Interactive Information Theory

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The BIRS workshop 12w5119 was a great success despite it being the coldest week of the year, with temperatures between -20 and -40 celsius the entire week. The workshop topic was "Interactive Information Theory." It is important to note that this title and topic may be interpreted in a variety of ways, i.e. there is no standard definition of what constitutes "interactive" information theory. The goal of our workshop was to bring together researchers from different areas, each with their own interpretation of this topic, to interact.

1 Overview of the Field

Information theory, while a mathematical theory, has had a powerful impact on practical communication systems. While many ideas in information theory such as random code constructions, iterative decoding and maximum entropy principles were originally developed as solutions to well defined mathematical problems, interestingly, today's practical engineering solutions that approach fundamental limits, are closely related to these ideas. In the 60-odd years of information theory, the dominant focus and most successes have emerged in "one-way" problems in which information flows in a single direction. When information flows in two-directions, not only does establishing the fundamental limits of communication become more challenging due to the interactive or two-way nature of the problems, but the need for new "directional" information metrics themselves arise. As a result, due to their difficulty, interactive communication problems have received less attention and are less well understood than more classical one way problems. This is unfortunate, as interactive problems come closer to modeling true communication than do one-way models: in a human conversational dialogue, we naturally adapt our speed, tone, and content to account for the other party's reactions in a fully interactive fashion rather than speak in monologues. Communicating two-way data by treating it as two one-way data flows ignores the ability of a communication system to adapt to the received messages, to have the terminals *interact*. An interactive model and theory for communication is not only more relevant and general, but also of immediate use in current and future applications such as distributed data storage in data centers and two-way video-conferencing, tele-presence and tele-medicine applications. In the past years, significant progress in network coding, on interactive function computation and compression, interference management, particularly in two-way relaying scenarios, and a renewed interest in feedback and directed information and its applications, have given information theory new insight into interactive problems; various disconnected pieces of the interactive communication puzzle have emerged in different areas, what remains to be done is to piece them together. The time was ripe for a meeting bringing together active, young, and established experts in interactive communication to exchange recent results and determine promising avenues to tackle the fundamental problem of interactive communications.

While the term "interactive information theory" encompasses many concepts, the key ingredient which sets our topic apart from more classical information theory is the concept of "interaction", in which communication nodes adaptively change their behavior (encoding, decoding, compression, etc.). Specifically, nodes are said to interact if their next actions (which may for example be what message to send or how to encode them) are based on what they have received from the other terminal nodes, forming a type of "closed-loop" information exchange process. State of the art results in both source and channel coding surprisingly involve very simple interactive protocols, often involving only one or two rounds of communication. Part of the challenge lies in the fact that it is difficult to find computable (single-letter) rate expressions for more sophisticated interactive schemes or show their optimality.

2 Recent Developments and Open Problems

Given the breadth of this topic, we formed four rough themes for the workshop (which by no means exhaust the definition of interactive information theory – notably missing is the interplay of control, interaction and information theory as discussed by several of the speakers), each addressed on a different day.

2.1 Focus area 1: Interactive source coding and computing

Source coding, or data compression, involves the process of encoding information using fewer bits. The lowest number of compressed bits needed on average to represent an information source X, modeled as a random variable which generates a sequence of symbols $\{X_i\}$ according to the distribution $p(x)$, is given by that source's entropy $H(X)$. From this compressed version, the source may be reconstructed in either a lossy or a lossless manner. Work on interactive source coding considers variants of the basic problem in which a source which generates symbols (X_i, Y_i) according to a joint distribution $p(x, y)$. Symbols X_i are known to node N_x and symbols Y_i are known to node N_y. If one of the nodes, say N_y, wishes to noiselessly reconstruct X given one compressed version of it over a noiseless channel we obtain the celebrated "one-way" Slepian-Wolf result [1] which states that the minimal rate at which X may be compressed such that perfect reconstruction is possible at node N_y is $H(X|Y)$. If both nodes wish to reconstruct each others' messages, or if multiple rounds of messages may be exchanged in order for one node to reconstruct the others' source, we obtain a fully "two-way" problem. Both lossless extensions of Slepian Wolf's result [1] to this two-way source coding problem [2], which may take place over numerous interactive - and half-duplex - rounds led by the work of Orlitsky [3, 4, 5, 6], as well as lossy source coding with [7] or without [8, 9] a " common helper" - again using K rounds of interactive messages, have been obtained. In a related vein, motivated by sensor networks and database privacy, recent work has focused on functional compression [10], where the goal is to separately compress possibly correlated sources in order to compute an arbitrary function of these sources given the compressed data. Of particular interest are the techniques used in proving these results, and the results themselves which outline when and how interactive source coding is beneficial. For example, Ma and Ishwar's very recent result indicates that *interaction* is strictly beneficial [9] in reducing the amount of bits needed to be communicated in certain scenarios. The recent work of Braverman and Rao [11, 12, 13, 14] have looked at similar problems from a theoretical computer-science perspective. However, in general only bounds on various interactive source-coding schemes exist under various constraints exist; open problems abound as we are only starting to understand the role of interaction in communicating sources.

2.2 Focus area 2: Interactive channel coding and networking

Source coding's so-called dual is channel coding, in which determining and attaining the channel capacity is the goal. A one-way channel is characterized by the set of conditional probability distributions $p(y|x)$ between channel input x and channel output y. The channel capacity corresponds to the maximal rate (in bits/channel use) at which a message may be reliably, i.e. with probability of error $\to 0$ as the number of channel uses increases, be communicated between the source and the destination, and is given by the maximal mutual information $I(X; Y)$ between the channel inputs and outputs. The simplest extension of this point-to-point channel to a two-way scenario was first proposed by Shannon in 1961 [15], where he introduced the "two-way channel" $A \leftrightarrow B$ in which A and B wish to exchange messages over a common channel characterized by the set of conditional distributions $p(y_A, y_B|x_A, x_B)$. While capacity is known for specific channels, it remains unknown in general. This is surprising given the model's simplicity, and highlights our lack of understanding on how to communicate in an interactive fashion. The inherent difficulty lies in the fact that each source is also a destination in a two-way channel, and as such its channel input at channel use n may depend on its message w as well as all past received channel outputs y_1^{n-1}, i.e. $x_n(w, y_1^{n-1})$. While the capacity region of a two-way channel may be expressed in terms of *directed-information,* $I(\mathbf{X} \rightarrow \mathbf{Y})$ between channel inputs and outputs, a directional version of the classical mutual information $I(X;Y)$ expression taking into account the causal, channel output-dependent structure of the channel inputs $x_{1n} = f(w_1, y_1^{n-1})$, and $x_{2n} = g(w_2, y_2^{n-1})$ [15, 16, 17, 18, 19], this is often deemed unsatisfactory, as it is, with our current techniques, prohibitively complex to evaluate. Nonetheless, general inner and outer bounds [15, 18, 20, 21, 22, 23, 24] for the two-way channel exist. By bringing together active contributors to this area we intend to exchange recent results and determine possible directions in which to attack this problem. Two-way communications have very recently been extended to network scenarios [25, 26]; two-way channel coding problems over networks highlight the interplay between classical two-way problems and those seen in networks, including how to relay information, combine information from various paths, and deal with interference between multiple streams. For example, the two-way relay channel is the logical extension of the classical relay channel [20, 27, 28] - a channel in which a transmitter send a message to a receiver with the help of a designated third "relay" node - to allow for two messages to be exchanged between the two nodes with the help of a relay [29, 30, 31]. A large body of work has emerged (see for example [32, 33] and references therein). The capacity region is in general still unknown, but exact and performance guaranteeing (finite-gap, generalized degrees of freedom) capacity results have been derived under different channel conditions. Much remains to be done to fundamentally understand the interplay of two-way techniques with interference networks which extend upon the simple two-way and two-way relay channels. Interaction may also be used to denote channels in which nodes cooperate, or channels in which feedback in present: in both cases nodes are able to adapt their inputs based on other inputs/outputs in the network.

2.3 Focus area 3: Adaptive hypothesis testing

Yet another area of interactive information theory involves the extension of hypothesis testing – a statistical method of making decisions using (noisy) data – to dynamic, adaptive settings. Connections between information theory and hypothesis testing, or decision theory, have long been made (see [35] and references therein); recent developments however have focussed on *sequential* hypothesis testing scenarios where some degree of control or interaction is available. For example, rather than being *given* a set of measurements from which we will make a decision about the underlying hypothesis, one could ask how one should best select observations (or control the sensing) in a multi-hypothesis test, as very recently done in [36, 37]; this is interactive in the sense that one controls and dynamically may change where one senses (or what types of observations are received), and information theory and its metrics (e.g. Kullback-Leilber divergence) are often useful in obtaining bounds on the performance. In a similar spirit, one may consider information theoretic limits of "active learning" schemes as in [38, 39]. While sequential hypothesis testing has been studied, its active or interactive counterpart has only recently been considered and much remains to be theoretically understood.

2.4 Focus area 4: Interactive information security

Security is a core requirement in any networked infrastructure system. The purpose of this focus topic was to understand fundamental limits of secret-key generation between two parties using interactive communication. Interaction between the legitimate parties is a fundamental requirement in secure communication. It provides advantage to the legitimate terminals by virtue of their active participation in the protocol. An adversary, despite having significantly better resources (computation power, communication channel etc) cannot decode the share secret-key if he/she does not participate in the protocol. While the foundation of interactive secret-key generation was laid out almost 20 years ago [40], in recent years a significant progress has been made on this area, due to the applications to wireless systems.

3 Presentation Highlights

The workshop was structured as follows: each of four days had a different "theme", which was opened by one (or sometimes two) one hour tutorial-style lectures in the morning, followed by a series of 30 minute talks on related topics. For brevity, we only highlight several of the talks; and simply list the title and speaker for all others.

3.1 Day 1: Interactive source coding and computing

Tutorial talk: MARK BRAVERMAN, PRINCETON UNIVERSITY, *Tutorial on information complexity in interactive computing*. Mark opened up the workshop by discussing interactive computing from a computer-science perspective. In particular, he discussed several new extensions of information-theoretic notions to the two-way communication setting, which he used to prove a direct sum theorem for randomized communication complexity, showing that implementing k copies of a functionality requires substantially more communication than just one copy. More generally, he showed that information cost $I(f)$ can be defined as a natural fundamental property of a functionality f, measuring the amount of information that the parties need to exchange in order to compute f. He described several new tight connections between $I(f)$, direct sum theorems, interactive compression schemes, and amortized communication complexity.

This tutorial was especially interesting in the context of this workshop as Mark is a computer scientist (one of the few at the workshop) and hence provided a different perspective on problems similar to those considered by information theorists. It became evident that translating results (and even vocabulary and notions) between the information and computer science theory communities, enabling the communities to exchange results more readily, could advance the state of the art in interactive communication complexity.

Tutorial talk: BABAK HASSIBI, CALTECH, *Control over Lossy Networks: The Interplay Between Coding and Control*. Babak followed up Mark's tutorial with another excellent one on tree codes for interactive communications and relationships with control theory. In particular, he asks how (and whether) one can design interactive codes in order to stabilize a plant in a closed-loop control system, where the controller and the plant are not necessarily co-located and must communicate over *noisy* channels. He developed a universal and efficient method for stabilizing plants driven by bounded noise over erasure channels; numerous open questions remain, including how to deal with unbounded noise, with other types of channels (beyond erasure), or how to optimize performance rather than only stabilizing the plant.

PRAKASH ISHWAR, BOSTON UNIVERSITY, *The Infinite-Message Limit of Interactive Source Coding*. Prakash considered distributed block source coding problems, where it is known that multiround interaction can improve the communication efficiency of distributed computation. One may then ask: what is the ultimate limit of this efficiency when the number of messages (in the multiround interaction) is unbounded – an asymptotic that has received relatively little attention in the literature. He outlined recent efforts to tackle this question for distributed computation in two-terminal and collocated networks.

NAN MA, UNIVERSITY OF CALIFORNIA - BERKELEY, *The benefit of interaction in lossy source coding*. Nan discussed the sum-rate-distortion function for a two-way lossy source coding problem in which two terminals send multiple messages back and forth with the goal of reproducing each other's sources. He constructed an example which shows that two messages can strictly improve the one-message (Wyner-Ziv) rate-distortion function, and that the multiplicative gain in terms of the ratio of the one-message rate to the two message sum-rate can be arbitrarily large, while simultaneously the ratio of the backward rate to the forward rate in the two message sum-rate can be arbitrarily small.

JIN MENG, UNIVERSITY OF WATERLOO, *Interactive Encoding and Decoding: Concept, Coding Theorems and Algorithm Design*.

ANUP RAO, UNIVERSITY OF WASHINGTON, *Towards Coding for Maximum Errors in Interactive Communication*.

3.2 Day 2: Interactive channel coding and networking

Tutorial talk: YOUNG-HAN KIM, UNIVERSITY OF CALIFORNIA - SAN DIEGO, *On the role of interaction in network information theory.* Young-Han opened the second day, on interactive *channel coding* (in contrast to the previous day's focus on *source coding*), by providing a high-level overview of interaction in channel coding problems. He discussed several channels with feedback: the posterior-matching scheme for the point-to-point feedback channel, the Cover-Leung coding scheme for the multiple access channel with feedback, Dueck's example for the broadcast channel with feedback. Interactive coding schemes for the two-way channel (and in particular the Shannon-Blackwell Binary Multiplying channel) were discussed before outlining another form of interaction – relaying in general networks.

YOSSI STEINBERG, TECHNION, *The broadcast channel with action dependent states*.

BOBAK NAZER, BOSTON UNIVERSITY, *Computation over Feedback Channels*.

DANIELA TUNINETTI, UNIVERSITY OF ILLINOIS - CHICAGO, *Cooperation in interference channels*. Daniela gave an overview of interaction in one of the classical multi-user information theoretic channels – the interference channel (not discussed in Young-Han's tutorial). In the cooperative interference channel, the two transmitting nodes are able to obtain causal, noisy versions of the others' transmission, allowing them to cooperate, or interact, in transmitting their messages to two independent destinations. She outlined recent capacity bounds for this challenging channel, emphasizing the role and potential benefits of cooperation.

HOLGER BOCHE, TECHNISCHE UNIVERSITÄT MÜNCHEN, *Conferencing Encoders for Compound and Arbitrarily Varying Multiple-Access Channel*. Continuing in the line of cooperation, Holger first discussed two coding theorems for the compound multiple-access channel with an arbitrary number of channel states, where conferencing – a type of cooperation or interaction – between the transmitters is possible. Next, he discussed the capacity region of arbitrarily varying multiple-access channels with conferencing encoders for both deterministic and random coding. Unlike compound multiple-access channels, arbitrarily varying multiple-access channels may exhibit a discontinuous increase of the capacity region when conferencing is enabled. Applications to wireless networks with cooperating base-stations were discussed.

TOBIAS OECHTERING, KTH, *Transmit strategies for the bidirectional broadcast channel & latest results*. Tobias discussed the bi-directional relay channel where two users exchange messages with the help of a relay. Here, two nodes transmit their messages to a relay in phase 1. In phase 2, the relay then forwards these two messages to nodes, allowing them to exchange messages. He discussed optimal transmit strategies for this second broadcast phase for Gaussian channels where the nodes (possibly) have multiple antennas.

PETAR POPOVSKI, AALBORG UNIVERSITY, *Protocol Coding for Two-Way Relay Communication*. Petar introduced a new type of coding which is backwards compatible in the sense that it encodes information through changing the actions taken in existing communication protocols (rather than more physical layer changes). He asks how much information may be extracted on top of the usual information by this protocol coding, or resource re-ordering. In particular, for the two-way relay communication channel he showed interesting links to interactive schemes initially suggested by Schalkwijk for the binary multiplier channel.

BESMA SMIDA, PURDUE UNIVERSITY - CALUMET, *On the utility of Feedback in Two-way Networks*. Besma spoke about developing a fundamental and practical – from a wireless communications perspective – understanding of the value of feedback in two-way, interactive networks. In general, feedback has been studied from a one-way perspective, meaning data travels in one direction, and feedback often assumed to be perfect in the other. Besma proposed a new unied framework which captures the key tradeoffs particular to two-way networks and presence of different types of feedback including quantized channel state information, Automatic Repeat reQuest, or extensions and combinations thereof; many open questions were discussed.

RAMJI VENKATARAMAN, YALE UNIVERSITY, *Interactive Codes for Synchronization from Insertions and Deletions*. Ramji discussed efficient codes for synchronization from insertions and deletions. As an example, he considered remotely located users who independently edit copies of a large file (e.g. video or text), where the editing may involve deleting certain parts of the file, and inserting new data in other parts. The users then want to synchronize their versions with minimal exchange of information, in terms of both the communication rate and the number of interactive rounds of communication. He focussed on the case where the number of edits is small compared to the file-size, and described an interactive synchronization algorithm which is computationally efficient and has near-optimal communication rate.

JEAN-FRANCOIS CHAMBERLAND, TEXAS A&M UNIVERSITY, *Challenges and Potential Approaches in Combining Channels with Memory, Block Codes and Queues*.

3.3 Day 3: Adaptive hypothesis testing and energy efficient green communications

Tutorial talk: ROB NOWAK, UNIVERSITY OF WISCONSIN - MADISON, *Interactive Information Gathering and Statistical Learning*. Rob opened up the third day with yet another aspect of interactive information theory: the notions of adaptive and non-adaptive information, in the context of statistical learning and inference. He considered a collection of models denoted by X and a collection of measurement actions (e.g., samples, probes, queries, experiments, etc.) denoted by Y. A particular model x in X best describes the problem at hand and is measured as follows. Each measurement action, y in Y, generates an observation $y(x)$ that is a function of the unknown model. The goal is to identify x from a set of measurements $y_1(x), ..., y_n(x)$, where y_i in Y, $i = 1, ..., n$. If the measurement actions $y_1, ..., y_n$ are chosen deterministically or randomly without knowledge of x, then the measurement process is non-adaptive. However, If y_i is selected in a way that depends on the previous measurements $y_1(x), ..., y_{i-1}(x)$, then the process is adaptive. The advantage of adaptive information is that it can sequentially focus measurements or sensing actions to distinguish the elements of X that are most consistent with previously collected data, and this can lead to significantly more reliable decisions. The key question of interest is identifying situations in which adaptive information is significantly more effective than non-adaptive information, which depends on the interrelationship between the model and measurement spaces X and Y. He covered the general problem, connections to channel coding and compressed sensing, and considered two illustrative examples from machine learning.

MATT MALLOY, UNIVERSITY OF WISCONSIN - MADISON, *Sequential testing in high dimensions*.

TE SUN HAN, NATIONAL INSTITUTE OF INFORMATION AND COMMUNICATIONS TECHNOLOGY, *Trade-off of data compression and hypothesis testing*.

SENNUR ULUKUS, UNIVERSITY OF MARYLAND, *Interacting with Nature: Information Theory of Energy Harvesting Communications*.

KAYA TUTUNCUOGLU, PENN STATE, *Interactive Transmission Policies for Energy Harvesting Wireless Systems*.

3.4 Day 4: Interactive information security

Tutorial talk: HIMANSHU TIYAGI, UNIVERSITY OF MARYLAND, *Function Computation, Secrecy Generation and Common Randomness*. This talk provided a comprehensive overview on the role of common randomness in secure function computation and secret-key generation. The setup presented included multiple source terminals, each observing a correlated source sequence. The terminals interactively exchanged messages over a public discussion channel and ultimately generated a common secret-key. The quantity of interest was the maximum secret-key rate that can be generated in this setup. The general problem remains open, but the state of the art was surveyed.

PRAKASH NARAYAN, UNIVERSITY OF MARYLAND, *Multiple-access channels, feedback and secrecy generation*. This talk presented a secret-key generation problem when the legitimate terminals do not have access to correlated sources, but instead can communicate over a noisy communication link. An adversary can also eavesdrop this communication. The problem was partially solved when there are two legitimate terminals. An open problem involving three terminals and a multiple access channel for communication was described.

FRANS WILLEMS, TU EINDHOVEN *Authentication based on secret generation*. This talk focused on an application of biometric encryption. A user who provides an enrolment biometric to the system wishes to authenticate by providing another biometric reading. The successive readings from the same user can be noisy. The system is required to authenticate legitimate user and reject a false user. An information theoretic formulation was presented and the conditions under which successful decoding is possible were derived.

AYLIN YENER, PENN STATE, *Secrecy: Benefits of Interaction*. A two-way wiretap channel with an external eavesdropper was presented and conditions under which secure communication is possible were derived. The two-way nature is particularly useful since the eavesdropper observes a super-position of two information signals and cannot separate the two individual signals from the sum.

ASHISH KHISTI, UNIVERSITY OF TORONTO, *Secret-Key Generation over Fading Channels*. This talk provided a review of secret-key generation over wireless fading channels. Fundamental limits of secret-key generation was of particular focus. The difference in the limits of various wireless systems was discussed and connections to the information theoretic formulations from earlier works were elaborated.

PULKIT GROVER, STANFORD UNIVERSITY, *Interactive communication in circuits: understanding Shannon's "magic trick"*.

STARK DRAPER, UNIVERSITY OF WISCONSIN - MADISON, *Reliability in streaming data systems with feedback*.

ADITYA MAHAJAN, MCGILL UNIVERSITY, *The stochastic control approach to real-time communication: an overview.*

ANDREW ECKFORD, YORK UNIVERSITY, *Models and Capacities of Molecular Communication*. Andrew spoke about a very different channel model: a diffusion-mediated molecular communication channel. This very recent channel model's capacity is surprisingly difficult to obtain, not only because the communication medium unfamiliar to communication engineers; but the mathematical details of the communication environment are complicated. Andrew discussed mathematical models for molecular communication, which are both information-theoretically useful and physically meaningful; discussed the difficulties of dealing exactly with these models; and presented some simplified scenarios in which capacity can be evaluated.

ERSEN EKREM, UNIVERSITY OF MARYLAND, *The Vector Gaussian CEO Problem*.

4 Scientific Progress Made and Outcomes of the Meeting

The central outcome of the meeting was bringing together various experts in theoretical computer science, information theory, wireless communications, and statistical signal processing who had tackled different aspects of the broad "interactive information theory" theme. Each person was given a speaking slot so they were able to expose their work to others, and there was plenty of time for discussion at the end of each talk, over meals and during coffee breaks. This group was particularly interactive, and many participants came away with useful comments on their work, and exposure to results in other areas which would be useful tools. I believe many collaborations have been initiated by this workshop on a unique topic (the first of its kind); it will take time for us to see the results of bringing together this particular and diverse group of researchers. In the meantime, we let the participants themselves attest to the scientific progress made and the outcomes of the meeting directly:

> "I was very fortunate to attend the BIRS workshop on interactive information theory. As a computer scientist I don't usually get to interact with information theorists and electrical engineering researchers. This workshop provided me with such an opportunity. I have been working on applications of information theory in the field of computational complexity – a subarea of computer science. The workshop has really impacted my research agenda in two ways. It exposed me to new ideas from information theory, which allowed me and my students to make progress on the problems we have been working on. At the same time, it made me aware of the problems and challenges in information theory to which the techniques we have been developing can be applied. I hope BIRS will hold another workshop in this area in the future years."
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> *– Mark Braverman, Princeton University and University of Toronto*

> The workshop was very beneficial in getting people from related fields together. This is valuable to our community as it created a better awareness of the specific problems people are currently addressing, and available mathematical methods to address these problems. It also gave me an opportunity to meet and exchange with new researchers in my area. This too is of great value. I think that the BIRS workshop will have a definite impact on the research in our area (interactive information theory). It may take a few months or a couple of years for the seeds planted during the workshop to come to fruition.
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> *– Jean-Francois Chamberland, Texas A&M University*

The wonderful BIRS Workshop on Interactive Information Theory (Jan 16-20, 2012) afforded several intellectual benefits. I met and learned of the research of several computer scientists who work on areas related to my own but who normally do not attend the same meetings as I do. The tutorial lectures organized by Profs. Devroye, Khisti and Blake, as part of the Workshop program, were very useful. Several new and exciting ideas were discussed. My student, Himanshu Tyagi, gave a well-received one-hour tutorial which, I think, will have a beneficial impact on his future career in academia.

– Prakash Narayan, University of Maryland

This has been perhaps one of the three best scientific gatherings I have ever been to. The small number of selected participants and the focused topic provided opportunity for intensive discussions. After this workshop there is a potential for at least 3-4 new collaborations, which is excellent and rare in other conferences.

– Petar Popovski, Aalborg University

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