Women in Noncommutative Algebra and Representation Theory (WINART) 2016

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This report summarizes the organization, presentation highlights, and scientific progress made at the first workshop for women in noncommutative algebra and representation theory at the Banff International Research Station in Banff, Canada. The workshop had 42 participants from 10 countries (Argentina, Australia, Austria, Brazil, Canada, France, Germany, Israel, USA, UK) in varying stages of their career, all with strong connections to the research themes of the workshop. The workshop featured several 45-minute introductory talks from world experts in noncommutative algebra and representation theory and a panel discussion on collaboration, while the rest of the time was dedicated to (group) research activities. Overall, this workshop was a great success and we look forward to having follow-up events.

1 Objectives

The goals of this workshop were the following.

- To have accessible introductory lectures by world experts in the themes of the workshop.
- To have each participant engaged in a stimulating research project and/ or be involved in a expansive research program in noncommutative algebra and/or representation theory.
- To have each participant provide or receive training toward this research activity (before and at the workshop) and to have made significant progress in such directions by the end of the workshop.
- To set-up mechanisms so that the collaborative research groups formed before/ at the workshop can continue research after the workshop, so that their findings will be published eventually.
- To provide networking opportunities and mentoring for its participants at and beyond the workshop.

2 Organization

In this section, we discuss the organization of WINART 2016 in great detail as organizers of future workshops at BIRS and other venues may be interested in this format.

In early July 2014, the organizers decided to submit a proposal to BIRS for the first workshop for women in noncommutative algebra and representation theory (WINART), modeled on past workshops at BIRS including women in number theory (WIN) and women in topology (WIT). Namely, the participants of the workshop were going to be split up into 7-8 research teams each led by two experts.

Soon after our proposal was accepted by BIRS (in mid-December 2014), we confirmed the participation of the group leaders and set up an external website

`https://math.temple.edu/events/winart/WINARTsite.html`

for our workshop. The website went live in mid-February 2015, and included

– the format of the workshop (later, this page contained the schedule of the workshop),
– descriptions of each group leaders' research interests and broad research topic,
– possible sources of funding, and
– an application for individuals to apply to participate in the workshop.

The process for selecting the non-leader participants was as follows. To ensure that we had a good number of top-notch graduate students and postdocs attending this workshop, we invited 10 of these individuals (with a wide range of interests) before the application was launched. Depending on the discretion of organizers of future WINART workshops, we may not do this next time in order to generate more open slots for applicants.

> It is very important that the organizers did not only invite women they knew, particularly as this is often how many women are excluded from conferences, workshops, and other research events as participants or speakers.

The participant application was available on the external website above, via a Google document; it requested the applicants full name, email address, university/ affiliation, position, year of Ph.D, top 3 choices of research group, reasons for research group preferences, and a description of previous work relevant to the themes of WINART. The site/ application was then heavily circulated through various email listserves for women in mathematics, algebraists, representation theorists, and social media. The deadline for the application was June 30, 2015; we received about 40 applicants for 12 open slots.

Soon after the application deadline, the organizers made use of the applicant data, including the preferences of the accepted junior participants, to form the research groups. The top priority was to ensure that each non-leader had a very strong interest/ connection to the leaders' research interests. Other factors for selecting participants included experience (we wanted each group to have a mix of junior, mid-career, and senior participants, if possible), location (preferably, the non-leaders should not already be at a leader's institution), and previous connections (we tried to avoid having junior members teamed up with their thesis or postdoc advisor, when possible). We also aimed to have a diverse collection of the geographic locations and the type of institutions from which the participants originate, and to have a significant number of members of racially/ culturally underrepresented groups.

> Even with all of the participants being women, the vast majority of these secondary goals was achieved without sacrificing our top priority of having first-rate mathematicians, mathematical training and research activity occur at our workshop.

The participants were notified of their group placement and received an introductory email in mid-July 2015; this email included the following information.

- Names and links to website of the group members.
- Timing/ mechanics of the group. (Leaders were encouraged to send out reading materials and non-leaders were encourage to meet regularly to discuss these materials before the workshop. But these were only suggestions, as everyone works differently.)
- Tentative schedule.
- Notification that participants must accept an official invitation from BIRS.
- Travel, accommodation, child care (which is exceptional at BIRS), and funding information.

Moreover, due to the overwhelming interest in the workshop, in July 2015 we also launched the website

`http://women-in-ncalg-repthy.org/`

to serve as a networking resource for women in noncommutative algebra and representation theory. Applicants who were not chosen as workshop participants were encouraged to join the site.

Closer to the workshop, the organizers needed to replace 3 participants with alternates (which worked out well, as there was a long list of applicants). Also, two research groups (led by Balagovic and Stroppel and by Gorelik and Serganova) merged into one 'supergroup' (pun intended), which again, worked out very well because they have common research goals.

The format of the workshop included introductory talks by some of the research leaders during the first two days, a panel discussion and a social activity at the end of the last full day, and the rest of the time was dedicated to group discussion/ research. The final schedule of the workshop is given below.

Sunday

19:30-21:00	Feel free to drop by and say Hello in Corbett Lounge

Monday

7:00-8:30	Breakfast
8:30-8:40	Introductory Remarks from BIRS Station manager
8:40-9:25	Anne Shepler
9:30-10:15	Susan Montgomery & Sonia Natale
10:15-12:15	Group Discussions 1 (with Tea 10:15-10:45)
12:15-13:30	Lunch
13:30-14:15	Rosa Orellana & Monica Vazirani
14:30-15:15	Karin Baur & Gordana Todorov
15:30-17:30	Group Discussions 2 (with Tea 15:20-15:50)
17:30-19:30	Dinner

Tuesday

7:00-8:30	Breakfast
8:45-10:15	Maria Gorelik & Vera Serganova & Catherina Stroppel
10:15-12:15	Group Discussions 3 (with Tea 10:15-10:45)
12:15-13:30	Lunch
13:30-15:30	Group Discussions 4 (with Tea 15:00-15:30)
15:30-16:15	Susan Sierra & Michaela Vancliff
16:30-17:15	Maria Redondo & Andrea Solotar
17:30-19:30	Dinner

Wednesday

7:00-8:30	Breakfast
8:45-11:45	Group Discussions 5 (with Tea 10:15-10:45)
11:45	Group Photo (meet in TCPL foyer)
12:00-13:30	Lunch
13:30-17:00	Group Discussions 6 (with Tea 15:20-15:50)
17:30-19:30	Dinner

Thursday

7:00-8:30	Breakfast
8:45-11:30	Group Discussions 7 (with Tea 10:15-10:45)
11:30-13:30	Lunch
13:30-17:00	Group Discussions 8 (with Tea 15:20-15:50)
17:30-19:00	Dinner

19:00-19:30	Group Summaries from 3 groups
19:30-20:30	Panel (Pamela Harris, Ellen Kirkman, Gail Letzter, Monica Vazirani, moderated by Van Nguyen)
20:30-22:00	Social time at Corbett Hall

Friday

7:00-8:00	Breakfast
8:00-9:15	Group Summaries from the remaining 4 groups
10:15-10:45	Tea

The groups always had the option to adjust the group discussion slots as they saw fit; the time of Group Discussions 6 was changed for most groups, for example. Many of the research groups met in the evenings, as well, and there was some collaboration across groups.

BIRS offers several rooms for break-out discussions and the organizers had initially planned a rotating day schedule for rooms for group discussion so that every group had access to some of the bigger rooms that were available. But on the first day, the group leaders expressed interest in keeping their room (so that mathematics can be left on the board, for instance); the group discussion rooms were fixed for the week as follows:

201: Balagovic-Gorelik-Serganova-Stroppel research group
202 : Shepler-Witherspoon research group
101 : Sierra-Vancliff research group
102 : Benkart-Orellana research group
105 : Baur-Todorov research group
106 : Redondo-Solotar research group
107 : Montgomery-Natale research group

Corbett Hall Reading Room and Lounge were free.

3 Introductory Lectures

There were several introductory lectures given at the workshop; we provide a brief description of each below.

From the Balagovic-Gorelik-Serganova-Stroppel research group: There were two introductory talks from this group: “Lie Superalgebras” by Maria Gorelik and Vera Serganova and “Representation Theory via Categorification” by Catharina Stroppel.

In the Lie Superalgebras talk, Maria Gorelik first introduced the notion of a Lie superalgebra $\mathfrak{g} = \mathfrak{g}_0 \oplus \mathfrak{g}_1$ and the examples of $\mathfrak{gl}(m|n)$, $\mathfrak{osp}(m|2n)$, and strange Lie superalgebras $\mathfrak{p}(n)$ and $\mathfrak{q}(n)$. For classical Lie superalgebras (whose even part $\mathfrak{g}_0$ is a reductive Lie algebra), she then discussed the structure theory: questions of existence of nondegenerate invariant bilinear forms (such as supertrace on $\mathfrak{gl}(m|n)$), triangular decompositions, root systems, and even and odd reflections.

Vera Serganova then discussed some aspects of the representation theory of Lie superalgebras. Let $\mathcal{F}$ be the category of finite dimensional integrable representations of $\mathfrak{g}$, that is finite dimensional representations which are semisimple over $\mathfrak{g}_0$. $\mathcal{F}$ is a monoidal category and in general it is not semisimple. It has enough projectives and injectives. In the special case of $\mathfrak{g} = \mathfrak{gl}(m|n)$, by a theorem of Zou, $\mathcal{F}$ is a highest weight category with irreducible objects $L(\lambda)$ parametrized by pairs of increasing m-tuples and decreasing n-tuples of integers. The module $L(\lambda)$ can be realised as a quotient of an induced Kac module $K(\lambda)$. The question of calculating the multiplicities $[K(\mu) : L(\lambda)]$ for $\mathfrak{gl}(m|n)$ has been solved by Serganova 1996. using geometry, then reproved by Brundan 2003, and then further refined via weight diagrams by Brundan-Stroppel 2008. who finally obtained a complete description of the endomorphism ring of a projective generator using categorifications. It is known that for $\mathfrak{g} = \mathfrak{q}(n)$ (Penkov-Serganova 1998. and Brundan 2004.) and $\mathfrak{g} = \mathfrak{osp}(m|2n)$ (Gruson-Serganova 2010, Ehrig-Stroppel 2016), $\mathcal{F}$ is not a highest weight category. For $\mathfrak{g} = \mathfrak{p}(n)$, $\mathcal{F}$ is a highest weight category, with many questions about it still open (see [32]).

In the Representation Theory via Categorification talk, Catharina Stroppel explained how the combinatorics of weights for the Lie algebra $\mathfrak{gl}(a + b)$ and for the Lie superalgebra $\mathfrak{gl}(m|n)$ leads to several different levels of categorification. Let $\mathcal{C}$ be the parabolic integral category $\mathcal{O}$ for the parabolic subalgebra of block upper triangular matrices with blocks of sizes (a, b) in $\mathfrak{gl}(a+b)$, and $\mathcal{F}$ the category of finite dimensional $\mathfrak{gl}(m|n)$-modules semisimple over $\mathfrak{g}_0$. In both categories, the irreducible objects are quotients of standard (induced) modules, which are parametrized by highest weights: in case of $\mathcal{C}$, the parabolic Verma modules parametrized by pairs of increasing sequences of integers of lengths a and b, and in case of $\mathcal{F}$, the Kac modules parametrized by pairs of an increasing sequence of integers of length m and a decreasing sequence of integers of length n. Let $W = \mathbb{C}^{\mathbb{Z}}$. The weakest level of categorification is to establish the bijection between (the complexification of) the Grothendieck group $K_0(\mathcal{C})$ and the vector space $\Lambda^a W \otimes \Lambda^b W$, and (the complexification of) the Grothendieck group $K_0(\mathcal{F})$ and the vector space $\Lambda^m W \otimes \Lambda^n W^*$, both obtained by sending the class of the standard module of highest weight λ to the standard basis vector labelled by λ.

The second level of categorification, which describes more of the structure of these categories and not just the labeling of the objects, relies on the observation that in both $\mathcal{C}$ and $\mathcal{F}$, every indecomposable projective-injective module is a summand of $M \otimes V^d$, where M is a standard module, V is the vector representation, and d is large enough. Studying the functors $- \otimes V$ and its adjoint $- \otimes V^*$ on $\mathcal{C}$, it is possible to construct endofunctors $\mathcal{F}_i$ and $\mathcal{E}_i$ of $\mathcal{C}$ which categorify the action of the elements E_i, F_i of the Lie algebra $\mathfrak{gl}_{\mathbb{Z}}$ on the space $\Lambda^a W \otimes \Lambda^b W$. This is Brundan's weak categorification; by introducing a grading on $\mathcal{C}$, it is possible to consider this as a categorification of a $U_q(\mathfrak{gl}_{\mathbb{Z}})$-module.

The third level of categorification is based categorification, where one looks for nice bases of $K_0(\mathcal{C})$ and $K_0(\mathcal{F})$ corresponding to standard/ projective modules/ simple. For $\mathcal{C}$, this leads to different bases (standard / Kazhdan-Lusztig / dual Kazhdan-Lusztig) of the Hecke algebra. A similar construction is available for $\mathcal{F}$.

Finally, the fourth level is to consider morphisms between compositions of the functors $\mathcal{F}_i$ and $\mathcal{E}_i$. This is given by some affine Hecke algebra (work of Brundan-Stroppel).

For $\mathfrak{g} = \mathfrak{p}(n)$, the categorification results do not yet exist, and were part of the discussions of the week.

From the Baur-Todorov research group: We recall the description of cluster categories ([6], [8]) and cluster algebras ([14]) and their relation. One of the guiding principles in cluster theory is the concept of mutation. We explain the beautiful connection of both of them to triangulations of polygons and show how these are related with frieze patterns in the sense of Conway and Coxeter ([10, 11]). This leads to many natural questions, in particular, the question of describing the effect of mutations on friezes.

From the Benkart-Orellana research group: In Part I, Rosa Orellana discussed Schur-Weyl duality and its role in connecting the representation theory of two centralizer algebras. The main example was the Schur-Weyl duality between the partition algebra and the symmetric group algebra acting on tensor powers of the permutation module and how this can help explain open problems related to the Kronecker product.

In Part II, Monica Vazirani described connections between the space of class functions of the symmetric group and the ring of symmetric functions. The group algebra of the symmetric group can be identified with functions on S_n, and the center of the algebra can be identified with the space of class functions. When these are glued together over all n, the resulting space is isomorphic as a Hopf algebra to the ring of symmetric functions. Under this isomorphism, the functors of induction and restriction are intimately tied to the product and coproduct. The two spaces are also isomorphic as highest weight representations of $\mathfrak{sl}_\infty$. The refinement of restriction (resp. induction) that gives the action of the Chevalley generators of $\mathfrak{sl}_\infty$ can be constructed from the Jucys-Murphy operators. This idea can also be extended to other highest weight representations (of higher level).

From the Montgomery-Natale research group: In Part I, we recall some definitions and results on fusion categories over an algebraically closed field of characteristic zero. In particular we discussed group extensions and equivariantizations of fusion categories, and their relation with the notions of solvability and nilpotency of a fusion category, due to Etingof, Gelaki, Nikshych and Ostrik, and some connections with the extension theory of Hopf algebras. We discussed as well some known classification results for semisimple Hopf algebras of certain dimensions which admit few prime numbers in their factorizations, posing the question of obtaining related classification results using methods and techniques from classification of fusion categories.

In Part II, we introduced an invariant to help study fusion categories. This invariant, the Frobenius-Schur indicators, were originally defined for representations of a finite group over the complex numbers.

They were extended to finite-dimensional semisimple Hopf algebras by V. Linchenko and the speaker, and eventually to any pivotal fusion category by Ng-Schauenburg, using work of Kashina-Sommerhauser-Zhu. Ng-Schauenburg show that the indicators are gauge invariants, that is invariants of the fusion category under equivalence of categories. Thus it is important to find for which fusion categories the indicators are a complete set of invariants. This is true for Tambara-Yamagami categories, in which there is only one non-invertible simple object, by recent work of Basak-Johnson. Note an old result of Seitz says that if Rep(G) has only one non-invertible simple, then G is an extra-special 2-group.

From the Redondo-Solotar research group: María Julia Redondo explained in the first part of the talk the Gerstenhaber algebra structure of the Hochschild cohomology $HH^*(A)$ of an associative algebra A, as well as a problem related to the concrete description of the cup product and of the Gerstenhaber bracket. She listed some families of algebras (radical square zero [30], toupie [2], string [29]) for which this problem has been solved, or at least the Lie structure of the first Hochschild cohomology group has been described (monomial algebras [34]).

In the second part of the talk, Andrea Solotar described in detail a new method developed by Mariano Suárez-Álvarez [35] that can be used to compute the Lie action of $HH^1(A)$ on $HH^n(A)$ via the Gerstenhaber bracket.

From the Shepler-Witherspoon research group: An associative algebra satisfies a Poincaré-Birkhoff-Witt property when its homogeneous version coincides with its associated graded version. However, the homogeneous version of a given algebra is somewhat in the eyes of the beholder. It depends on the choice of generators for the ideal of relations. When generators are taken with appropriate degrees, the PBW property often allows us to identify the algebra with a graded deformation of the homogeneous version. We discuss examples of this phenomenon in combinatorics and representation theory. We then contrast in particular the Drinfeld Hecke algebras defined by Drinfeld with the graded affine Hecke algebras defined by Lusztig. We end with an example to show how a new kind of PBW deformation arises over fields of positive characteristic.

From the Sierra-Vancliff research group: In their opening talks, the group leaders Michaela Vancliff and Sue Sierra introduced workshop participants to projective algebraic geometry à la Artin, Tate and Van den Bergh. Vancliff's talk touched on the history and motivation for this area, including the ideas of *point module*, *line module* and *Artin-Schelter regular algebra*. Artin-Schelter regular algebras are often viewed as a non-commutative analog of the polynomial ring, and geometric methods have been used in classifying Artin-Schelter regular algebras of global dimension three. Sierra's talk focused on providing particular motivation for the study of point modules, outlining how they can be used–under appropriate restrictions–to construct the underlying ring and thus have provided a tool in the classification of Artin-Schelter regular rings. Sierra also described joint work with Špela Špenko, in which they used similar ideas to study certain "point-like" representations of the Witt algebra, an important infinite-dimensional Lie algebra.

4 Scientific Progress Made

4.1 Balagovic-Gorelik-Serganova-Stroppel research group on Lie superalgebras and categorification

Group members (leader):* **Martina Balagovic*** (Newcastle University), **Zajj Daugherty** (City College of New York), **Maria Gorelik*** (Weizmann Institute), **Iva Halacheva** (University of Toronto), **Johanna Hennig** (University of Alberta), **Mee Seong Im** (United States Military Academy), **Gail Letzter** (Department of Defense), **Emily Norton** (Kansas State University), **Bea Schumann** (University of Cologne), **Vera Serganova*** (University of California, Berkeley), **Catharina Stroppel*** (Universität Bonn).

Schur-Weyl duality for $\mathfrak{gl}_n$. Let V be the vector representation of the Lie algebra $\mathfrak{gl}_n$. The symmetric group S_d acts on $V^{\otimes d}$ by permuting the tensor factors, and this action commutes with the $\mathfrak{gl}_n$ action. The action of the transposition $s_{ij} \in S_d$ coincides with the action of the Casimir element $\Omega \in \mathfrak{gl}_n \otimes \mathfrak{gl}_n$ on the i-th and

j-th tensor factor, $s_{ij} = \Omega_{ij}$. This gives a map of algebras

$$\mathbb{C}[S_d] \to \mathrm{End}_{\mathfrak{gl}_n}(V^{\otimes d}).$$

The classical Schur-Weyl duality theorem states that this map is always surjective, and is injective if $n \geq d$.

Higher Schur-Weyl duality for $\mathfrak{gl}_n$. Let M be any representation of $\mathfrak{gl}_n$, and consider the analogous problem for the module $M \otimes V^{\otimes d}$. Label the tensor factors as $0, 1, 2, \ldots, d$. The degenerate affine Hecke algebra H_d of S_d now acts on $M \otimes V^{\otimes d}$ (see [1]): the action of transpositions in S_d is given as above by $s_{ij} = \Omega_{ij}$, and the action of $x_i \in H_d$ is given by $x_i = \sum_{j=0}^{i-1} \Omega_{ji}$. This action commutes with the action of $\mathfrak{gl}_n$, and hence gives a map of algebras $H_d \to \mathrm{End}_{\mathfrak{gl}_n}(M \otimes V^{\otimes d})$. For $p(x_1)$ the minimal polynomial of the action of x_1, the map factors through the quotient and gives a map of algebras

$$H_d / \langle p(x_1) \rangle \to \mathrm{End}_{\mathfrak{gl}_n}(M \otimes V^{\otimes d}).$$

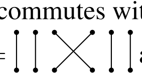
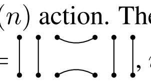
In general this map is neither surjective nor injective, except for special choices of M (eg. a certain projective-injective generator of a block of a parabolic category $\mathcal{O}$, see [4] Theorem B, and [5] Corollary 8.6).

In the special case of $M = \mathbb{C} =$ the trivial representation, the action of x_1 is by 0, the action of x_i is by Jucys-Murphy elements in $\mathbb{C}[S_d]$, $p(x_1) = x_1$, and the map reduces to the classical Schur-Weyl duality map

$$H_d / \langle x_1 \rangle \cong \mathbb{C}[S_d] \to \mathrm{End}_{\mathfrak{gl}_n}(\mathbb{C} \otimes V^{\otimes d}).$$

Schur-Weyl duality for $\mathfrak{p}(n)$. Consider the strange Lie superalgebra

$$\mathfrak{p}(n) = \left\{ \begin{bmatrix} A & B \\ C & -A^{\tau} \end{bmatrix} \mid A, B, C \in \mathrm{Mat}_{n \times n}(\mathbb{C}),\ B^{\tau} = B,\ C^{\tau} = -C \right\},$$

and let $V = \mathbb{C}^{2n}$ be its natural representation. The paper [24] constructs an action of the signed Brauer algebra $\mathrm{Br}_d^{\mathfrak{p}}$ (see [19], [3]) on $V^{\otimes d}$, which commutes with the $\mathfrak{p}(n)$ action. The signed Brauer algebra $\mathrm{Br}_d^{\mathfrak{p}}$ is a certain diagram algebra generated by $s_i =$  and $e_i =$ , $i = 1, \ldots, d-1$, which agrees with the classical Brauer algebra over $\mathbb{F}_2$. The action on $V^{\otimes d}$ is given by s_i and e_i acting on the i-th and $i+1$-st tensor factor as the super permutation $s(v \otimes w) = (-1)^{\bar{v}\bar{w}} w \otimes v$ and $e : V \otimes V \xrightarrow{ev} \mathbb{C} \xrightarrow{coev} V \otimes V$. The map

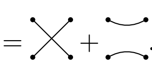
$$\mathrm{Br}_d^{\mathfrak{p}} \to \mathrm{End}_{\mathfrak{p}(n)}(V^{\otimes d})$$

is an isomorphism if n is large enough in comparison with d.

Higher Schur-Weyl duality for $\mathfrak{p}(n)$. The question we want to answer is:

Can we describe $\mathrm{End}_{\mathfrak{p}(n)}(M \otimes V^{\otimes d})$ for V the vector representation of $\mathfrak{p}(n)$ and M some "nice" representation (e.g. the Kac module $K(\lambda)$ or $\tilde{K}(\rho)$)?

The strategy and progress so far:

- *Constructing the map* $\mathrm{Br}_d^{aff,\mathfrak{p}} \to \mathrm{End}_{\mathfrak{p}(n)}(M \otimes V^{\otimes d})$.
  - Define the affine signed Brauer algebra $\mathrm{Br}_d^{aff,\mathfrak{p}}$. (Should be a straightforward generalisation of the affine Brauer algebra and the signed Brauer algebra, generated by $\mathrm{Br}_d^{\mathfrak{p}}$ and a polynomial algebra $\mathbb{C}[y_1, \ldots, y_d]$.)
  - Let $\Omega^{\mathfrak{p}}$ be the equivalent of Casimir element. Show that $\Omega^{\mathfrak{p}}|_{V \otimes V} =$ . (Done).
  - Define the action of $\mathrm{Br}_d^{aff,\mathfrak{p}}$ on $M \otimes V^{\otimes d}$ by letting $\mathrm{Br}_d^{\mathfrak{p}}$ act as before, and setting $y_i = \sum_{j=0}^{i-1} \Omega_{j,i}^{\mathfrak{p}}$. Show that this satisfies the relations of $\mathrm{Br}_d^{aff,\mathfrak{p}}$ and commutes with the action of $\mathfrak{p}(n)$. For $M = \mathbb{C} =$ trivial, y_1 acts by 0 and the action of $\mathrm{Br}_d^{aff,\mathfrak{p}} / \langle y_1 \rangle$ coincides with the action of $\mathrm{Br}_d^{\mathfrak{p}}$ from before. (We have shown some relations, others should be straightforward.)

- Calculate the minimal polynomial p of the action of $y_1 = \Omega^{\mathfrak{p}}_{0,1}$ on $M \otimes V$. (To be done. It is enough to solve the problem for M=Kac module or $M = \tilde{K}(\rho)$.)
- This gives a map $\mathrm{Br}_d^{aff,\mathfrak{p}} / \langle p(y_1) \rangle \to \mathrm{End}_{\mathfrak{p}(n)}(M \otimes V^{\otimes d})$. (Follows immediately from above.)

- *Showing that the map* $\mathrm{Br}_d^{aff,\mathfrak{p}} / \langle p(y_1) \rangle \to \mathrm{End}_{\mathfrak{p}(n)}(M \otimes V^{\otimes d})$ *is injective for* $M = \tilde{K}(\rho)$.
  - Find a nice basis and the dimension of $\mathrm{Br}_d^{aff,\mathfrak{p}} / \langle p(y_1) \rangle$. (To be done.)
  - Using the explicit description of the action of the generators of $\mathrm{Br}_d^{aff,\mathfrak{p}} / \langle p(y_1) \rangle$ on $M \otimes V^{\otimes d}$, show that $\mathrm{Br}_d^{aff,\mathfrak{p}} / \langle p(y_1) \rangle \hookrightarrow \mathrm{End}_{\mathfrak{p}(n)}(M \otimes V^{\otimes d})$. (To be done.)

- *Showing that the map* $\mathrm{Br}_d^{aff,\mathfrak{p}} / \langle p(y_1) \rangle \to \mathrm{End}_{\mathfrak{p}(n)}(M \otimes V^{\otimes d})$ *is surjective for* $M = \tilde{K}(\rho)$.
  - Describe the composition series of $K(\lambda) \otimes V$ for any (suitable) λ. (To be done.)
  - Describe the composition series or a filtration by Kac modules of $M \otimes V^{\otimes d}$; or a recursive rule for deducing it from those for $M \otimes V^{\otimes d-1}$. (Should follow from the previous point.)
  - Find (or bound from above) $\dim \mathrm{End}_{\mathfrak{p}(n)}(M \otimes V^{\otimes d})$. (Using the previous point.)
  - Comparing $\dim \mathrm{Br}_d^{aff,\mathfrak{p}} / \langle p(y_1) \rangle$ and $\dim \mathrm{End}_{\mathfrak{p}(n)}(M \otimes V^{\otimes d})$, deduce that the injective algebra homomorphism between them is an isomorphism. (Follows immediately from above.)

4.2 Baur-Todorov research group on cluster categories

Group members (leader):* **Karin Baur*** (Universität Graz), **Eleonore Faber** (University of Michigan), **Sira Gratz** (Leibniz Universität Hannover), **Khrystyna Serhiyenko** (University of California, Berkeley), **Gordana Todorov*** (Northeastern University).

We study the omnipresent concept of mutation in cluster theory. There is a close link between cluster algebras and cluster categories. In type A, there is furthermore a link to triangulations of polygons which are naturally related to friezes of positive numbers as studied by Conway and Coxeter ([12], [10, 11]): clusters correspond to cluster-tilting objects and both correspond to triangulations of polygons. In particular, the initial cluster corresponds to the initial cluster-tilting object which corresponds to the initial triangulation which in turn corresponds to the 1's in the frieze.

There is a direct route from cluster categories (or cluster variables) to friezes via the specialized Caldero-Chapoton map (see [7] and [16])

$$M \mapsto \sum_{\underline{e}} \chi(Gr_{\underline{e}}(M))$$

where the sum is over all dimension vectors of submodules of M and $\chi(Gr_{\underline{e}}(M))$ is the Euler characteristic of the corresponding Grassmannian.

In our case, this amounts to understanding the submodule structure of the indecomposable module M. We have a formula for the number of submodules in terms of the shape of the module. The shape of M is determined by the position of the entries 1 in the frieze which is the same as the shifted projectives in the Auslander-Reiten quiver of the cluster category. Note that all the indecomposables are string modules. So we can write M as tuple $(k_1, \dots, k_m)$ of the lengths of its maximal uniserial submodules (read in order).

Results

- Let $s(M)$ be the number of submodules of M. Then we have

$$s(M) = \left(\sum_{j=0}^{m} \sum_{\substack{I \text{ admissible}, \\ |I| = m-j}} \prod_{i \in I} k_i \right) + 1$$

- Description of $s(M)$ in terms of various natural submodules and quotients.

- We combine the above formula with these descriptions and develop methods to determine all entries in the friezes by the positions of the 1's in the frieze.
- Using all of the above, we are then able to describe the effect of cluster mutations on friezes which was one of the goals of our project.

4.3 Benkart-Orellana research group on diagram algebras, tensor invariants, representation theory

Group members (leader):* **Georgia Benkart*** (University of Wisconsin-Madison), **Ana Ros Camacho** (Institut de Mathmatiques de Jussieu-Paris Rive Gauche), **Pamela Harris** (United States Military Academy), **Rosa Orellana*** (Dartmouth College), **Rebecca Patrias** (University of Minnesota), **Monica Vazirani** (University of California, Davis).

Background: Spherical functions for a group-subgroup pair (G, H) are idempotents (up to scalars) in the group algebra $\mathbb{C}[G]$ that determine the center of the centralizer algebra $\mathbb{C}[G]^H$ of H acting on $\mathbb{C}[G]$ by conjugation. When the restriction of the irreducible G-modules to H is multiplicity free, the nonzero spherical functions form an orthogonal basis for the centralizer algebra $\mathbb{C}[G]^H$. This is true for the symmetric group-subgroup pair $(\mathsf{S}_n, \mathsf{S}_{n-1})$ and for the alternating group-subgroup pair $(\mathsf{A}_n, \mathsf{A}_{n-1})$. The Jucys-Murphy elements, which are defined for $k = 2, 3, \ldots, n$ as a sum of transpositions by the formula $X_k = (1\ k) + (2\ k) + \cdots + (k-1\ k)$, belong to $\mathbb{C}[\mathsf{S}_n]^{\mathsf{S}_{n-1}}$ and play a critical role in the representation theory of S_n, as indicated by the following results:

• (Jucys) The center $\mathsf{Z}(\mathbb{C}[\mathsf{S}_n])$ of $\mathbb{C}[\mathsf{S}_n]$ is generated by the symmetric polynomials in the X_k, and the elements X_k satisfy the following polynomial identity in $\mathbb{C}[\mathsf{S}_n][t]$:

$$(*) \qquad (t + X_1)(t + X_2)\cdots(t + X_n) = \sum_{\sigma \in \mathsf{S}_n} t^{\text{number of cycles of } \sigma}\, \sigma, \qquad (X_1 = 0).$$

• (Okounkov-Vershik) The subalgebra of $\mathbb{C}[\mathsf{S}_n]$ generated by the centers $\mathsf{Z}(\mathbb{C}[\mathsf{S}_1]), \mathsf{Z}(\mathbb{C}[\mathsf{S}_2]), \ldots, \mathsf{Z}(\mathbb{C}[\mathsf{S}_n])$ is exactly the subalgebra generated by the Jucys-Murphy elements X_k.

• The standard Young tableaux of partition shape λ form a basis for the irreducible S_n-module labeled by λ and are eigenvectors for the Jucys-Murphy elements. Each tableau is uniquely determined by the eigenvalues of the X_k.

Group Investigations: The goal of the group discussions was to identify Jucys-Murphy elements for the pair $(\mathsf{A}_n, \mathsf{A}_{n-1})$ (or more generally for other pairs (G, H)) by finding elements that satisfy many (all) of the important properties above. These elements should live in $\mathbb{C}[\mathsf{A}_n]^{\mathsf{A}_{n-1}}$, hence be linear combinations of spherical functions, and should be able to separate the A_{n-1}-irreducible summands of an irreducible A_n-module by acting as a different scalar on each summand. Ultimately, the analogues of Jucys-Murphy elements for the Hecke algebras associated to the alternating groups having properties similar to the above ones would be desirable.

• Various candidates and their representation-theoretic properties were examined during the workshop and afterwards. Some seem promising.

• The spherical functions for A_n for values of $n \leq 6$ have been explicitly computed.

• Analogues of equation $(*)$ for A_n are being investigated, and work on all these items has continued since the BIRS WINART workshop.

4.4 Montgomery-Natale research group on fusion categories and semisimple Hopf algebras

Group members (leader):* **Luz Adriana Mejía Castaño** (Universidade federal de Santa Catarina-Brazil), **Susan Montgomery*** (University of Southern California, **Sonia Natale*** (Universidad Nacional de Córdoba and CIEM-CONICET), **Maria Vega** (United States Military Academy), **Chelsea Walton** (Temple University).

Let k be an algebraically closed field of characteristic 0 and Z_n denote the cyclic group of n elements. Let p and q be distinct prime numbers. The direction of our research is the classification of finite-dimensional Hopf algebras of specific dimensions (See Problems 1-3 below).

Pertaining to the semisimple case, there are important results of Etingof, Nikshych and Ostrik [13] on fusion categories of dimension $p^a q^b$, for $a, b > 0$, but the full classification of semisimple Hopf algebras of these dimensions is not known in the case where $a + b > 3$.

Here is a list of some known facts about semisimple Hopf algebras H:

- Hopf algebras of dimension p or p^2 are classified in [36, Theorem 2] and [23], and are commutative and co-commutative.
- Semisimple Hopf algebras of dimension p^3 are classified by Masuoka in [22, Theorem 3.1]. The non-commutative, non-co-commutative Hopf algebras are of the form $H = A_{\zeta,g}$ where ζ is a primitive p-th root of unity and g is a certain group-like element; these Hopf algebras are also self-dual. Furthermore, H arises from an exact sequence of Hopf algebras (an abelian cocentral extension, in particular). Therefore, such H are *group-theoretical*.
- Semisimple Hopf algebras of dimension 2^4 are classified by Kashina in [18]. Kashina uses the fact that the Galois objects over semisimple Hopf algebras of dimension 2^3 are trivial (cf. Problems 4 and 5 below) and that every nontrivial semisimple Hopf algebra of dimension 16 has a commutative Hopf subalgebra of dimension 8 [18, Theorem 9.1] . These algebras H also arise as abelian cocentral extensions.
- On the other hand, semisimple Hopf algebras of dimension $2^2 3^2$ do not necessarily arise via extension [15]. These algebras are semi-solvable up to cocycle deformation [26], and there exists non-group-theoretical examples [28]. (Compare to Problem 8 below.)
- Hopf algebras of dimension pq^2 are group-theoretical and are classified by Natale in [25]. However, there are examples of fusion categories of FP-dimension pq^2 which are not group-theoretical, see [17].

Problems that we are considering; the problems marked with (*) are long-term goals.

1. (*) Classification of semisimple Hopf algebras of dimension p^4: these can be obtained by central Z_p-extensions from k and are group-theoretical.
2. (*) Classification of semisimple Hopf algebras of dimension $p^3 q$, for p and q odd.
3. (*) Classification of semisimple Hopf algebras of dimension $p^2 q^2$. These are not necessarily group-theoretical; see Nikshych [28] for dimension $4p^2$. Moreover, these Hopf algebras are not all extensions.
4. Classification of Galois objects for semisimple Hopf algebras of dimension p^3.
5. Classification of Galois objects for semisimple Hopf algebras of dimension pq^2.
6. Study cocycle deformations/ coMorita equivalences of semisimple Hopf algebras of dimension p^3: In [31], Schauenburg, establishes a coMorita equivalence between two Hopf algebras H and L, via (L, H)-biGalois objects R, where all monoidal equivalences between the categories $\mathsf{Comod}(H)$ and $\mathsf{Comod}(L)$ are given by $R\square_H -$. So, this is related to Problem 4.
7. Study of cocycle deformations/ coMorita equivalences for semisimple Hopf algebras of dimension pq^2. For the reasons given above, this is related to Problem 5.
8. Study the conditions under which fusion categories of FP-dimension $p^2 q^2$ are group-theoretical, where p and q distinct odd primes.
9. Classify the pointed module categories over the fusion categories of Problem 8.
10. (*) Applications to physics.

4.5 Redondo-Solotar research group on Hochschild (co)homology

Group members (leader):* **Joanna Meinel** (Universität Bonn), **Van Nguyen** (Northeastern University), **Bregje Pauwels** (Australian National University), **Maria Redondo*** (Universidad Nacional del Sur), **Andrea Solotar*** (Universidad de Buenos Aires).

The aim of this group is to consider a particular family of algebras, compute their Hochschild cohomology and describe its structure as Gerstenhaber algebras, that is, the cup product and the Lie bracket. These structures were defined by Gerstenhaber using the bar resolution. When the Hochschild cohomology is computed using a more convenient projective resolution instead of the bar resolution, a general formula for these structures is not known yet. Clearly, one can use comparison morphisms between the convenient projective resolution and the bar resolution to relate these structures. However, in general these formulas are awkward and the computations are rather technically difficult. Our objective is to try two different approaches, developed in [27, 35].

We consider the algebras $A_{m,N} := \mathbf{k}Q/I$ over a field $\mathbf{k}$, where $Q = (Q_0, Q_1)$ is the quiver given by

$$Q_0 = \{1, \ldots, m\},$$

and

$$Q_1 = \{a_i : i \to i+1, \overline{a_i} : i+1 \to i\}_{i=1,\ldots,m},$$

that is, Q has m vertices and $2m$ arrows. The ideal I is generated by the relations

$$\{a_{i+1}a_i, \overline{a_i a_{i+1}}, (a_i\overline{a_i})^N - (\overline{a_{i+1}}a_{i+1})^N\}.$$

It is known that for $N = 1$, the algebra $A_{m,N}$ is Koszul. When applying the approaches [27, 35], we thus have some assurance knowing that we can always use the Koszul complex in our computations, though we haven't actually used the Koszulness yet. We first focus in the particular case $m = 3$ and $N = 1$. Let $A := A_{m=3,N=1}$. We construct an alternative projective resolution of A as an $A - A$-bimodule, which is easier to handle than the bar resolution. This resolution is given in terms of "n-ambiguities" A_n, that is, A_n is generated by words $s^n_{i,k}$ of length n containing exactly k arrows of form $\overline{a}$ and ending at either the vertex i (if $s^n_{i,k}$ ends with arrow $\overline{a_i}$) or the vertex $i+1$ (if $s^n_{i,k}$ ends with arrow a_i). Using this resolution, we compute the Hochschild cohomology groups of A in low degrees $HH^i(A),\ i = 0, 1, 2$, and we try to get results on the Lie brackets. Our first approach was to use the method introduced by Negron and Witherspoon [27], but we concluded that the needed computations were as difficult as in the comparison morphism method. Finally, we turned to the method introduced by Suárez-Álvarez [35]. We prove that $[HH^1(A), HH^1(A)] = 0$ and that any representative f of an element in $HH^1(A)$ can be characterized by two elements $C, \gamma \in \mathbf{k}$, such that the map

$$[f, -] : HH^n(A) \to HH^n(A)$$

has a natural basis of eigenvectors associated to eigenvalues involving C, γ and n.

After WINART at Banff, we will continue working on an explicit description of $HH^n(A)$ for all $n \geq 0$, as well as the cup product in relation with the Gerstenhaber bracket. Our future direction is to extend this work for cases $N = 1$ and $m > 3$, or more generally for any $N \geq 1$ and any $m \geq 1$.

4.6 Shepler-Witherspoon research group on algebraic deformations

Group members (leader):* **Sian Fryer** (University of Leeds), **Tina Kanstrup** (Hausdorff Center for Mathematics), **Ellen Kirkman** (Wake Forest University), **Anne Shepler*** (University of North Texas), **Sarah Witherspoon*** (Texas A&M University).

We examined algebraic deformations that arise from groups acting on quantum polynomial rings. The quantum polynomial ring $S_Q(V)$ is a noncommutative analog of the commutative polynomial ring $S(V)$ on a finite dimensional vector space V with each pair of indeterminates commuting up to a nonzero scalar. Specifically, let $x_1, \ldots, x_n$ be a basis of V and let $Q = \{q_{ij} \mid 1 \leq i < j \leq n\}$ where each q_{ij} is a nonzero element of the underlying field k. Then $S_Q(V)$ is the associative algebra generated by $x_1, \ldots, x_n$ subject to the relations $x_ix_j = q_{ij}x_jx_i$. Certain finite subgroups G of GL(V) act by graded automorphisms on the quantum polynomial ring $S_Q(V)$ giving rise to the natural semi-direct product algebras $S_Q(V) \rtimes G$. PBW deformations of $S_Q(V) \rtimes G$ are algebras of the form

$$\mathcal{H}_{Q,\kappa} = T_k(V) \rtimes G/(x_i x_j - q_{ij} x_j x_i - \kappa(x_i, x_j) \mid 1 \leq i < j \leq n)$$

where $T_k(V)$ is the tensor algebra of V over k and $\kappa : V \times V \to kG \oplus (V \otimes_k kG)$ is a bilinear quantum anti-symmetric map satisfying some additional conditions (see, e.g., [33]). These conditions are well understood and many examples are known in the classicial setting where all $q_{ij} = 1$; some examples are the Drinfeld Hecke algebras and graded affine Hecke algebras. We explored these conditions more generally, and investigated how different kinds of deformations impose different kinds of constraints on the group action.

We focused on analogs of universal enveloping algebras in particular. The original Poincare-Birkhoff-Witt theorem gives a canonical basis for the universal enveloping algebra $\mathcal{U}(V)$ of a Lie algebra V that identifies $\mathcal{U}(V)$ with a polynomial ring. We studied analogs of this theorem involving a quantum polynomial ring carrying the action of a finite linear group G by graded automorphisms. Some analogs correspond to new PBW deformations. We found a framework for identifying and understanding these algebras for some classes of examples. We discussed work towards a classification.

4.7 Sierra-Vancliff research group on noncommutative projective algebraic geometry

Group members (leader):* **Susan Sierra*** (University of Edinburgh), **Spela Spenko** (University of Edinburgh), **Michaela Vancliff*** (University of Texas at Arlington), **Padmini Veerapen** (Tennessee Tech University), **Emilie Wiesner** (Ithaca College).

During the workshop, the group focused on connections between Lie-theoretic structures and projective algebro-geometric structures. In particular, for a finite-dimensional Lie algebra $\mathfrak{g}$, consider the homogenization, $\mathcal{H}(\mathfrak{g})$, of the universal enveloping algebra of $\mathfrak{g}$ and its linear subspaces (i.e. point modules, line modules, etc). Building on the work of Le Bruyn and Smith [20] for $\mathfrak{sl}(2, \mathbb{C})$, Le Bruyn and Van den Bergh [21] showed that for any finite-dimensional Lie algebra $\mathfrak{g}$, a correspondence exists between d-linear subspaces associated to $\mathcal{H}(\mathfrak{g})$ and pairs $(\mathfrak{h}, f)$, where $\mathfrak{h}$ is a subalgebra of $\mathfrak{g}$ of codimension d and $f \in \mathfrak{h}^*$ such that $f \mid_{[\mathfrak{h},\mathfrak{h}]} = 0$.

The group's goal was to pursue possible extensions of these results to other Lie-theoretic settings, including Lie superalgebras and, more generally, color Lie algebras. Color Lie algebras (including Lie superalgebras) are graded by some abelian group G, and the techniques outlined in [20, 21] appeared likely to respect this grading. Thus, our central research question was to determine, for $\mathfrak{g}$ a Lie superalgebra or a color Lie algebra, links between G-graded subalgebras of $\mathfrak{g}$ and G-graded linear subspaces associated to $\mathcal{H}(\mathfrak{g})$.

The group began work in this direction by investigating the color Lie algebra $\mathfrak{sl}_2^c$ (cf. [9]) and the Lie superalgebra $\mathfrak{sl}(1 \mid 1)$. In the case of $\mathfrak{sl}_2^c$, we determined all subalgebras of $\mathfrak{sl}_2^c$; however, these subalgebras were not graded, in general, and thus do not appear to be connected in a clear way to the linear subspaces associated to $\mathcal{H}(\mathfrak{sl}_2^c)$.

In the case of $\mathfrak{sl}(1 \mid 1)$, the presence of nilpotent elements in the standard construction of $\mathcal{H}(\mathfrak{sl}(1 \mid 1))$ presented a challenge in adapting the ideas from [21]. To address this, we considered a modified version $\widetilde{\mathcal{H}}(\mathfrak{sl}(1 \mid 1))$ of the homogenized algebra, where the nilpotent relations were removed. We were able to establish a correspondence between the $\mathbb{Z}_2$-graded two-dimensional subalgebras of $\mathfrak{sl}(1 \mid 1)$ and the $\mathbb{Z}_2$-graded line modules of $\widetilde{\mathcal{H}}(\mathfrak{sl}(1 \mid 1))$. However, we believe that the presence of nilpotent elements cannot be handled in this way for Lie superalgebras in general, and thus different techniques (or research questions) will likely be needed in the general Lie-superalgebra setting.

5 Outcome of the Meeting

As described above, very significant scientific progress was made before and during the WINART workshop. Moreover, a number of common themes emerged across groups (e.g. involving certain homological, Lie-theoretic, diagrammatic techniques), which will be explored between members of different groups after the workshop. In any case, every group set concrete plans to continue research activities, and all look forward to stay in touch with their group members and other participants in the future. We close with some testimonials from the workshop participants.

“Organizers planned conference more than a year in advance contacting potential participants very early so that the workshop was on everyone’s calendar. Then stayed in frequent contact with participants after advertising and attracting large numbers of female mathematicians applicants. Better organized than most conferences by far!!!”

“I wasn’t sure how this would all work before coming here, but any expectations I had have been far exceeded. My research group is going quite well, and we likely will write a paper. The organization has been superb, just the right mix of organized activities and leaving the groups to themselves to get their work done. And best of all, everyone I’ve talked with seems to be having a great time as well as a very productive time.”

“It would be great to have a 2nd WINART [workshop] at BIRS. We’ve made a lot of progress on our project in these days. The environment was perfect: time to discuss, a few introductory lectures, [then] time to discuss, many hours. And [what] a great group! It helped that as group leaders we met up before to discuss possible projects.”

“It was great to be able to talk about family & pregnancy with worrying about people thinking that ‘she’s thinking of family– she’s not career-oriented enough’.”

“It was wonderful to see so many women of color participating. Although all participants were women, there was more cultural diversity than is usually present at research conferences I attend.”

“Lots of strong women in mathematics can’t travel much because of caring for aging parents or young children. But they make a point of coming to BIRS for women-centered workshops for opportunity to interact with leaders in the field while giving and getting support and encouragement. We are often invisible at other conferences! This workshop has been an exceptional mix of senior and junior researchers and resulted in new collaborations and the start of a new paper.”

“Extremely interested in WINART at BIRS in 2019!!!”

“This was a great workshop. The organizers did a wonderful job in putting this all together. Everything was well thought out, from the choice of topics, to the schedule that made it so easy for people to work with each other. It was nice to see that we could indeed have a workshop with just women and, at the same time, with such top notch mathematicians as group leaders and participants. Seeing all these impressive young women in algebra and representation theory was particularly heartwarming to me. I got a chance to work closely on mathematics with a few of them this past week and it was truly a blast. I am hoping that this cohort will find support with each other as they progress through their careers.

There were many things about the workshop that would work in general. I liked the idea of really learning new math and working on problems in this kind of collaborative environment. Everybody belonged to a group so all had a chance to participate. There were many networking opportunities. At the same time, it was clear how important it is to sometimes just have women get together. It gave us all an opportunity to share difficult stories of working in mostly male environments – something most of us are not comfortable doing at regular conferences and workshops. My sense was that this was just as important as the chance to do mathematics together.”

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