BIRS 12W5067: LINKING REPRESENTATION THEORY, SINGULARITY THEORY AND NON-COMMUTATIVE ALGEBRAIC GEOMETRY

Contents

All lectures in this workshop discuss various connections between representation theory and other parts of mathematics with the focus on singularity theory, commutative ring theory, commutative and non-commutative algebraic geometry.

1. Orlov's Theorem and Applications

The most important recent achievement in commutative ring theory is Orlov's theorem [O2], which is a key ingredient to relate two rather different areas, Cohen-Macaulay representation theory and (non-commutative) projective geometry. Orlov's theorem relates two completely different triangulated categories that can be associated to an isolated graded Gorenstein singulary. One of these categories is the derived category of coherent sheaves on smooth projective scheme arising by sheafification, that is, by Serre construction. The other one is the singularity category introduced by Buchweitz [Bu] and Orlov [O1], that is, the stable category of graded Cohen-Macaulay modules.

The main intension of the two introductory lectures by Ragnar Buchweitz was to introduce all participants to these new developments. More precisely, the relationship between these two categories is determined by the sign of an integral invariant, the Gorenstein parameter. The two triangulated categories are equivalent if the Gorenstein parameter is zero. If the invariant is positive, then the singularity category sits inside of the derived category of coherent sheaves as the perpendicular category with respect to an exceptional sequence. If it is negative, then the relationship between these categories is reversed. Buchweitz's explanation of Orlov's correspondence was based on an analysis of so called Betti tables. He also discussed the hypersurface case, relating the subject to the theory of matrix factorizations, a subject with interesting links to physics.

David Favero approached the same problem by different methods from the perspective of geometric invariant theory, reporting on a joint work with Ballard and Katzarkov [BFK]. In his exposition, he related weighted projective stacks and Landau-Ginzburg models. This again was relating the subject to mathematical physics.

Finally Dirk Kussin discussed a very explicit special instance of Orlov's theorem in the context of weighted projective lines and Nakayama algebras.

2. Singularity theory, Mirror symmetry and Weighted projective lines

Weighted projective lines were introduced by Geigle-Lenzing [GL] in 1980s and provide one of the most important classes of abelian categories in representation theory. They form a special class of non-commutative projective schemes in the sense of Artin-Zhang [AZ] obtained by Serre construction.

In reporting on joint recent work with Atsushi Takahashi [ET], Wolfgang Ebeling explained the mirror symmetry between orbifold curves and cusp singularities equipped with a group action. He did put Arnold's strange duality into the mirror symmetry context, bringing together Landau-Ginzburg models and Fukaya categories. These aspects were complemented by his extension of Arnold's strange duality from joint work with E.T.C Wall [EW]. These topics were perfectly complemented by Atsushi Takahashi's talk reporting on a new lane of access to understand aspects of classical mirror symmetry between orbifold projective lines and cusp singularities. Things were then put in an even wider context by Kazushi Ueda when dealing with the surface case by relating the mirror symmetry for K3 surfaces to singularity theory by means of the Fukaya-Seidel category, equipped with an A_∞-structure [KMU]. This yielded, among others, a categorification of the Milnor lattice from singularity theory. Ueda did further bring together Kontsevich's mirror symmetry conjecture with recent work of Kapranov-Vasserot and discussed in this context an application of Orlov's machinery, that was — as previously discussed — a central objective of this meeting.

There were two further talks relating the sheaf category and singularity theory aspects of weighted projective lines, and their coordinate algebras, by Lidia Angeleri-Hügel and Hagen Meltzer. Angeleri-Hügel dealt with the category of quasi-coherent sheaves on a weighted projective line and the related category of possibly infinite-dimensional modules over a canonical algebra when studying large tilting sheaves, respectively, large tilting modules. In her talk, she followed the approach of [AS]. Tilting sheaves for categories of coherent sheaves, respectively tilting modules for categories of finite dimensional modules, and their endomorphism rings, form a very central topic in current representation theory; in particular, these objects are responsible for inducing triangle equivalences between the respective derived categories and the derived categories of modules over their endomorphism rings. Concerning large tilting modules, the situation is to some extent — but not completely — analogous. Here, the investigation afforded a deep knowledge on quasi-coherent sheaves, respectively infinite dimensional modules.

Hagen Meltzer reported on a quite surprising class of triangle equivalences between certain Nakayama algebras $A_n(r)$ given by an equi-oriented quiver of Dynkin type $\mathbb{A}_n$, all arrows denoted x, and equipped by all nilpotency relations $x^r = 0$. From joint work with Kussin and Lenzing [KLM] he deduced a number of surprising triangle equivalences, termed *Happel-Seidel symmetry*, showing that always the algebras $A_{(a-1)(b-1)}(a)$ and $A_{(a-1)(b-1)}(b)$ are triangle-equivalent, a fact he derived from the clever construction of appropriate tilting objects in the triangle singularity category associated to a weighted projective line of weight type $(2, a, b)$.

3. Representation theory

Auslander-Reiten theory makes representation theory unique among other subjects [ARS, ASS, Ha, Y]. It reveals deep homological symmetry properties of module categories and enables us to visualize their structure by using Auslander-Reiten sequences and quivers. Its key ingredient is the classical Auslander-Reiten duality

$$\underline{\mathrm{Hom}}_\Lambda(X,Y) \simeq D\,\mathrm{Ext}^1_\Lambda(Y, \tau X)$$

which is an analog of Serre duality

$$\mathrm{Hom}_{\mathbb{X}}(X,Y) \simeq D\,\mathrm{Ext}^d_{\mathbb{X}}(Y, X \otimes \omega)$$

for a smooth projective variety $\mathbb{X}$. Actually Auslander-Reiten and Serre dualities agree for important cases like weighted projective lines. Auslander-Reiten theory works for an arbitrary finite dimensional algebra, and makes especially the representation theory of algebras with global dimension one, that is path algebras of quivers, very well understood. In this sense Auslander-Reiten theory has a one dimensional characteristic from a structure theoretic point of view.

On the other hand, the structure of the module category $\mathsf{mod}\,\Lambda$ of a finite dimensional algebra Λ is two dimensional because the functor category $\mathsf{mod}(\mathsf{mod}\,\Lambda)$, which is basic in Auslander-Reiten theory, is an abelian category of global dimension two. As an important consequence of this interpretation, we can describe the structure of $\mathsf{mod}\,\Lambda$ by using Auslander-Reiten quivers, which have the structure of two dimensional simplical complexes as Bongartz, Gabriel and Riedtmann have observed. In this sense Auslander-Reiten theory has a two dimensional characteristic in a representation theoretic sense.

The two dimensional features become clearer by looking at the Auslander-Reiten theory for the category $\mathsf{CM}\,\Lambda$ of Cohen-Macaulay modules over orders Λ (e.g. Λ is a commutative Cohen-Macaulay ring), a theory found by Auslander and Reiten in the 70s [Au1, Y]. The category $\mathsf{CM}\,\Lambda$ is especially nice in Krull dimension two because it has fundamental sequences completing Auslander-Reiten sequences. Thanks to this property, the complete classification of representation-finite orders could be given in Krull dimensional two by Reiten and Van den Bergh [RV]. For example, quotient singularities in dimension two are known to be representation-finite [He], and their Auslander-Reiten quivers are given by the McKay quiver of the group

[Au2]. In this case the stable category of Cohen-Macaulay modules is equivalent to what is called, in recently developped cluster theory, the '1-cluster category' of the corresponding Dynkin quiver. This equivalence is called Auslander's algebraic McKay correspondence.

3.1. **Higher dimensional Auslander-Reiten theory.** The appearence of higher dimensional Auslander-Reiten theory marks a completely new development [I1, I2]. Concerning the relationship to Cohen-Macaulay representation theory and non-commutative algebraic geometry, this new theory is still changing rapidly. The key notion is d-cluster tilting subcategory $\mathscr{C}$, which is defined by vanishing conditions of higher extension groups. The category $\mathscr{C}$ enjoys nice properties which are higher dimensional analogs of module categories by having long exact sequences called d-Auslander-Reiten sequences, and the functor category $\mathsf{mod}\,\mathscr{C}$ is an abelian category of global dimension $d+1$. Hence the study of d-cluster tilting subcategories can be regarded as Auslander-Reiten theory in dimension $d+1$ from the representation theoretic viewpoint discussed above.

Algebras having d-cluster tilting modules are called *d-representation finite*. In fact, in the classical case $d = 1$ they are precisely the representation finite algebras, which have been one of the central subjects in representation theory. The case $d = 2$ has a strong connection with cluster theory, which is one of the most popular subjects in present representation theory. In fact the 2-cluster tilting objects in a 2-Calabi-Yau triangulated categories play a crucial role to categorify cluster algebras of Fomin-Zelevinsky [FZ]. This is a reason why we use the name "cluster tilting".

From a theoretic viewpoint, a higher dimensional analog of path algebras of quivers is algebras of global dimension d. A distinguished class of algebras of global dimension d is given by d-representation finite algebras with global dimension d, often simply called d-representation finite algebras [I3, IO1, HI1, HI2]. For example, 1-representation finite algebras are precisely path algebras of Dynkin quivers. In this case many nice properties are satisfied. In particular, higher preprojective algebras appear as important objects in the theory. They are an analog of classical preprojective algebras. Moreover they are the 0-th cohomology of Keller's derived higher preprojective algebras [Ke] which plays an important role in cluster theory [Am, KY]. Higher preprojective algebras of d-representation finite algebras of global dimension d satisfy many fundamental properties, for example, they are selfinjective and their stable categories are $(d+1)$-Calabi-Yau and triangle equivalent to certain $(d+1)$-cluster categories [IO2].

In the lectures by Martin Herschend and Steffen Oppermann, their recent work [HIO] was explained. They introduced a notion of *d-representation infinite algebras*, being another distinguished class of algebras of global dimension d forming a counterpart of d-representation finite algebras. 1-representation infinite algebras are precisely path algebras of non-Dynkin quivers. As a generalization of their representation theory, d-representation infinite algebras have three important classes of modules, d-preprojective modules, d-preinjective modules and d-regular modules. Moreover the higher preprojective algebras of d-representation finite algebras are an especially nice class of graded algebras, and this gives a strong relationship between representation theory and non-commutative algebraic geometry. In fact d-representation infinite algebras form a nice class of Minamoto's Fano algebra, which will be discussed below. There is a bijection between d-representation infinite algebras Λ and graded bimodule $(d+1)$-Calabi-Yau algebras Π of Gorenstein parameter 1. We have derived equivalence

$$\mathsf{D}^{\mathrm{b}}(\mathsf{mod}\,\Lambda) \simeq \mathsf{D}^{\mathrm{b}}(\mathsf{qgr}\,\Pi)$$

between d-representation infinite algebras Λ and the non-commutative projective schemes associated with their higher preprojective algebras Π. As an application we can describe d-regular modules Λ as 0-dimensional sheaves in the Serre quotient $\mathsf{qgr}\,\Pi$.

3.2. **Cohen-Macaulay representation theory.** Higher dimensional Auslander-Reiten theory naturally appears in Cohen-Macaulay representation theory in Krull dimension $d+1$. In this case, d-cluster tilting subcategories $\mathscr{C}$ have an extremely nice structure since they have d-fundamental sequences completing d-Auslander-Reiten sequences. In particular the endomorphism algebras of d-cluster tilting module are non-singular orders in the sense of Auslander [Au3]. Moreover they are $(d+1)$-Calabi-Yau algebras when the ring is commutative Gorenstein.

An important class of d-representation finite algebras in Krull dimension $d+1$ is given by isolated quotient singularities. In fact they have natural d-cluster tilting modules whose endomorphism algebras are skew group algebras of the groups. It is a natural question to ask what are their stable categories of Cohen-Macaulay modules, and this was highlighted in the lectures of Idun Reiten and Osamu Iyama. Recently Amiot, Iyama and Reiten [AIR] gave a very promising higher dimensional generalization of Auslander's algebraic McKay correspondence. They start with an d-representation infinite algebra Λ and their higher preprojective algebra Π explained above. In this case, for a certain nice idempotent e of Λ, they showed that the stable category of Cohen-Macaulay $e\Pi e$-modules is triangle equivalent to the d-cluster category of $\Lambda/\langle e\rangle$. In the special case $d=1$, this precisely gives Auslander's algebraic McKay correspondence discussed above.

In the lecture by Izuru Mori, the range of algebraic McKay correspondence was extended by replacing the polynomial algebra by Artin-Schelter regular algebras [Mo].

3.3. **Recent development on Representation theory.** In the lecture by Lutz Hille, he discussed the Auslander algebra of $k[T]/(T^t)$ (cf. [BHRR]), which appears in many different contexts, for example, a certain category of sheaves on a chain of (-2)-curves on a rational surface. He explained a clasification of important classes of modules containing spherical modules, exceptioal modules, rigid modules, and tilting modules.

In the lecture by Markus Schmidmeier, certain varieties associated with invariant subspaces of nilpotent operators [RS] are discussed. There are reductive algebraic groups acting on these varieties, and he described joint work with Kosakowska [KS] yielding a combinatorial characterization of the partial order given by degenerations.

Henning Krause discussed in his lecture the category of strict polynomial functors introduced by Friedlander and Suslin. He exposed a very fundamental result showing deep similarities between fundamental duality theories: the Koszul, Ringel and Serre duality [Kr].

In the lecture by Sefi Ladkani, he discussed an aspect of an important operation of quivers called mutation introduced in cluster theory [FZ]. He characterized quivers whose numbers of arrows do not change under mutation [L].

In the lecture by Charles Paquette, Auslander-Reiten theory for the representations of infinite quivers with relations was discussed [P]. When the relations are nice, it is possible to get all the Auslander-Reiten sequences. When there are no relations, we obtain a complete description of the Auslander-Reiten components of the category of fnitely presented representations.

4. Non-commutative algebraic geometry

Around 1990, Michael Artin's school initiated a new area called non-commutative algebraic geometry. This subject is based on the notion of a non-commutative projective scheme [AZ], which is obtained from a (not necessarily commutative) graded algebra A by applying Serre construction, that is, taking the quotient category of the category $\mathsf{mod}^{\mathbb{Z}} A$ of graded A-modules by its Serre subcategory $\mathsf{mod}_0^{\mathbb{Z}} A$ of finite dimensional graded A-modules.

In the lecture by Paul Smith, he discussed an application of non-commutative algebraic geometry to Penrose tilings [S]. The key role is played by the non-commutative projective curve associated with the graded algebra $A = k\langle x, y\rangle/(y^2)$, and he explained that the points on this curve parametrize the Penrose tilings. In the closely related lecture by Xiao-Wu Chen, irreducible representations of Leavitt path algebras are discussed [C].

Also further aspects of non-commutative algebraic geometry were discussed. For example, in the lecture by Colin Ingalls, Iskovskih's conjecture about rationality of a conic bundle over a surface was generalized to the case of projective space bundles of higher dimension by using maximal orders and toric geometry. As a corollary he showed that the Brauer-Severi variety of a Sklyanin algebra is rational.

4.1. **Fano algebras.** There is a strong similarity between finite dimensional algebras and projective varieties. The bounded derived categories of finite dimensional algebras of finite global dimension and smooth projecive varieties both satisfy Auslander-Reiten-Serre duality, which plays an important role in representation theory. In a very important lecture by Hiroyuki Minamoto, he explained his recent work on *Fano algebras* [Mi]. This is a new important class of finite dimensional algebras formalizing properties of Serre functors on Fano varieties for finite dimensional algebras. Main examples of Fano algebras are given by endomorphism algebras of tilting bundles on Fano varieties. Moreover, the work by Minamoto and Mori [MM] gives a systematic construction of Fano algebras by giving a bijection between isomorphism classes of certain d-Fano algebras and graded Morita-equivalence classes of Artin-Schelter regular algebras of dimension $d+1$. Suprisingly this result could be applied in higher dimensional Auslander-Reiten theory yielding essential new results [HIO]. These links deserve further studies.

4.2. **Non-commutative resolutions.** Recently people including Van den Bergh initiated a non-commutative analog of resolutions of singularities in birational geometry resulting in the important concept of non-commutative crepant resolutions (NCCR for short) [V]. There is a strong connection between d-cluster tilting modules discussed above and non-commutative crepant resolutions. If an isolated singularity R in dimension $d+1$ has a d-cluster tilting modules M, then the endomorphism algebra $\mathrm{End}_R(M)$ gives an NCCR. Van den Bergh gave a sufficient condition for a singularity R to have an NCCR Λ in terms of the resolution X of the singularity R, and moreover gave a derived equivalence between Λ and X in this case.

Recently people including Broomhead, Davison and Ishii-Ueda [Br, D, IU] found a systematic construction of NCCRs of affine toric Gorenstein 3-fold in terms of dimer model, a kind of quivers with potential. This theory was the central topic in the lecture by Nathan Broomhead.

Michael Wemyss was adding a new accent to NCCRs by introducing the new concept of modifying modules [IW1]. This is a non-commutative counterpart of partial crepant resolutions, and motivated by the minimal model program in birational geometry. It was shown that maximal modifying modules give a non-commutative counterpart of $\mathbb{Q}$-factorial terminalizations [IW2].

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