BIRS Workshop #11w5025 — Algebraic Combinatorixx

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1 Overview of the Field

Algebraic combinatorics is a broad discipline with substantial connections to many areas of mathematics and physics such as representation theory, algebraic geometry, number theory, knot theory, mathematical biology, statistical mechanics, symmetric functions, invariant theory, and quantum computing. These connections are reflected in the topics that were featured at the workshop:

I. **Combinatorics of Representations**

  (i) representations of groups and algebras
  (ii) combinatorial objects that arise in the study of representations such as crystal bases, Littelmann paths, tableaux, quivers, Littlewood-Richardson coefficients, knots and tangles, and alternating sign matrices

II. **Geometry and Combinatorics**

  (i) Schubert varieties, Grassmannians, cluster algebras, and tropical geometry
  (ii) simplicial complexes, polytopes, discrete Morse theory, Whitney stratification, phylogenetic trees
  (iii) reflection and braid groups and hyperplane arrangements

III. **Combinatorial Functions**

  (i) symmetric functions such as group characters, Schur functions, Schur P-functions, k-Schur functions, and Macdonald polynomials (both symmetric and nonsymmetric)
  (ii) quasisymmetric functions, combinatorial Hopf algebras, and noncommutative Schur functions,
  (iii) Kazhdan-Lusztig polynomials.

The topics are interconnected and the workshop focused on many interesting, open problems. Among them:

  (a) study the composition poset that arises from considering quasisymmetric functions and find a tower of algebras connected with it;
  (b) provide a straightforward combinatorial description for Kazhdan-Lusztig polynomials; and
  (c) give a combinatorial interpretation for the Kronecker product of Schur functions.

The answers would impact representation theory, algebraic geometry, mathematical physics, and computer science.

2 Goals of the Workshop

The workshop brought together forty-two senior and junior female mathematicians to collaborate on cutting-edge research problems in algebraic combinatorics and related fields and to forge mentoring networks, with the long-term goal of increasing and strengthening the participation of women in mathematics.

The workshop also had a major mentoring component. Almost two-thirds of the participants were early-career (pre-tenure or recently tenured) women faculty members, postdoctoral fellows, or graduate students.

3 Format

The program for the workshop consisted of seven 45-minute survey lectures on the latest research developments, which were given by more established researchers, and twelve 25-minute shorter research talks. In addition, eleven participants presented posters and gave a brief 2-3 minute preview of their poster before it was displayed. Roughly half the time was spent working in smaller groups organized according to research interests, where actual research problems were discussed. Groups gave brief daily updates to the entire workshop audience on the topics and problems they had worked on and posed open questions. As a result, there was much collaboration and discussion. At the end of the workshop, the groups presented reports (which are attached below). During two of the evenings there were panel discussions on building the tools to succeed: from professional development to work-life balance. BIRS Scientific Director, Professor Nassif Ghoussoub, and chair of the BIRS Board of DIrectors, Attorney Karen Prentice, visited during the workshop, spoke with participants, and led them on a tour of the new TransCanada PipeLines Pavillion scheduled to open in the fall.

4 Participants

Before submitting a formal proposal to BIRS, the organizers sent out a description of the proposed workshop to women working in algebraic combinatorics to gauge interest. Within 24 hours, the organizers had received positive responses from over half of the women contacted, and within less than a week's time, they had heard from all but two. Some of the women offered to prepare reading lists and to help apply for grants.

To encourage more participation from early-career mathematicians and from those working at smaller colleges, the organizers advertised for applications in various venues including the Association for Women in Mathematics newsletter and website, through the Canadian Mathematical Society and its Women in Mathematics Committee, and by sending out announcements to several combinatorics email lists and to mathematics departments across the U.S. The response was overwhelming. Less than a third of the junior-level mathematicians who applied could be accommodated. The majority of the workshop participants were quite junior (26 of the 42 participants received their doctoral degrees after 2001 or are currently enrolled as graduate students).

5 Group Research Projects and Scientific Progress

5.1 Group I: Alternating Sign Matrices, Crystal Bases, and Tableaux

Members: Julie Beier, Angèle Hamel, Gizem Karaali, Anne Schilling, and Jessica Striker. After the first day Sophie Morier-Genoud and Karola Meszaros also participated.

The group spent most of its time on two separate projects related to common themes that interested the whole group. The first project attempted to interpret the poset of alternating sign matrices as a modified crystal graph. The second project aimed to compute the order of the promotion map on hooks. At this time they have a conjecture for the latter and some computational evidence that supports it. In the former project, too, the group has made some progress. The group is excited about these two questions and is continuing to work on them together.

5.2 Group II: k-Derangements and the Sandpile Problem

Members: Hélène Barcelo, Camillia Smith Barnes, Heather Dye, Susanna Fishel, Kristina Garrett, Kathryn Nyman, Bridget Tenner

The group began the week investigating k-derangements and eventually turned its attention to the Sandpile Problem.

A *k-derangement* is a permutation in S_n such that the induced permutation on the set of all unordered k-tuples leaves no k-tuple fixed. A permutation $\sigma \in S_n$ is a k-derangement if and only if the cycle decomposition of σ does not contain a set of cycles whose lengths partition k.

The group found a recursive formula for the number of 2-derangements in S_n and used this to verify an existing exponential generating function (A. Fraticelli, Missouri State REU 2009). However, finding a closed form for the number of 2-derangements of n requires counting partitions which avoid subpartitions, and this turns out to be very difficult.

A *sandpile* is a partition $\lambda = (\lambda_1, \ldots, \lambda_t)$ represented by its Ferrers shape. The group studied the following operation on sandpiles: a grain of sand can "fall" from column i to column $i+1$ provided $\lambda_i - \lambda_{i+1} \geq 2$. This Sandpile Model induces a directed graph on the set of partitions of n: place an edge from μ to λ if λ arises from a grain of sand falling in μ. Note that this graph is generally not connected. A partition in which no "sand" can fall further is a *fixed point* (a sink of the graph), and a partition which could not have arisen from a dynamic operation on any previous partition is a *garden of eden* (a source).

The group raised the following questions about the Sandpile Problem and made progress on several of them.

1. How many partitions are both fixed points and gardens of eden?
2. The directed sandpile graph is contained in the (non-graded) dominance poset, $\mathcal{D}_n$ and as such is a poset itself. Is this sandpile poset graded?
3. Given a fixed point, can we determine its connected component?
4. What are the equivalent questions for the more generalized models? (Jumping Pile, Ice Pile, Theta Model, etc.)

Partitions which are both fixed points and gardens of eden were easily characterized. The group is now working on recursive formulas, and enumerating these "*fixed gardens*" may be possible with generating functions. They believe they can describe the covering relations in $\mathcal{D}_n$ which do not survive in the sandpile subposet, and that the resulting connected sandpile components are graded. Finally, they suspect that the connected components are obtained as follows. Take a highest-ranked fixed point and its upper order ideal. This ideal corresponds to one of the graph components. Removing this ideal and repeating this process with the remaining partitions would give the next component, and so on, until all components have been recovered.

The group plan is to collect and then disseminate our individual notes in mid to late June. The group hopes to meet via video conferencing later in the summer to continue work on these questions with an eye on extending the theory to the models mentioned above (Jumping Pile, Ice Pile, Theta Model, etc.).

5.3 Group III: Symmetric and Quasisymmetric Functions

Members: Christine Bessenrodt, Soojin Cho, Huilan Li, Sarah Mason, Vidya Venkateswaran, Stephanie van Willigenburg, Martha Yip, Meesue Yoo

The group investigated quasisymmetric and Schur P-functions. For a composition α, F is said to be a composition tableau of shape α if it has α_i cells in the i-th row from the top, and the diagram is filled with positive integers such that the following three conditions are satisfied:

1. left-most column is strictly increasing from top to bottom
2. rows are weakly decreasing from left to right

3. triple rule: supplement F by adding enough cells with zero valued entries to the end of each row so that the resulting supplemented tableaux, $\hat{F}$, is of rectangular shape $l \times m$. Then for $1 \leq i < j \leq l$, $2 \leq k \leq m$,
$$\left(\hat{F}(j,k) \neq 0 \text{ and } \hat{F}(j,k) \geq \hat{F}(i,k)\right) \Rightarrow \hat{F}(j,k) > \hat{F}(i,k-1).$$

Then the quasisymmetric Schur function [8] indexed by a composition α is defined to be

$$\mathcal{S}_\alpha = \sum_{\substack{F \text{ a composition} \\ \text{tableaux of} \\ \text{shape } \alpha}} x^F,$$

where $x^F = \prod x_i^{\#i}$. These functions refine the Schur functions indexed by partitions λ as follows:

$$s_\lambda = \sum_{\tilde{\alpha}=\lambda} \mathcal{S}_\alpha. \tag{1}$$

The Schur P-function is defined by

$$P_\lambda = \sum_T 2^{-l(\lambda)+b(T)} x^T,$$

where T are certain SSYTx of shape λ, $b(T)$ is the number of positions where i occurs in a given column but not in the next column, and λ is a strict partition (see [9]).

Research Collaboration: The group investigated the following natural question: just as Schur functions decompose into quasisymmetric Schur functions, as given in [8], is it possible to decompose the Schur P-functions into a sum of quasisymmetric Schur P-functions? The group was able to answer this question in the affirmative and will study several other aspects of these quasisymmetric Schur P-functions. In particular, they would like to express these functions in other bases, find a Pieri rule, and see if there is a representation-theoretic interpretation.

5.4 Group IV: Diagram Algebras and the Representation Theory of the QSYM Poset

Members: Georgia Benkart, Christine Bessenrodt, Maud Devisscher, Rosa Orellana, Alison Parker, Monica Vazirani

Group discussions focused on two problems. The first, which was inspired by Kronecker products for symmetric groups, explored the following question. Given two diagram algebras A_k and A_ℓ of the same kind (perhaps both Temperley-Lieb algebras or both partition algebras or both Brauer algebras), and an irreducible module M_k and M_ℓ for each, determine the decomposition for the induced module

$$\mathrm{Ind}_{A_k \times A_\ell}^{A_{k+\ell}} M_k \boxtimes M_\ell$$

as a module for $A_{k+\ell}$ in the generic semisimple case and the structure of the induced module in the nongeneric case. The second problem the group discussed involved the composition poset that arose from considering quasisymmetric functions. The group viewed this poset as the labels for the irreducible modules of a tower of algebras in the same spirit as Young's lattice of partitions is the poset corresponding to irreducible modules of the symmetric groups $\mathcal{S}_k$ (or their group algebras in characteristic 0). They computed what might be dimensions for these algebras. At first the numbers seemed to be matching with the number of connected planar maps with k edges, but the numbers diverged starting with compositions of 6. Paths in the poset correspond to certain tableaux. If p_α is the number of paths from the unique composition of 0 to a given composition α, and if $\tilde{\alpha}$ is the partition corresponding to α, then $\sum_{\alpha:\tilde{\alpha}=\lambda} p_\alpha = \dim S^\lambda$, where S^λ is the Specht module labelled by λ. It is an interesting problem to look for patterns in these path numbers and for a possible analogue of the hook length formula.

5.5 Group V: Möbius Function of the QSYM Poset

Members: Heather Dye, Patricia Hersh, Karola Meszaros, Bridget Tenner, Lauren Williams

The group studied an analogue of Young's lattice, determining the Möbius function and homotopy type of each interval. The posets considered were recently introduced by Christine Bessenrodt, Kurt Luoto, and Stephanie van Willigenburg [2] in their development of a Pieri Rule for noncommutative symmetric functions (NSYM). A composition v covers a composition u in this poset if multiplication of the composition u by a single box yields a positive combination of compositions in which v appears with a positive coefficient. The group became interested in the question of how strong an analogy exists between these posets and the Pieri Rule poset for traditional symmetric functions, namely Young's lattice.

Unlike Young's lattice, these new posets are not distributive lattices, are not lattices, and in fact are not shellable (or even Cohen-Macaulay). Nonetheless, the group proved that each interval is homotopy equivalent to a ball or sphere using lexicographic discrete Morse functions. This implies that the Möbius function of each interval is 0, 1 or -1, something these posets do have in common with Young's lattice (where again each interval is homotopy equivalent to a ball or a sphere).

Another result shown during the workshop was a closed form description of which pairs of elements u, v satisfy u less than or equal to v. The previous description by [2] was of the covering relations, and did not make apparent which pairs u, v would be comparable. Further collaboration on a closely related poset, one giving the Pieri Rule for quasisymmetric functions is planned for the near future. Ongoing group work involves collecting Möbius function data using Stembridge's posets software package. The group expects to complete a paper this summer with an analysis of the NSYM poset, and hopefully also the QSYM poset.

In a different direction, two members of the group (Patricia Hersh and Lauren Williams) also continued work in their ongoing collaboration to determine the homeomorphism type of the totally nonnegative part of the Grassmannian. In particular, they identified further properties of reduced expressions for permutations that are needed to extend to the setting of Postnikov's plabic graphs, and made definite progress towards the proofs. They also analyzed fibers over 0-cells in Postnikov's map from a polytope of plabic graphs to the totally nonnegative part of the Grassmannian. The time at BIRS provided a good opportunity for them to make further headway on this fairly involved project.

5.6 Group VI: Chip Firing

Members: Melody Chan, Caroline Klivans, Megan Owen, Josephine Yu

A chip firing game consists of a graph with its vertices labelled by integers, which can be thought of as the number of chips allocated to that vertex, along with a distinguished vertex called the *bank*. A vertex *fires* by sending one chip to each of its neighbouring vertices. Two group members, Josephine Yu and Caroline Klivans, were already interested in different variations of the chip firing game, so they first presented their work, and then the group focused on a generalization of the chip firing game to a higher dimensional complex, instead of a graph.

Josephine Yu told the group about a variant of the chip firing game, in which the underlying graph has lengths on its edges, and the definition of chip firing moves is expanded to make them equivalent to the set of *tropical rational functions* on that metric graph [7]. In this scenario, the set of all chip firing moves that makes a chip configuration non-negative forms a *tropical linear system* or *tropical module*. The set of all such functions also forms a cell complex.

Caroline Klivans explained the *dollar game*, in which the bank is the only vertex allowed a negative value and can only fire when no other vertex can fire. A chip configuration is *stable* when only the bank can fire, *recurrent* when there exists some sequence of fires that bring the configuration back to this starting one, and *critical* when it is both stable and recurrent. The set of critical configurations is particularly interesting, because it forms a finite Abelian group, called the *critical group* (or *sandpile group* or *Picard group*), which is isomorphic to the quotient group of $\mathbb{Z}^{n-1}$ modulo the reduced Laplacian. Using this definition of the critical group, it is possible to generalize these results to higher dimensional simplicial complexes [6]. However, in this context, the original meaning of a critical configuration is lost.

The group spent the remaining group time trying to figure out a combinatorial definition of a critical configuration in this higher dimensional setting. In particular, to bridge the two perspectives, they tried to generalize the idea of tropical rational functions on a graph to tropical rational functions on a simplicial complex.

5.7 Group VII: Flag Homology Spheres and the Gamma Vector

Members: Margaret Bayer, Margaret Readdy, Michelle Wachs

The group used Gal's conjecture as a starting point for their discussions. Recall that for an n-dimensional simplicial complex with its h-vector encoded as the polynomial $h(t) = h_0 + h_1 t + \cdots + h_n t^n$, the *gamma vector* $\gamma = (\gamma_0, \ldots, \gamma_n)$ is defined by the change of basis formula

$$h(t) = \sum_{i=0}^{n} \gamma_i \cdot t^i (1+t)^{n-2i}.$$

In 2005 Gal conjectured that for flag homology spheres the entries of the γ-vector are nonnegative.

There are five questions the group is pursuing.

1. Which h-vectors occur from flag homology spheres? A special case is to first study simplicial polytopes whose Stanley-Reisner ring has its face ideal generated by squarefree degree 2 monomials.
2. It is known that flag homology spheres occur from the operations of taking the barycentric subdivision of a flag homology sphere or by barycentrically subdividing an edge of a flag homology sphere. Do these two operations capture all flag homology spheres?
3. Given a γ-vector with entries $\gamma_i \geq 0$, $i = 0, \ldots, n$, construct a poset whose order complex has that γ-vector.
4. Given a flag homology sphere not arising from the order complex of a poset, is there another poset construction or generalization of the order complex construction?
5. It is known that substituting $\mathbf{cv} = \mathbf{1}$ and $\mathbf{dv} = \mathbf{2t}$ into the $\mathbf{cd}$-index of an Eulerian poset P recaptures the gamma vector of the order complex of P. Purtill showed the coefficients of the $\mathbf{cd}$-index of the face lattice of the n-simplex and the n-cube enumerate the "$\mathbf{cd}$-variation" of André and signed André polynomials. Do the resulting coefficients of the γ-vector in these two cases have meaning?

5.8 Group VIII: Quantum Schubert Calculus

Members: Elizabeth Beazley, Anna Bertiger, Nicole Lemire, Anne Shepler, Kelli Talaska, Julianna Tymoczko

The group studied a problem in quantum Schubert calculus. The goal was to better understand the quantum cohomology ring of G/B for groups G of all types. The tools available are using the isomorphism

$$QH^*(G/B) \simeq H^*(G/B) \otimes_{\mathbb{Z}} \mathbb{Z}[q_1, \ldots, q_r]$$

and the fact that $H^*(G/B)$ has a basis of the Schubert classes, indexed by the Weyl group of G. The big open problem in quantum Schubert calculus is to find the closed formulas for the numbers $c_{u,v}^w$ and monomials $q^d = q_1^{d_1} \cdots q_r^{d_r}$ such that

$$\sigma_u * \sigma_v = \sum_{d,w} c_{u,v}^w q^d \sigma_w.$$

Postnikov has provided a combinatorial formula for all the monomials q^d that appear with nonzero coefficient in any product $\sigma_u * \sigma_v$ in terms of certain admissible paths in the quantum Bruhat graph in type A. Since $QH^*(G/B)$ is isomorphic to $\mathbb{Z}[q_1, \ldots, q_r][x_1, \ldots, x_r]/I_q^W$ where I_q^W is the ring of invariants under the action of the Weyl group W, the key was to define an explicit quantum Pieri rule. This week, the group produced a conjectural type-free quantum Pieri rule, from which a type-free admissibility condition and hence a closed combinatorial formula for the monomials q^d can easily be derived. Their conjecture matches the known formulas when specialized to type A, and the group's next step will be to test the conjecture in other types and then either modify or prove the conjecture. In future work, they would consider the case of G/P for an arbitrary parabolic subgroup P.

6 Outcome of the Meeting

The organizers were inspired to propose this workshop after the highly successful "Connections for Women Workshop on Combinatorial Representation Theory and Representations of Finite Groups and Related Topics" at the Mathematical Sciences Research Institute in January 2008 and the "WIN" (Women in Numbers) Workshop at BIRS in November 2008. Both programs highlighted accomplishments of women researchers while introducing younger participants to role models, potential mentors and collaborators, and important problems in the field. We expect the momentum generated by "Algebraic Combinatorixx" to have long-lasting effects far beyond the 5-day meeting just as the "WIN" activities have continued since the time of their 2008 BIRS meeting and include a conference proceedings, special sessions at national and sectional AMS meetings, and continued collaboration. As yet another indication of the successful impact of "WIN" as well as the continued need, "WIN2: Women in Numbers 2" was among the 48 workshops chosen (out of 142 proposed) to be held at BIRS in 2011.

The long-term benefits of the Algebraic Combinatorixx workshop are expected to be an increase in the participation of women in research activities related to algebraic combinatorics and related fields, a research network of potential collaborators, and visibility and connectivity for younger researchers especially those at smaller colleges or isolated in departments not having a strong research presence in combinatorics. Given that almost two-thirds of the participants will have received their PhDs after 2001, they (and their experiences at this workshop) will play a critical role in shaping the field for a long time to come.

7 Testimonials

The following quotes are taken from testimonials from workshop participants. Some participants comment on the mentoring component of the workshop; others mention the high level of talks; and others stress the importance of the working groups and successful research projects started while at BIRS.

> The BIRS workshop greatly impacted my research. In general, over the past year I have wanted to begin a successful collaborative mathematical project in algebraic combinatorics, with an aim of broadening the scope of my active research beyond the immediate sphere of my thesis. Geographical and financial constraints make forging such collaborations difficult, and I am incredibly indebted to the organizers for helping to make this professional goal a reality during the BIRS workshop. In particular, there was a new project that I was interested in working on, for which I did not feel as though I was individually equipped with the requisite background. I proposed this problem to my designated research group, we generated a conjecture which would solve the problem, and we currently have a working draft of a joint paper. Of the four other group members, three of the four are individuals with whom I likely would not have had a chance to collaborate otherwise, and whose energy and expertise were vital to the movement of the project. My group members will certainly remain important contacts for me, and without the workshop several of these professional relationships would not have been made. Moreover, given more time at the workshop, I am confident that one or two additional collaborations might have begun, both with people in and outside of my designated working group. —

> The participation in the workshop was an amazing experience. The quality of the talks was exceptional, and the discussion between talks was very lively; I met many mathematicians (in particular younger ones) whom I hadn't known before. The 'research in teams' initiated by the organizers will probably lead to a joint paper with members of the team; it was a fruitful discussion on quasisymmetric analogs of Schur P-functions which both are among my current research interests (apart from the organizer Steph van Willigenburg I had not met any other member of the team before). A talk of one of the principal speakers (Anne Schilling) on the rather new topic of k-Schur functions together with direct exchange on her data on k-Schur functions already led to a conjecture on the determinants of the corresponding tables and a refined one on the invariants; this is closely related to my own work on character tables of the symmetric groups. We have exchanged emails after the meeting and will keep in touch on these questions. A poster and short presentation by a younger participant (Kelli Talaska) on determinants for special graphs led to an exchange on determinants and invariants for paths in certain quiver algebras, where formulae of a similar structure occurred in work of mine; it is not clear yet

what can be learned from this observation. Also, data on the composition poset appearing in work with Steph van Willigenburg and collaborators were discussed also with Georgia Benkart; here, the question is whether the data helps in finding an algebra that is connected with the composition data. —

This was, by far, the best conference I have ever been to. Everything was organised as to maximize interactions between (junior and senior) participants. And it worked brilliantly! I have come back with several new research projects with new collaborators. More specifically, I have started a project with Rosa Orellana on the restriction/induction rules for the Partition algebra (and other diagram algebras). This should give a new approach to the longstanding problem of describing the Kronecker product of Schur functions. With my "working group" at the conference (Benkart, Vazirani, Parker, Orellana), we have started investigating possible representation theoretic interpretations of the quasi-Schur functions introduced by van Willigenburg, Mason, Bessenrodt and al. and we plan to continue this work in the coming months. Following my talk at the conference, I have also had very useful discussions with Georgia Benkart, who suggested new directions for my own work, such as investigating the representation theory of the derragement algebra, and the relationship between the decomposition of tensor space for the orthosymplectic super Lie algebra and the Brauer algebra. I would also like to mention another aspect of the conference which (indirectly) influences my research as well. I did not know there were so many women in this field. I usually only meet a few at conferences. And many of them face the same challenge of juggling research and family life. I felt really encouraged hearing about other (more senior) people's stories and sharing experiences. —

Participation in the BIRS workshop Algebraic Combinatorixx led to new research ideas, new research collaborations, and meeting new people. This was a fantastic workshop, well scheduled to really take advantage of being at BIRS. We had many interesting talks, but also plenty of time dedicated to small working groups. This workshop had a secondary theme beyond the research topic. It was an all women's workshop and time was set aside for panel discussions and general group discussion of the issues and difficulties facing women in mathematics. I truly appreciate BIRS supporting this additional aspect of the workshop. It is hard to describe the need and positive effect such gatherings have for female mathematicians. Among other things, I was "re-charged" to come home and face the challenges that arose while I was away. —

I found this conference to be well-organised with a surprisingly refreshing format. The organisers decided to have more expository talks - from which one learns more anyhow and fewer talks in general in order to put an emphasis on group discussions. My particular group was a good mix of people from different backgrounds and stages in their career. Our group started a project in a area that is new to all of us but for which our common background allowed us to discuss. I learned quite a lot from these discussions as well as from the talks. We hope that a paper (at least one) will result from this collaboration and are in the process of working further on it. This collaboration most likely would not have started up without this conference. Although this workshop did not personally affect my job prospects as I am tenured, it was a valuable opportunity for mentoring as most of the participants were in the early stages of their career. I had some doubts about the all-woman format before attending, but was pleasantly surprised on how the whole thing worked out. I would say that it was one of the most effective workshops that I have attended and the organisers are to be congratulated. —

The workshop impacted my current research. In our group, we worked on quasi-symmetric Schur P-functions. We conjectured the formula for that and plan to work on more problems related to quasi-symmetric Schur P-functions. We hope to write a paper together. In this workshop I worked with some professor from Korea. This was the first time I could collaborate a Mathematician from a such far place. The workshop affected my job prospects. The other participants told us that women in Math usually are more productive as age grows. —

This was the best conference I have ever attended. The mathematics was superb, and I have started (at least) two new collaborative projects as a result. Additionally, the opportunity for panels and other discussions about women's issues in academia/mathematics was very valuable. I hope we can do this again!!! —

I generally do not like to rank events, but I must confess that if I was asked what is the best math workshop you ever attended, this would be it! I am so thankful for the opportunity you gave us to learn about some of the ”hot topics” in combinatorics, as well as for this unique occasion to start new and exciting collaborations. I felt revived, and so much energised. I am so thankful that there are so generous and thoughtful people such as yourselves. Your services to the combintorics as well as the combinatorixx communities are immense! Let me know if you would need help for the next one! —

This was a fantastic workshop. The scholarship was outstanding. The focus on professional development was at an exceptionally high level. The research that I began during that workshop has continued with a dedication that I have never experienced in a research group this large, or from research arising out of an impromptu discussion. The professional contacts that I made and solidified promise to be exceedingly useful, both to my research and career more broadly (mine as well as theirs, I hope). The organizers are to be commended for the time, energy, and thoughtfulness that they put into this conference. I hope that BIRS will support more workshops of this form—I can only imagine the scientific impact if every research area had a group of organizers who could bring together women in their discipline this effectively. —

Every time I have the opportunity to attend BIRS I have an outstanding experience. This workshop was no exception. The opportunity to meet a wide range of women algebraic combinatorialists led to my making many new valuable contacts across the globe. Furthermore the breadth of talks and high quality of exposition meant that I cemented a number of concepts and deepened my knowledge of many more. The poster sessions for graduate students helped them to bring their research to the attention of many senior mathematicians, and the panels gave much valuable insight to life in academia for all participants. One of the most amazing aspects of the workshop was the collaboration groups, which brought together researchers of similar interests in groups to discuss open problems. By the end of the workshop at least four papers were originated (my group included), with the amenities that BIRS provides being crucial to the success of these projects: breakout rooms to work in, the library for resources, the lounge for continuing discussions late into the night, the mealtimes for swapping ideas with other groups. As an organizer, I was thrilled to hear from a number of participants that this was the best conference they had *ever* attended, and that BIRS was a fundamental contributor to this. —

8 Future Plans

We expect to maintain a web site and mailing list so that communication and collaboration can continue far beyond the five days of the workshop. Many of the groups continue to work on the projects begun at BIRS and several papers are expected to result from these collaborative efforts. Plans were discussed for possible future meetings and for subsets of the participants to meet in research groups either via teleconferencing or by research visits.

9 Acknowledgments

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