ranks in big extensions of number fields.
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http://videos.birs.ca/2016/16w5043/201606281641-Sprung.html

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