Arithmetic geometry of orthogonal and unitary Shimura varieties (12w5011)

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In June 2012, a workshop dedicated to the arithmetic geometry and applications of orthogonal and unitary Shimura varieties was held at the Banff International Research Station, located in Banff in the Canadian Rockies. The workshop, attended by about 40 participants, from three continents and many countries, represented a surge of interest in this topic. At the time we had applied for the workshop, that is a few years earlier, one could say that the study of these particular Shimura varieties was undergoing revival and so we had thought it wise to take toll of recent and undergoing research concerning these varieties, with special emphasis on subjects that had received attention recently. For example, special cycles, generating series and special values of L functions (the Kudla Programme); integral models and arithmetic intersection theory; compactifications and Arakelov theory with "singularities at infinity".

In the time that had passed since, the impression that these particular areas are blooming has proven itself. Many important developments have been made exactly along the lines mentioned above and many of those by the participants of the workshop. Certain collaborations had their origins in this workshop, for instance the work by Andreatta, Goren, Howard and Madapusi-Pera. As many of the people attending the conference were already involved in collaborative research with other participants, the workshop certainly encouraged the continuation of these collaborations, for example: Goren-de Shalit, Bruinier-Howard-Yang, Kudla-Rapoport, Terstiege-Rapoport, Burgos-Gil-Kramer and more.

The following is a selection of titles and abstract for some of the talks. This selection gives an excellent outline of the contents of the workshop and its focus.

- Steve Kudla: "Special cycles for unitary groups: the unramified case".

  In the first part of this lecture I will review the definition of arithmetic special cycles on Shimura varieties for unitary groups of signature $(n-1,1)$ and explain how the arithmetic 0-cycles arise in the computation of their height pairings. In the second part of talk, I will review the structure of the Rapoport-Zink space used in the p-adic uniformization of the supersingular locus of such a varieties in the case of an inert prime. Finally, I will give the definition of the analogous special cycles in this RZ space and explain their properties.

- Ben Howard: A Gross-Zagier theorem for higher weight modular forms.

  I'll talk about an extension of the Gross-Zagier theorem to higher weight modular forms, expressing the height pairings of special cycles on unitary Shimura varieties to the central derivatives of Rankin-Selberg L-functions. This is joint work with Jan Bruinier and Tonghai Yang.

- Keerthi Madapusi: Regular integral models for orthogonal Shimura varieties and the Tate conjecture for K3 surfaces in finite characteristic.

  We construct regular integral canonical models for Shimura varieties of orthogonal type with maximal parahoric level, and we show that certain moduli spaces of polarized K3 surfaces can be viewed as open sub-schemes of such integral models. Using a result of Kisin, this then implies the Tate conjecture for K3 surfaces in odd characteristic p, as long as they admit a polarization of degree indivisible by p^2. The same methods also work to prove the Tate conjecture for cubic fourfolds in odd characteristic.

- Michael Rapoport: On the geometry of unitary Shimura varieties in the ramified case.

  I will explain structure theorems for the formal moduli space of p-divisible groups of Picard type of signature $(1, n-1)$ for a ramified quadratic extension of $\mathbb{Q}_p$. The underlying reduced scheme possesses a stratification by Deligne-Lusztig varieties for symplectic groups over $\mathbb{F}_p$; the strata are parametrized by simplices in the Bruhat-Tits building of a p-adic unitary group. This is joint work with U. Terstiege and S. Wilson; the results are analogous to the results of I. Vollaard and T. Wedhorn in the case of an unramified quadratic extension of $\mathbb{Q}_p$.

- Brian Smithling: Moduli descriptions of some local models for Shimura varieties.

  Local models are certain schemes introduced to model the étale-local structure of p-adic integral models of Shimura varieties. A general definition of them in the setting of PEL Shimura varieties was given by Rapoport and Zink; recently Pappas and Zhu have formulated a general group-theoretic definition of them for tamely ramified groups and established many good properties for them. Unfortunately the local models do not in general admit ready moduli-theoretic interpretations. To facilitate applications, it is therefore of interest to describe to describe them in moduli-theoretic terms when possible. In the case of orthogonal and ramified unitary groups, such a description has been proposed by Pappas and Rapoport. I will report on some progress towards proving their conjecture.

- Jurg Kramer: Arithmetic intersections on modular curves.

  In our talk we will report on asymptotic formulas for the arithmetic self-intersection of the relative dualizing sheaf equipped with the Arakelov metric on modular curves attached to congruence subgroups as the level tends to infinity. In case of the modular curve $X_0(N)$ (N squarefree and not divisible by $2, 3$) such results are due to A. Abbes and E. Ullmo. We will present analogous results for the modular curve $X_1(N)$ (for suitable squarefree N), which then enable us to compute the Faltings height of the associated Jacobian $J_1(N)$ asymptotically (as N tends to infinity).

- Ulf Kuhn: Modularity of generating series for arithmetic Hecke correspondences.

  The aim of the talk is to explain our approach to Kudla's conjectures for the case of the product of two modular curves. The major difficulties in this situation are of analytical nature. We present a mild modification of this Green function that satisfies the requirements of being a Green function in the sense of Arakelov theory on the natural compactification in addition. Only this allows us to define arithmetic special cycles and to show that the generating series of those modified arithmetic special cycles is as predicted by Kudla's conjectures a modular form with values in the first arithmetic Chow group. Moreover its intersection with the arithmetic canonical class yields essentialy the derivative of an Eisenstein series. This is joint work with Rolf Berndt: http://xxx.uni-augsburg.de/abs/1205.6417 http://www.math.uni-hamburg.de/home/kuehn/berndt-kuehn-part-II.pdf

- Ellen Eischen: p-adic families of Eisenstein series for unitary groups

  Special values of certain L-functions can be expressed in terms of values of Eisenstein series at points on the Shimura variety for U(n,n). One approach to p-adically interpolating values of these L-functions relies on construction of a p-adic family of Eisenstein series. In this talk, I will explain how to construct such a family of Eisenstein series, and I will explain how to p-adically interpolate certain values of both holomorphic and non-holomorphic Eisenstein series on U(n,n).

- Matthew Greenberg: Triple product p-adic L-functions for balanced weights.

  The conjecture of Gross and Prasad and its refinements constitute a framework for generalizing the famous formula of Gross and Zagier, relating the height of a Heegner point on an elliptic curve to the central derivative of an associated L-function, to the context of orthogonal and unitary Shimura varieties. In this talk, I would like to discuss work-in-progress with Marco Seveso dealing with p-adic analogues of these conjecture in a low-dimensional (though very rich!) test scenario. Specifically, I'll describe a construction of a triple product p-adic L-function in the case of "balanced weights." The key tools are existing classical special value formulae due to Gross-Kudla, Boecherer-Schulze-Pillot, and Ichino, and the Ash-Stevens theory of p-adic deformation of arithmetic cohomology.

Examination of these abstracts shows recurring themes that intersect and branch over and over again: integral models, cycles, L-functions, Hecke correspondences, automorphic forms. As such, one could say that this pattern, together with the all important sibling Galois representations, is a fair image of much of the arithmetic geometry of Shimura varieties. However, a closer examination, shows that the particular results discussed in these talks, and in other talks of the workshops, form a net of closely related and inter-dependent results. This explains well the relevance of this workshop to the area of orthogonal and unitary Shimura varieties in general, but more than that, it explains the service for the particular group that attended - a group that is responsible for many of the exciting developments in these area (unfortunately, the limit on the number of participants, as well as unavailability of some of the people we have invited, resulted in a partial representation of the key players).