Harmonic Analysis in Convex Geometry

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1 A short overview of the field

Convex geometry is an old subject that can be traced at least to Archimedes. The problems are usually very easy to formulate, nevertheless, the methods and approaches to these "easy" problems are very diverse, different from one problem to another, and sometimes to solve the problem one has to use the ideas from Topology, Analysis, Differential Geometry and even Ergodic Theory. The diversity and the mixture of methods are not the only reasons why people are still interested in Convexity. Other reasons are that completely new methods keep coming into play, and as a result new applications are discovered.

Harmonic Analysis methods are among them. Despite the fact that the Fourier coefficients and Parseval identity were first used by Hurwitz more than a century ago in the solution of the isoperimetric problem, the methods of Harmonic Analysis received a new breath only at the end of the last century, when they were applied to problems related to sections and projections of Convex bodies. In particular, the method of the Fourier transform of distributions were applied to the solution of celebrated Busemann-Petty problem, Shephard problem, the problem of the local characterization of zonoids, and to many other problems.

The use of harmonic analysis in the study of problems in convex geometry has been recently becoming more and more standard. Behind each class of bodies in question (such as zonoids, intersection bodies, centroid bodies, etc.) there are certain objects from Harmonic Analysis. The study of the underlying properties of these objects leads to an understanding of the properties of the associated bodies in question.

2 Presentation Highlights

The topics of the workshop included harmonic analysis on the sphere and special classes of bodies, theory of valuations, discrete geometry and tomography, probability and random matrices, quantum information theory, and Mahler conjecture.

We start the description with a harmonic analysis type result proved by Paul Goodey in his joint work with Wolfgang Weil. They studied certain properties of the operators on the sphere and their applications to geometric problems. To formulate the results we introduce some definitions.

A linear operator $T : C^\infty(S^{n-1}) \to C^\infty(S^{n-1})$ is said to be standard if it is linear, continuous, bijective and intertwining.

T has the local positivity property, if T satisfies:

(LP) If $f \in C^\infty(S^{n-1})$ is a function such that, for each $x \in S^{n-1}$, there is $\epsilon = \epsilon(x) > 0$ and a function $g = g_{x,\epsilon} \in C^\infty(S^{n-1})$, $g \geq 0$, with $Tf = Tg$ on the (open) ϵ-neighborhood $U_\epsilon(x)$ of x, then it follows that $f \geq 0$.

T has the equatorial positivity property, if the following holds:

(EP) If $f \in C^\infty(S^{n-1})$ is a function such that, for each $x \in S^{n-1}$, there is $\epsilon = \epsilon(x) > 0$ and a function $g = g_{x,\epsilon} \in C^\infty(S^{n-1})$, $g \geq 0$, with $Tf = Tg$ on the (open) ϵ-neighborhood $U_\epsilon(x^\perp)$ of $x^\perp$, then it follows that $f \geq 0$.

Furthermore, T has the local support property, if:

(LS) For every $f \in C^\infty(S^{n-1})$, we have supp $f \subset$ supp Tf or supp $f \subset$ supp Tf^*, where f^* is the reflection of f in the origin.

T has the equatorial support property, if:

(ES) For $f \in C^\infty(S^{n-1})$ with supp $Tf \subset U_\epsilon(x)$, $\epsilon > 0$, $x \in S^{n-1}$, we have supp $f \subset U_\epsilon(x^\perp)$.

Goodey and Weill prove the following results.

Theorem 2.1 *A standard operator T on $C^\infty(S^{n-1})$ has the local positivity property, if and only if it has the local support property.*

Theorem 2.2 *A standard operator T on $C^\infty(S^{n-1})$ has the equatorial positivity property, if and only if it has the equatorial support property.*

The importance of these theorems is in the description of the phenomenon lying behind the local / equatorial characterization of zonoids and intersection bodies. As an application, the authors also obtain local and equatorial characterizations of L_p-intersection bodies, mean section bodies, and their associated spherical transforms.

Gabriel Maresch presented his joint work with Franz Schuster on The Sine Transform of Isotropic Measures.

Recall that a non-negative finite Borel measure μ on the unit sphere S^{n-1} is said to be isotropic if for all $x \in \mathbb{R}^n$,

$$\|x\|_2 = \int_{S^{n-1}} \langle x, u\rangle^2 d\mu(u).$$

The sine transform $\mathcal{S}\mu$ of a finite Borel measure μ on S^{n-1} is the continuous function defined by

$$(\mathcal{S}\mu)(x) = \int_{S^{n-1}} \|x|u^\perp\| d\mu(u), \quad x \in \mathbb{R}^n.$$

The latter defines a norm on $\mathbb{R}^n$ whose unit ball is denoted by S^*_μ and its polar by S_μ.

Let κ_n denote the volume of the Euclidean unit ball in $\mathbb{R}^n$ and define

$$\alpha_n := \frac{n(n-1)^{2n}}{\Gamma(n)^{1/(n-1)}}, \text{ and } \gamma_n := \frac{(n-1)\kappa_{n-1}^2}{\kappa_{n-2}\kappa_n}.$$

Their main result is

Theorem 2.3 *If μ is an even isotropic measure on S^{n-1}, then*
1)

$$\frac{\kappa_n}{\gamma_n^n} \leq V(S_\mu^*) \leq \frac{\kappa_n \gamma_n^n}{\alpha_n},$$

with equality on the left if and only if μ is normalized Lebesgue measure.
2)

$$\frac{\kappa_n \alpha_n}{\gamma_n^n} \leq V(S_\mu) \leq \kappa_n \gamma_n^n,$$

with equality on the right if and only if μ is normalized Lebesgue measure.

Rolf Schneider gave a talk on zonoids with isotropic generating measures, based on a joint work with Daniel Hug.

A convex body $Z \subset \mathbb{R}^n$ is a zonoid if its support function has a representation

$$h(Z,u) = \int_{S^{n-1}} |\langle u, v\rangle| \mu(dv), \qquad u \in \mathbb{R}^n,$$

with an even, finite Borel measure μ on the unit sphere S^{n-1}.

They proved

Theorem 2.4 *If $j \in \{1, ..., n\}$ and if $Z_1, ..., Z_j \subset \mathbb{R}^n$ are zonoids with isotropic generating measures, then*

$$V(Z_1, ..., Z_j; B_2^n[n-j]) \geq 2^j \kappa_{n-j}.$$

For $j = 1$, the latter inequality holds with equality. For $j \geq 2$, equality holds if and only if $Z_1 = \cdots = Z_j$ is a cube of side length 2.

As a corollary, they obtained

Theorem 2.5 *Let $Z \subset \mathbb{R}^n$ be a zonoid with isotropic generating measure μ. If $j \in \{1, ..., n\}$, then*

$$V_j(Z) \geq 2^j \binom{n}{j}.$$

For $j = 1$, the latter inequality holds with equality. For $j \geq 2$, equality holds if and only if Z is a cube of side length 2.

A few talks at the workshop were devoted to the theory of valuations.

Let $\mathcal{K}^n$ be the set of convex bodies in an n-dimensional Euclidean vector space V and let A be an abelian semigroup. A function $\phi : \mathcal{K}^n \to A$ is called a valuation if

$$\phi(K) + \phi(L) = \phi(K \cup L) + \phi(K \cap L)$$

whenever $K,\ L,\ K \cup L \in \mathcal{K}^n$.

Valuations on convex bodies have been actively studied. A famous classical result in this area is Hadwigers classification of rigid motion invariant real valued continuous valuations as linear combinations of the intrinsic volumes. Among many applications, this result gives an effortless proof of the famous Principal Kinematic Formula from integral geometry.

In his talk, Franz Schuster presented a joint work with Semyon Alesker and Andreas Bernig where they obtained the decomposition of the space of continuous and translation invariant valuations into a sum of $\mathrm{SO}(n)$ irreducible subspaces. To describe their result, we will recall some definitions and notation.

A valuation ϕ is called translation invariant if $\phi(K+x) = \phi(K)$ for all $x \in V$ and $K \in \mathcal{K}^n$ and ϕ is said to have degree i if $\phi(tK) = t^i \phi(K)$ for all $K \in \mathcal{K}^n$ and $t > 0$. We call ϕ even if $\phi(-K) = \phi(K)$ and odd if $\phi(-K) = -\phi(K)$ for all $K \in \mathcal{K}^n$. We denote by Val the vector space of all continuous translation invariant complex valued valuations and we write $\mathrm{Val}_i^{\pm}$ for its subspace of all valuations of degree i and even/odd parity. An important result by McMullen is that

$$\mathrm{Val} = \bigoplus_{0 \le i \le n} \mathrm{Val}_i^{+} \oplus \mathrm{Val}_i^{-}.$$

We need the following basic fact from the representation theory of the group $\mathrm{SO}(n)$: The isomorphism classes of irreducible representations of $\mathrm{SO}(n)$ are parametrized by their highest weights, namely sequences of integers $(\lambda_1, \lambda_2, ..., \lambda_{\lfloor n/2 \rfloor})$ such that

$$\begin{cases} \lambda_1 \ge \lambda_2 \ge ... \ge \lambda_{\lfloor n/2 \rfloor} \ge 0, & n \text{ odd}, \\ \lambda_1 \ge \lambda_2 \ge ... \ge \lambda_{n/2-1} \ge |\lambda_{n/2}|, & n \text{ even}. \end{cases}$$

The natural action of the group $\mathrm{SO}(n)$ on the space Val is given by

$$(\theta\phi)(K) = \phi(\theta^{-1}K), \qquad \theta \in SO(n),\ \phi \in \mathrm{Val}.$$

The authors prove the following decomposition of the space Val into irreducible $\mathrm{SO}(n)$-modules.

Theorem 2.6 *Let $0 \le i \le n$. The space Val_i is the direct sum of the irreducible representations of $\mathrm{SO}(n)$ with highest weights $(\lambda_1, \lambda_2, ..., \lambda_{\lfloor n/2 \rfloor})$ precisely satisfying the following additional conditions:*
(i) $\lambda_j = 0$ for $j > min\{i, n-i\}$;
(ii) $|\lambda_j| \ne 1$ for $1 \le j \le \lfloor n/2 \rfloor$;
(iii) $|\lambda_2| \le 2$.
In particular, under the action of $\mathrm{SO}(n)$ the space Val_i is multiplicity free.

They give also give applications of this theorem to geometric inequalities.

Judit Abardia presented her joint work with Andreas Bernig on projection bodies in complex vector spaces.

Let V be a real vector space of dimension n, and Let K be a convex body in V. The projection body of K is denoted by ΠK and is defined by its support function:

$$h_{\Pi K}(u) = \mathrm{vol}_{n-1}(K|u^{\perp}) = \frac{n}{2}V(K,...,K,[-u,u]), \qquad u \in S^{n-1}.$$

Theorem 2.7 *Let W be a complex vector space of complex dimension m, $m \geq 3$. If the operator $Z : \mathcal{K}(W) \to \mathcal{K}(W)$ is*

1) translation invariant,

2) $SL(W,\mathbb{C})$-contravariant,

3) continuous Minkowski valuation,

then $Z = \Pi_C$, where $C \in \mathbb{C}$ is a convex body and

$$h(\Pi_C K, u) = V(K,...,K,Cu), \qquad u \in S^{2m-1},$$

$Cu = \{cu : c \in C \subset \mathbb{C}\}$.

The converse also holds for every $C \in \mathcal{K}(\mathbb{C})$.

This is a complex version of the result, proved earlier by Monika Ludwig: If an operator $Z : \mathcal{K}^n \to \mathcal{K}^n$ is

1) translation invariant,

2) $SL(V,R)$-contravariant,

3) continuous Minkowski valuation,

then $Z = c\Pi$, $c \in \mathbb{R}^+$.

The dual notion of the projection body is the intersection body. It was introduced by E. Lutwak in 1988 and played a crucial role in the solution to the Busemann-Petty problem. Let K be a star body in $\mathbb{R}^n$. Its intersection body IK is the star body whose radial function is given by

$$\rho_{IK}(\xi) = \mathrm{vol}_{n-1}(K \cap \xi^{\perp}), \qquad \xi \in S^{n-1}.$$

If K is origin-symmetric and convex, then Busemann's theorem asserts that IK is also convex. However, this is not true without the symmetry assumption on K.

Mathieu Meyer jointly with Shlomo Reisner introduced the notion of the convex intersection body $CI(L)$ of L. It is defined by its radial function

$$\rho_{CI(L)}(u) = \min_{z \in P_u(L^{*g(L)})} \mathrm{vol}_{n-1}\left(\left[P_u(L^{*g(L)})\right]^{*z}\right).$$

In this formula, $g(L)$ is the centroid of L, P_u denotes the orthogonal projection from $\mathbb{R}^n$ onto $u^{\perp}$, and if $E \subset \mathbb{R}^n$ is an affine subspace, $M \subset E$ and $z \in E$,

$$M^{*z} = \{y \in E; \langle y - z, x - z\rangle \leq 1 \text{ for every } x \in M\}.$$

They prove that the body $CI(L)$ obtained from this construction is actually convex!

If K is symmetric and convex, then IK is convex. But what can we say about IK if K is merely a star body?

Jaegil Kim presented his work (joint with V. Yaskin and A. Zvavitch), where they extend Busemann's theorem to p-convex bodies. Recall that given a star body K and $p \in (0,1]$, we say that K is p-convex if, for all $x, y \in \mathbb{R}^n$,

$$\|x+y\|_K^p \leq \|x\|_K^p + \|y\|_K^p,$$

or, equivalently $t^{1/p}x+(1-t)^{1/p}y \in K$ whenever x and y are in K and $t \in (0,1)$.

Theorem 2.8 *Let K be a p-convex symmetric body in $\mathbb{R}^n$ for $p \in (0,1]$. Then the intersection body IK of K is q-convex for every $q \leq [(1/p-1)(n-1)+1]^{-1}$.*

The sharpness of this result, its generalizations to some general measure spaces with log-concave or s-concave measures, as well as other geometric implications were also discussed.

Hermann König presented a joint work with Alexander Koldobsky "On the maximal measure of sections of the n-cube".

They study the analogues of Ball's cube slicing theorem for the Gaussian measure and more general measures.

Let $h : [-1,1] \to \mathbb{R}_{>0}$ be even and in C^1. Then

$$d\mu_h(s) := \prod_{j=1}^{n} h(s_j)ds_j / \left(\int_{-1}^{1} h(r)dr\right)^n, \quad s = (s_j)_{j=1}^n \in B_\infty^n,$$

defines a probability measure on the n-cube B_∞^n. For $a \in S^{n-1}$ let

$$A(a,h) := \mu_h\{x \in B_\infty^n | \langle x, a\rangle = 0\}$$

be the $(n-1)$-dimensional measure of the central section orthogonal to a. For $k \in \{1, ..., n\}$, let

$$f_k := \frac{1}{\sqrt{k}}(\underbrace{1, ...1}_{k}, 0, ..., 0) \in S^{n-1}.$$

Theorem 2.9 *Let $h : [-1,1] \to \mathbb{R}_{>0}$ be even and in C^3 with $h' \leq 0$, $h'' \leq 0$, $h''' \geq 0$ on $[0,1]$ and $h(0) \leq \frac{3}{2}h(1)$. Suppose further that*

$$\pi \left(\int_0^1 r^2 h(r)dr\right)\left(\int_0^1 h(r)^2 dr\right)^2 \geq \left(\int_0^1 h(r)dr\right)^5.$$

Consider

$$d\mu_h(s) := \prod_{j=1}^{n} h(s_j)ds_j / \left(\int_{-1}^{1} h(r)dr\right)^n, \quad s = (s_j)_{j=1}^n \in B_\infty^n.$$

Let $a = (a_j)_{j=1}^n \in S^{n-1}$ *with* $a_1 \geq \cdots \geq a_n \geq 0$*. Then, if* $a_1 \leq 1/\sqrt{2}$*,*

$$A(a,h) \leq A(f_2,h).$$

This theorem applied to the Gaussian measure gives

Corollary 2.10 *For* $\lambda > 0$ *consider the Gaussian measure with* $h(r) = exp(-\lambda r^2)$*,*

$$d\mu(s) = exp(-\lambda \|s\|_2^2)ds/\left(\int_{-1}^{1} exp(-\lambda r^2)dr\right)^n, s \in B_\infty^n.$$

Then for $\lambda \leq 0.196262$ *and* $a_1 \leq 1/\sqrt{2}$*,*

$$A(a,h) \leq A(f_2,h),$$

while for $\lambda > 0.196263$ *and large* n*,*

$$A(f_n,h) > A(f_2,h).$$

Alex Iosevich and Eric Grinberg presented their results in discrete geometry and tomography.

Alex Iosevich spoke about distribution of lattice points near families of convex surfaces. He used the operator bounds for generalized Radon transforms to obtain lattice point bounds previously approached using hands on number theoretic methods.

Eric Grinberg presented his joint results with David Feldman.

In the standard mathematical model of tomography, an unknown function in Euclidean space is to be recovered from data regarding its integrals over certain families of lines, planes, etc. The treatment of this problem involves both the geometry of the collection of lines, planes etc., and the analysis of function spaces that model the data. Grinberg and Feldman replaced the Euclidean space by an affine or projective space over a finite field, so as to focus the recovery and inversion problem on the collection lines involved. They also gave a series of properties of the Radon transform in this context culminating in a Gelfand-style admissibility theorem, which characterizes minimal sets of lines whose x-rays determine a function.

Several speakers presented their results on probability and random matrices.

Rafal Latala talked about the tail inequalities for order statistics of log-concave vectors and their applications. He presented the new tail estimates for order statistics of isotropic log-concave vectors and showed how they may be applied to derive deviation inequalities for lr norms and norms of projections of such vectors. Part of the talk was based on his joint work with Radoslaw Adamczak, Alexander Litvak, Alain Pajor and Nicole Tomczak-Jaegermann.

Mark Rudelson studied the following question: to which extent the spectral and geometric properties of the row product of independent random matrices resemble those properties for a matrix with independent random entries, (the row product of K matrices of size d by n as a d^K by n matrix, whose rows are entry-wise products of rows of these matrices). In particular, he showed that while the general volume ratio property does not hold for these matrices, it still holds in case of a cross-polytope.

Peter Pivovarov presented his joint work with G. Paouris on the rearrangements and Isoperimetric Inequalities . They studied the rearrangement inequalities and their use in isoperimetric problems for convex bodies and classes of measures.

Let $\mathcal{P}_{[n]}$ be the class of probability measures on $\mathbb{R}^n$, absolutely continuous with respect to Lebesgue measure.

For $N \geq n$, $x_1, ..., x_N \in \mathbb{R}^n$, consider the $n \times N$ matrix $[x_1...x_N]$. If $C \subset \mathbb{R}^N$ is a convex body, then

$$[x_1...x_N]C = \left\{ \sum_{i=1}^{N} c_i x_i : (c_i) \in C \right\} \subset \mathbb{R}^n.$$

Theorem 2.11 *Suppose*

1) $N \geq n$ *and* $\mu_1, \ldots , \mu_N \in \mathcal{P}_{[n]}$*;* $f_i = \frac{d\mu_i}{dx}$*;*

2) $C \subset \mathbb{R}^N$ *is a convex body. Set*

$$\mathcal{F}_C(f_1, ..., f_N) = \int_{\mathbb{R}^n} ... \int_{\mathbb{R}^n} \mathrm{vol}([x_1...x_N]C) \prod_{i=1}^{N} f_i(x_i) dx_n ... dx_1.$$

If $\|f_i\|_\infty \leq 1$ *for* $i = 1, ..., N$*, then*

$$\mathcal{F}_C(f_1, ..., f_N) > \mathcal{F}_C(\mathbb{1}_{D_n}, ..., \mathbb{1}_{D_n}),$$

where $D_n \subset \mathbb{R}^n$ *is the Euclidean ball of volume one.*

There were two talks on the quantum information theory. This theory is now one of the most active fields in science since the prospect of building quantum computers becomes more and more concrete.

Elisabeth Werner spoke about her joint results with S. Szarek and K. Zyczkowski. They investigated the nested subsets of a convex body formed by the set of trace preserving, positive maps acting on density matrices of a fixed size. Working with the measure induced by the Hilbert-Schmidt distance they derived asymptotically tight bounds for the volumes of these sets.

Deping Ye gave a talk about his joint results with G. Aubrun and S. Szarek. He discussed the problem of the detecting quantum entanglement , which is a central problem in the quantum information theory. First discovered by Einstein-Podolsky-Rosen in 1935, quantum entanglement serves as fundamental and key ingredients for many objects in quantum information, such as, quantum algorithms, quantum key distributions, and quantum teleportation.

A quantum state ρ on the N dimensional system $\mathcal{H}_N$ may be identified as a density matrix, i.e., an $N \times N$ positive semi-definite matrix with trace 1. It can be obtained by partial tracing over the K dimensional environmental system $\mathcal{H}_K H$; namely, $\rho = MM^\dagger$ where M is a $N \times K$ (complex) matrix and $M^\dagger$ denotes its complex conjugate. Deping presented the recent progress on estimating the threshold K, such that a random induced quantum state being separable and/or entangled.

The vast majority of the participants were interested in problems related to duality. The duality problems discuss the relations between the properties of a given convex body K, and the properties of its polar K^*. In particular, many questions about sections and projections of convex bodies fall into this category. Several conjectures stipulate that a direct duality connection between projections and sections, if found, would lead to a significant progress in the area of convex geometry.

Mahler conjecture, asking for the minimum, among all convex K, of the volume product $vol_n(K)vol_n(K^*)$ is, in a way, a step to resolve the mystery. Despite many important partial results, the problem is still open in dimensions 3 and higher.

At the workshop, several participants reviewed the known results related to the conjecture. Carsten Schütt explained that the minimum of the volume product may not be reached for the body having a positive curvature at a point. Yehoram Gordon presented a proof of the functional version of the above result. The approach was extensively discussed by a group of participants.

An interest has been expressed in discrete versions of results related to duality and volumes of polytopes. Shlomo Reisner discussed the relations of the volume product of polygons, and presented a method that allows to prove the following result: the volume product of polygons in $\mathbb{R}^2$ with at most n vertices is bounded from above by the volume product of regular polygons with n vertices.

A classical bound on the minimum of the volume product, given by Mahler himself in the two-dimensional case, is based on a beautiful procedure of "erasing vertices". The analogue of this idea in the three (and higher)-dimensional space is not known, and it would be very interesting to understand the structure of "neighbouring" polytopes that has the same vertices plus(minus) one additional vertex. Viktor Vigh gave a talk on on the sewing construction of polytopes, which allows one to construct a wide variety of neighbourly polytopes that are not necessarily cyclic. He also presented some new results on the sewing construction and, as a corollary, a fast algorithm for sewing in practice.

There were other results on lattice polytopes. Ivan Soprunov explained how the bound on the number of interior lattice points of a lattice polytope P, in terms of the volume of P, is related to zeroes of polynomial systems.

David Alonso-Gutiérrez gave a talk on the factorization of Sobolev inequalities through classes of functions, based on a joint work with J. Bastero and J. Bernués.

For $1 \le p < \infty$ and a function $f : \mathbb{R}^n \to \mathbb{R}$, define

$$\|f\|_{\infty,p} = \left(\int_0^\infty (f^{**}(t) - f^*(t))^p \frac{dt}{t^{p/n}} \right)^{1/p},$$

where f^* is the decreasing rearrangement of f, and f^{**} is the Hardy transform of f^* defined by $f^{**}(t) = \frac{1}{t}\int_0^t f^*(s)ds$.

Let

$$\mathcal{E}_p^+(f) := \frac{2^{1/p}}{I_p} \left(\int_{S^{n-1}} \|D_u^+ f\|_p^{-n} du \right)^{-1/n},$$

where $D_u^+ f := \max\{\langle \nabla f(x), u\rangle, 0\}$, and $I_p^p := \int_{S^{n-1}} |u_1|^p du$.

The authors use tools from classical real analysis and recent advances in convex geometry to establish the correct relation between $\mathcal{E}_p^+(f)$ and $\|f\|_{\infty,p}$.

Theorem 2.12 *Let $1 \le p < \infty$ and $\frac{1}{q} = \frac{1}{p} - \frac{1}{n}$, $q \in (-\infty, -n) \cup \left[\frac{n}{n-1}, \infty\right]$. Then*

$$\mathcal{E}_p^+(f) \ge \left(1 - \frac{1}{q}\right) n\omega_n^{1/n} \|f\|_{\infty,p}, \qquad \forall f \in W^{1,p}(\mathbb{R}^n).$$

Moreover the constant is sharp.

Igor Rivin talked about the "limit" convex sets of finite volume in hyperbolic space. He indicated some results on the dimension (Minkowski and Hausdorff) of such sets, and gave some geometric corollaries. He also presented an analogue of Dvoretzky's Theorem in the context of Hyperbolic Geometry.

3 Outcome of the meeting

The meeting was very successful, we were lucky to bring together mathematicians from many countries and many research areas, such as harmonic analysis, theory of valuations, discrete geometry and tomography, probability and random matrices, quantum information theory. Besides the leading scientists, we also had 14 graduate students and recent PhDs participating in the workshop. The friendly atmosphere created during the workshop helped many participants not only to identify the promising ways to attack the old problems but also to get acquainted with many open new ones.