Renaissance Banff II: Mathematics, Music, Art, Culture (09w5134)

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In July 2009, more than 150 architects, artists, computer scientists, dancers, educators, hobbyists, mathematicians, musicians, and assorted creative thinkers assembled in Banff for the 12th annual Bridges conference. The conference, which has been travelling across North America and Europe since its genesis in Winfield, Kansas, was hosted this year by BIRS with the cooperation of the Banff Centre. The conference was previously held in Banff in 2005, and we were delighted to have an opportunity to return.

In this report, we introduce the Bridges Organization and discuss its aims, and then describe the 2009 conference and summarize its outcomes.

1 Bridges

The Bridges conference is an annual international event that has been running since 1998. It has provided a remarkable model of how seemingly unrelated and even antipodal disciplines, such as mathematics and art, can be brought together. During the conference, practicing mathematicians, scientists, artists, educators, musicians, writers, computer scientists, sculptors, dancers, weavers, and model builders have come together in a lively and highly charged atmosphere of mutual exchange and encouragement. It is important in this event to provide a number of outlets for dissemination of ideas, in order to support a wide diversity of backgrounds and presentation skills. In addition to our technical paper presentations, we offer an exhibition of visual art, hands-on workshops hosted by practitioners, educators and artists, and evening cultural events. Furthermore, a lasting record of each Bridges conference is its Proceedings, a resource book of the papers and the visual presentations of the meeting.

For the past few years the conference has brought about 250 participants annually from more than twenty-five countries together. Moreover, since the conference reaches the public by providing mathematical art exhibits, musical events, public lectures, and theater shows, the number of attendees can reach 500 in some cases. Most participants are faculty members from colleges and universities who are interested in interdisciplinary, multidisciplinary and trans-disciplinary research. Many are seeking new ways of teaching mathematics and science using tools empowered or borrowed from disciplines such as art and music.

The overall goals of the Bridges conferences can be summarized as follows:

1. Introduce participants to innovative and integrative techniques that promote interdisciplinary work in the fields of mathematics and art.

2. Allow mathematicians and artists who are crossing the mathematics/arts boundaries opportunities to present their work, meet each other, and exchange ideas.

3. Offer encouragement and inspiration to teachers of mathematics, science, and art at all levels by revealing relationships between mathematical subjects and their artistic/aesthetic presentations.
4. Provide participants with an opportunity to improve their understanding of fields and disciplines outside their primary area of study.
5. Offer visual displays and performances of mathematically-themed art.
6. Produce interdisciplinary resource materials such as conference proceedings books, CDs and DVDs for documenting the works of artists and scientist, or for educational purposes.

1.1 Planning and organization of Bridges 2009

Preparations for Bridges 2009 began almost immediately after the end of the 2005 conference, when Craig Kaplan agreed to serve as the general chair. This early in the process, the work consisted primarily of applying to BIRS to host the 2009 event. There was also a brief period of activity in 2007 when Bridges Organization board members happened to be at BIRS for a 5-day workshop. We took the opportunity to meet with several key people around the Banff Centre (such as Barry Schiffman from music and Jim Olver from customer services), in order to solicit their participation and support for the conference and discuss opportunities for joint events.

Over the past decade, the Banff International Research Station (BIRS) has grown to become a focal point for mathematical interaction within North America and worldwide, and has made The Banff Centre an ideal place to explore ideas that lie in the intersection of art and mathematics. We were thrilled to be invited back to Banff for 2009.

As in the past few years, we created an online submission website for papers and workshops, using the free service EasyChair (`www.easychair.org`) offered by the University of Manchester. Each submission was assigned to two members of our international Program Committee for review. In some cases one or more additional reviews were provided by other Program Committee members or by external experts. The quality of the reviewing was quite high, with many reviewers providing fine-grained, detailed advice for improving a paper's content or presentation. In several cases authors wrote back to express their gratitude to the reviewers who helped improve their papers significantly.

Once the reviewing process was complete, Craig Kaplan, the chair of the Program Committee, worked with the reviewers and the Bridges organizers to arrive at a decision for each paper. Occasionally the decision process called for additional opinions or communication with the authors. Submissions of all kinds were either accepted as-is, accepted with revisions, or rejected. In addition, some regular paper and workshop submissions were recommended for inclusion in the short papers program. In the end, we arrived at a total of 38 regular papers (including four contributed papers from invited speakers), 24 short papers (including two contributed paper from invited speakers), and 11 workshops. These papers represent a wide range of multidisciplinary work in mathematics, art, culture and pedagogy, from an equally diverse group of authors from around the world.

The Bridges Visual Art Exhibition is always a highlight of the conference. Robert Fathauer, the curator, collected submissions. He then assembled a Jury that decided on the works to include in the exhibition. This year the exhibit featured the work of 40 artists from eight countries working in a variety of visual media. One design by Brian Evans was chosen as the cover image for the Proceedings.

2 Invited speakers

In order to guaranty a quality conference that inspires the participants to discover new ideas and connections between mathematics and other disciplines, we invited a diverse range of creative and talented plenary speakers. The following people were among the invited speakers:

- **Jim Bumgardner** is a senior technical guru at Topspin Media, a teacher at Pasadena's Art Center College of Design, and the creative mind behind `CoverPop.com` and `KrazyDad.com`. An expert in graphics and music software, Jim makes mashups, software toys, and experimental user interfaces using Flash, Perl and other tools.

- **Erik D. Demaine** (MIT), International Francqui Chair of Belgium, MacArthur Fellow, Alfred P. Sloan Research Fellow. Erik studies computational aspects of geometric operations such as paper folding and cutting.
- **Robert J. Lang**, One of the world's leading origami masters, with over 500 designs catalogued and diagrammed. In addition to creating intricate origami paper sculptures, Dr. Lang applies his knowledge of folding to solve complex problems in science and engineering.
- **Carlo H. Séquin** (U.C. Berkeley), Computer Scientist and Virtual Sculptor, IEEE Fellow, Member of the Swiss Academy of Engineering Sciences. Carlo studies the generation and construction of artistic forms by computer algorithms.
- **Dmitri Tymoczko** (Princeton University), composer and music theorist. His article "The Geometry of Musical Chords" was the first music theory article ever published by the journal *Science*.

3 Themes

Naturally, a conference as broad as Bridges can not be said to focus on any one theme. Nevertheless, examining the program after the fact it becomes possible to discern some high-level topics that recurred throughout the week.

3.1 Folded structures

The mechanical transformation of paper by bending, creasing and (occasionally) cutting offers countless opportunities for exploration in both art and mathematics. Even better, there seem to be many situations in which paperfolding lies at the intersection of both topics—some of the most ingenious artistic creations in paper rely on deep mathematical statements about the expressive range of folding systems.

In his plenary talk, **Erik D. Demaine** presented examples of how mathematics and art have inspired and interacted with each other. In his joint paper with **Martin L. Demaine**, he posits that pursuing the mathematical and artistic angles of any problem in tandem is both more productive and more fun, leading to new interdisciplinary collaborations. They gave several examples of an alternating process of mathematical discovery and artistic creation, each side informing the next step in the other.

Demaine introduced us to the "hypar" (short for hyperbolic paraboloid), a mathematical form that has enjoyed occasional periods of popularity in architecture. There is a famous folding process, based on a network of concentric square folds in the square piece of paper, which causes the paper to collapse into a seeming approximation of a hypar (see Figure 1). He then presented two surprising facts about this structure:

> First, the first appearance of the model is much older than we thought, appearing at the Bauhaus in the late 1920s. Second, together with Vi Hart, Greg Price, and Tomohiro Tachi, we proved that the hypar does not actually exist: it is impossible to fold a piece of paper using exactly the crease pattern of concentric squares plus diagonals (without stretching the paper). This discovery was particularly surprising given our extensive experience actually folding hypars. We had noticed that the paper tends to wrinkle slightly, but we assumed that was from imprecise folding, not a fundamental limitation of mathematical paper. It had also been unresolved mathematically whether a hypar really approximates a hyperbolic paraboloid (as its name suggests). Our result shows one reason why the shape was difficult to analyze for so long: it does not even exist!

Robert J. Lang, who holds a doctorate in physics, believes that the marriage of art and mathematics has been widespread and productive, but almost nowhere more productive than in the world of origami. In the past 25 years or so, origami has experienced what can only be characterized as a revolution. A group of enthusiasts, including Lang, developed a suite of computational tools and algorithms that can automatically construct a folding pattern for the gross structure of an arbitrary origami model. Given an arbitrary tree graph (representing, say, the limbs of an insect), they use a disc-packing algorithm to embed the tree in such a way that a folding pattern can then be computed. This tree-based approach has allowed origami artists to

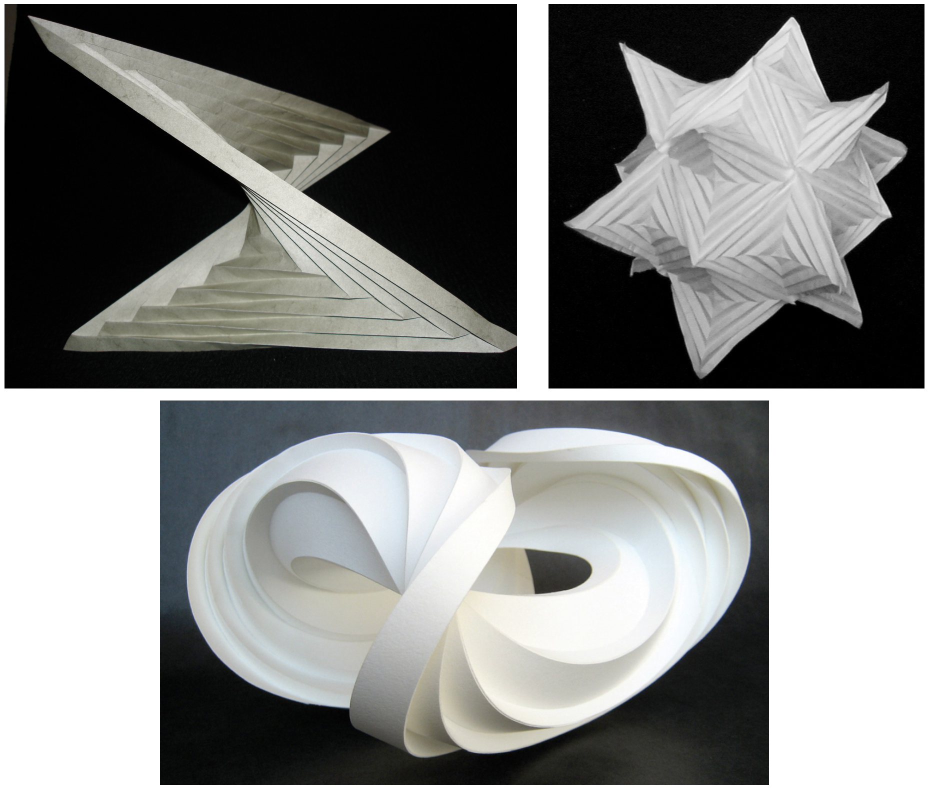

Figure 1: Three examples of paper construction based on hypars by Erik Demaine et al. The hypar shown in the top left is constructed using concentric square folds on a single sheet of paper. Hypars can be assembled into 3D sculptures based on arbitrary polyhedra, as shown on the top right. The bottom image demonstrates a similar process based on elliptical folds.

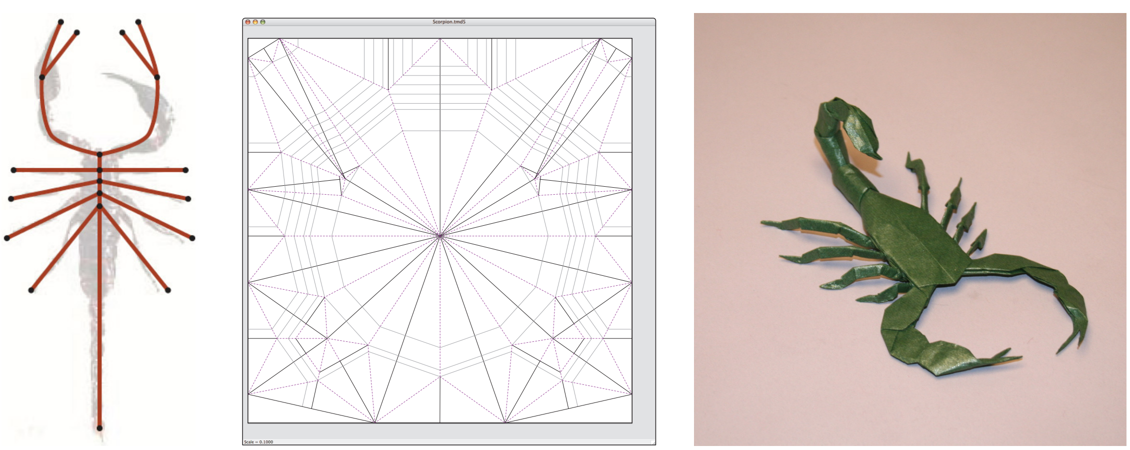

Figure 2: An example of using contemporary tree-based construction for origami sculptures. The conceptual tree for a scorpion is shown superimposed on an image on the left. The plan of mountain and valley folds that achieve this tree is shown in the centre. The right image shows the completed scorpion sculpture.


Figure 3: Two samples of pleated paper constructions by Goran Konjevod.

produce a new generation of models of immense scope and complexity (leading, for example, to an era they refer to affectionately as "The Bug Wars"). An example is shown in Figure 2. In his paper, Lang discussed how mathematical ideas led to the development of powerful tools for origami design and presented a step-by-step illustration of the design and realization of a representational origami figure using mathematical design algorithms. He also spoke about the implications of origami in science and engineering. For example, he recently worked with Lawrence Livermore National Laboratory in California to develop a lens system for a space telescope. The lens can be folded into a compact structure that fits into a launch vehicle, then deployed to its full size once in orbit.

The regular papers program featured an excellent paper by **Goran Konjevod** and **Ana Maria Kuprešanin** (Arizona State University) on paperfolded models. Goran Konjevod was the presenter. He mentioned that in origami, or paperfolding, there are models that require complex fold steps. In order to describe the folding process in written form, over time an intricate system of diagramming symbols and conventions has been developed. Even so, there have been models that simply defy diagramming, for which the authors (designers, or composers) prefer to teach the folding process only in person, or not at all. Instead, they provide only the crease pattern that shows the location (and sometimes sense) of all the folds actually present in the finished model. Of course, a crease pattern alone may not uniquely define the model. In typical cases it allows for an educated guess at how, for example, the layers of paper should be arranged, though in full generality this problem is known to be NP-complete.

Konjevod introduced pureland origami, in which folds are restricted to simple mountain and valley folds, and then moved on to paperfolded tessellations, made up of highly structured arrangements of pleats He presented a new notation for these pleat tessellations that can account for a broad class of attractive, abstract paperfolded models (two examples are shown in Figure 3). One such model allowed Konjevod to capture the first prize at the 2009 joint AMS-MAA Art Exhibition. Finally, he noted that although the notation defines a combinatorial range of models, it is less clear how to predict the final shape assumed by the paper when the pleats are executed.

Among our various evening cultural events, we included a screening of the award-winning documentary film *Between the Folds*, about the art and science of paperfolding. The film was an homage to the beauty of folding, and a survey of some of the experts in the field, including Erik Demaine and Robert Lang (and **Chris Palmer**, who was also present at the conference). After the film, Demaine, Lang and Palmer agreed to answer some audience questions. We advertised the screening to the Banff Centre in general, and attracted about twenty interested visitors from outside the conference. They enjoyed the film, and participated in the discussion afterwards.

3.2 Music and mathematics

The inextricable link between mathematics and music is probably self-evident to any participant at the Bridges conference, and always a source of inspiration to our interdisciplinary audience. This year's conference

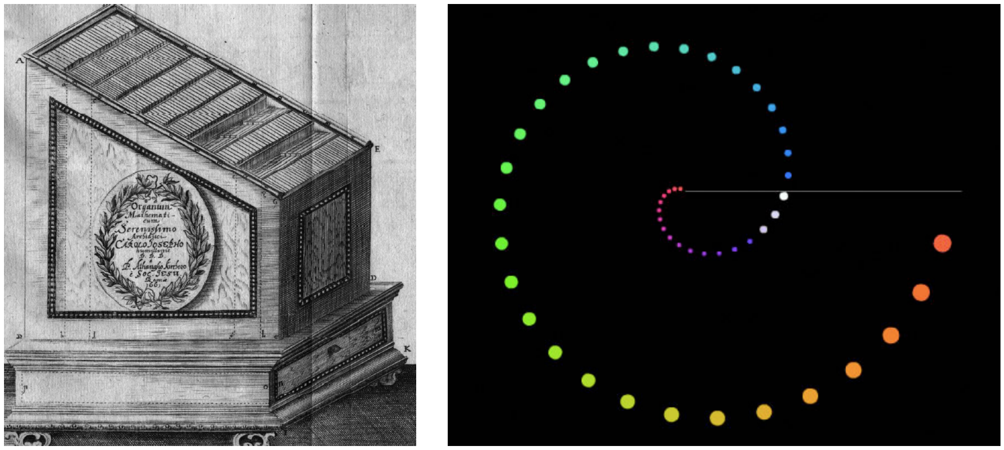

Figure 4: The two components of Jim Bumgardner's plenary talk. The image on the left is an engraving showing Kircher's original *Organum Mathematicum*. The image on the right is a computer rendering of one of Bumgardner's Whitney music box simulations in operation.

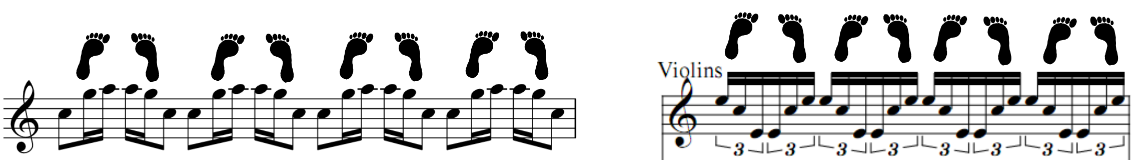

Figure 5: Two examples of musical pieces with frieze symmetry, as presented by Vi Hart. Both examples are accompanied by a visualization of the symmetry group in footprints, as popularized by Conway.

featured exciting presentations from first-time Bridges speakers.

Jim Bumgardner discussed two separate musical explorations in his plenary talk *Arca Musarithmica: A Pair of Curious Music Boxes*. First, he introduced the audience to Athanasius Kircher, the great 17th-century Jesuit polymath. Among contributions in many areas of science, Kircher investigated the automated generation of musical sequences (similar in spirit to, but much more complex than, Mozart's more famous *Musikalisches Würfelspiel*). Bumgardner had re-engineered the contents of Kircher's *Organum Mathematicum* in software, and played hymns that had been generated automatically by his system. He also provided one of the conference's most entertaining moments, when he described one of Kircher's other inventions, the *Katzenkalvier* or "cat piano".

The second half of Bumgardner's presentation concerned variations on the Whitney music box. In this project, Bumgardner investigated algorithmic music that can be produced by dots moving at different rates in orbit around a central point. When any given dot crosses an axis it produces a tone. He experimented with many arrangements of dots that produce both attractive animated patterns and corresponding music.

The *Organum Mathematicum* and a Whitney music box simulation are shown in Figure 4.

Dmitri Tymoczko presented an invited paper entitled "Three Conceptions of Musical Distance". Tymoczko is interested in defining abstract metrics that can offer a mathematical notion of distance between musical keys. The paper defines three such notions of distance: one based on orbifolds, a second based on acoustic models, and a third based on interval content of groups of notes. He describes these three models in isolation, but also shows how they are interconnected.

Tymoczko's talk was a deep and informative look into the modern frontier of music theory. He played several relevant recordings of musical performances by himself and other musicians, coupled with mathematical visualizations of the paths these pieces take through abstract musical spaces.

In her regular paper presentation, **Vi Hart** offered her view of the relationship between discrete planar symmetry groups and musical composition. She interpreted the seven frieze groups as transformations in the

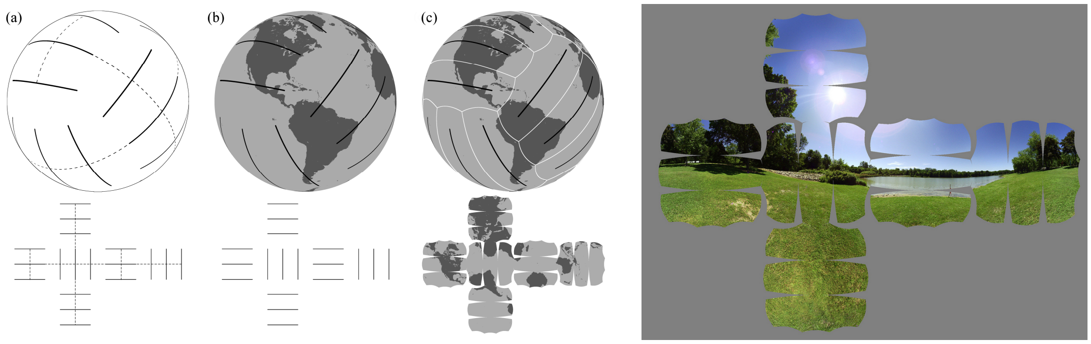

Figure 6: The left diagram shows Swart's process for dividing the sphere into Voronoi cells and constructing corresponding cells in the plane in order to create an unwrapping. The right image gives an example of this process applied to a photographic panorama. The unwrapping in this case is based on the panels of a volleyball.

musical plane and gave examples from classical composition where these frieze groups appear (see Figure 5). She then extended these ideas to the wallpaper groups that can be realized in music, and to two-colour counterchange symmetries. Hart capped off her talk by giving examples of how the mental model of planar symmetry influenced her own musical compositions.

No Bridges conference would be complete without a night of musical performance. This year, we took advantage of our location within The Banff Centre to engage with on-site musicians. Barry Schiffman helped organize the music department to send us a handful of incredibly talented young musicians who offered us virtuosic performances on the trumpet and piano. In the second half of the event, Bridges participants were invited to contribute performances of their own; as always, there were many volunteers and the performances were well received. Many people remained in the hall for hours after the end of the formal program to improvise and jam.

In this category, we might also mention a few additional contributions to the conference in topics that relate mathematics to other performing arts. **Hartmut Höft** applied techniques from combinatorial enumeration to calculate sets of end rhyme patterns in sonnets. He also offered a high-level visualization technique for these rhyming patterns based on arrangements of coloured bands. In the world of mathematics and dance, **Karl Schaffer** discussed some of his recent dance works that incorporate overt mathematical themes—dances based on geometrical forms, and demonstrations of famous equations.

3.3 Computer graphics

As computer graphics has matured as a discipline, it has become a natural medium for the expression of many ideas in mathematical art. The conference always features many excellent presentations that take advantage of computer graphics techniques. We highlight two such papers here.

David Swart presented captivating work on how to create two-dimensional projections of spherical imagery. This topic has been the obsessive focus of cartographers for centuries; Swart is more interested in the artistic possibilities that arise in the attempt to unwrap photographic spherical panoramas into planar designs. To that end, he created a simple turtle graphics language for placing geometric primitives on a sphere. The same turtle graphics commands can then be executed in the plane to create analogous primitives in the final design. He then maps the Voronoi regions surrounding those spherical primitives to corresponding regions on the plane, and carries the photographic information from the spherical panorama into the flat domains. The result can be viewed as a unique piece of two-dimensional art, or cut out and reassembled into an approximation of a sphere, giving the appearance of the famous spherical paintings by Dick Termes.

Swart demonstrated his technique using a number of classical decomposition of the sphere. Some were based on regular polyhedra or well-known cartographic projections; others were drawn from the world of sports! He showed us decompositions of the sphere by the panels of baseballs, soccer balls, basketballs, and volleyballs. One design, based on the volleyball, is shown in Figure 6. In order to support the creation of a

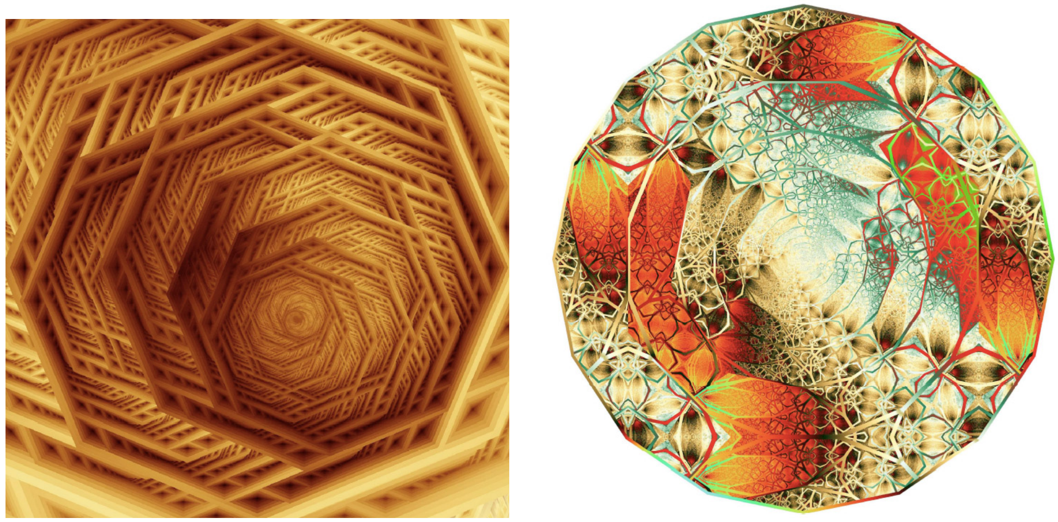

Figure 7: Two examples of fractal images by Philip Van Loocke's, based on iterated function systems with expanding functions.

future commemorative design, he also captured a lovely spherical panorama of the BIRS lecture room with the Bridges audience in attendance.

Related to Swart's work was the paper by Brosz et al. on the history of non-linear perspective in art. The authors provided a survey of the many uses of non-traditional projections, discussed their computer graphics "flexible projection" framework for achieving similar effects in a 3D graphics environment, and discussed one particular art installation based on flexible projection. The paper is an excellent demonstration of the harmonious balance that can arise between artistic intention and mathematical technique.

Sculptures with polyhedral symmetry always figure prominently at the Bridges conferences. **Vladimir Bulatov** presented a suite of techniques for creating 3D models of polyhedral sculptures. Many of his creations feature intricate, organic, interwoven forms that make for compelling art. Bulatov has used direct metal printing to fabricate many of his models as physical objects. **Robert Rollings** contributed a paper that described his polyhedral creations as a woodworker of many years.

Rinus Roelofs gave a plenary talk that was, as always, filled with charming mathematical ideas, all executed as enthralling 3D animations. He explored a topic he called "Non-flat Tilings with Flat Tiles", novel 3D forms that arise by considering spatial transformations of tiles in a flat tiling of the plane.

Philip Van Loocke (University of Ghent) presented a paper entitled "The Art of Iterated Function Systems with Expanding Functions". A great deal of art has been created using iterated function systems, in particular those in which the functions are contractions. Van Loocke's paper explores iterated function systems consisting of *expanding* linear functions (i.e., linear functions whose inverses are contractions). An area in the plane is divided in parts which are defined implicitly by an inverse function technique. An expanding function is associated with each part. A coloring technique is proposed that produces textures suggestive of sophisticated patterns of depth and light. He also briefly describes how a rendering technique for recurrent origami can be obtained as a special case of this method. The results are lovely, organically textured fractal designs. Two fractal designs by Van Loocke are given in Figure 7.

4 Other events

A few other events held at the conference are worth noting. We summarize them here.

1. **Steve Abbott** (Middlebury College) organized a Theatre Night, in which a set of Bridges participants were drafted into a short-lived acting troupe and staged scenes from several mathematically-themed

Figure 8: The monumental Zometool construction. The photo on the left shows a dedicated team of builders assembling modules in the lobby of Max Bell. On the right, a group photo from the conference, posed outside Rolston Recital Hall in front of the finished sculpture.

dramatic works. We enjoyed scenes from Stoppard's well-known play *Arcadia* and other wonderful plays.

2. **Paul Hildebrandt** (Zome corporation) supervised the construction of a monumental, record-setting Zome construction in honour of the late Chris Kling. The sculpture, based on the rhombic triacontahedron, used an unprecedented 50000 pieces. Dozens of Bridges participants (and staff from BIRS and The Banff Centre!) spent at least part of their time assembling individual modules, which were then transported to the patio outside the Rolston Recital Hall for final assembly. Many Banff Centre residents had an opportunity to witness the sculpture under construction and in its final form. We held an informal celebration of its completion, set against a dramatic mountain sunset. A photo from the event, showing the monumental sculpture, appears in Figure 8.

3. The conference had an opportunity to take in the region's natural beauty with an excursion to Yoho National Park. We stopped at Spiral Tunnel, Takkakaw Falls, and Emerald Lake. The only downside of the excursion is that we wished we had more time to take in each of these amazing sights.

5 Outcomes and future

By all accounts, Renaissance Banff II was a tremendous success. The participants offered uniformly positive feedback about the organization, the facility, and the quality of the invited and contributed work. First-timers, including some of our invited speakers, were incredibly enthusiastic about the conference and have promised to return in future years for their own enjoyment.

Planning is already well underway for Bridges 2010, which will be held in Pécs, Hungary (the city will be a European Capital of Culture for the year). We are planning expanded music and theatre programs and look forward to another year of inspirational invited and contributed presentations. We have also just begun organization of Bridges 2011, which we hope to hold at the University of Coimbra in Portugal. We also know that we'll be holding the 2015 conference in conjunction with the MAA centennial in Washington, DC. The intervening years are still to be determined; of course, we would be very excited to consider a return to Banff in the future.