09w5055 Statistical Mechanics on Random Structures

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1 Overview of the Field

The theme of the workshop has been equilibrium and non-equilibrium statistical mechanics in a random spatial setting. Put differently, the question was what happens when the world of interacting particle systems is put together with the world of disordered media. This area of research is lively and thriving, with a constant flow of new ideas and exciting developments, in the best of the tradition of mathematical physics.

Spin glasses were at the core of the program, but in a broad sense. Spin glass theory has found applications in a wide range of areas, including information theory, coding theory, algorithmics, complexity, random networks, population genetics, epidemiology and finance. This opens up many new challenges to mathematics.

2 Recent Developments and Open Problems

The workshop brought together researchers whose interest lies at the intersection of disordered statistical mechanics and random graph theory, with a clear emphasis on applications. The multidisciplinary nature of the proposed topics has attracted research groups with different backgrounds and thus provided exchange of ideas with cross-fertilisation. As an example, we mention two problems on which we focused during the workshop.

The first problem has its origin in the many fundamental issues that are still open in the theory of spin glasses. Even tough today we have a rigorous proof, in the context of mean-field models, of the solution for the free energy first proposed by G. Parisi, certain relevant properties of this solution (e.g. ultrametricity) still lack a firm mathematical understanding. Moreover, when considering spin-glass models on the lattice with short-range interaction, the mean-field picture that predicts the existence of many different equilibrium states is challenged by the droplet scenario, where only a few relevant equilibrium states are predicted (related via the spin-flip symmetry).

The second problem concerns the application of ideas and methods from statistical mechanics to the social sciences. Dichotomic social issues (yes or no matters) are known to manifest sudden changes, very much like phase transitions. An approach that has appeared recently is to use mean-field models (i.e., non-local interactions) to describe peer-to-peer relations, possibly extended to social interaction networks carrying the interesting small-world and scale-free features of random graphs. The final aim of this investigation is to establish conditions for opinion spreading, nucleation, cluster formation, and other observed social phenomena, especially within the enormous migratory fluxes nowadays present in developed countries. The use of statistical mechanics for social purposes will also lead to the necessity of statistical parameter estimation via polls, a newly emerging field with several applications.

The workshop took advantage of an introductory meeting on the same topic organized at EURANDOM, Eindhoven, The Netherlands, in March 2008. Some of the students and postdocs who attended YEP-V (the fifth in a successful series aiming at Young European Probabilists) had the occasion to meet again in Banff, together with leading scholars in the field.

3 Presentation Highlights

Here we summarize the themes that were discussed during the workshop.

• Edwards-Anderson model

Monday was largely devoted to the Edwards-Anderson (EA) model, which describes short-range spin glasses.

In a series of four lectures, presented by *Chuck Newman*, *Dan Stein*, *Louis-Pierre Arguin* and *Michael Damron*, the notion of "metastate" was introduced, and the question was addressed whether or not the EA-model (with a continuous, bounded and symmetric disorder distribution) has one or more pairs of ground states. These two options distinguish between the "droplet picture" and the "mean-field picture". It was shown that for the two-dimensional EA-model in the half-plane there is only one pair of ground states. The proof combines ideas from two-dimensional dependent percolation with a detailed study of how ground states are affected by local spin-flips.

Claudio Giberti presented results of numerical simulations on the three-dimensional EA model which indicates that overlap is a good order parameter, in the sense that states conditioned to a given value of the overlap are clustering. He also discussed analytically the effect of flipping the interaction in a subvolume of the lattice: from bounds on the fluctuation, new integral identities involving the overlap quenched expectation are derived.

Hidetoshi Nishimori spoke about the EA-model in the setting of the replica method, and showed that on self-dual lattices a recursion formula for the replicated partition function combined with a gauge argument yields a zero-temperature phase transition.

• Disordered statistical mechanics model on graphs

Tuesday was largely devoted to statistical mechanics on sparse random graphs.

Andrea Montanari gave an introduction to the Ising model on graphs that are locally tree-like, showing that interesting behavior occurs at the critical line. He proved that in the low-temperature phase the Gibbs measure converges locally to a symmetric linear combination of the 'plus' and 'minus' state.

Amir Dembo pulled the theory into a more general context and reviewed a number of results for statistical mechanical models away from criticality.

Sander Dommers spoke about the Ising model on a sparse random network obtained by randomly connecting vertices, called the "configuration model". He argued that the critical behavior is sensitive to the exponent in the tail of the distribution of the number of connected neighbors of a typical point.

Lenka Zdeborova spoke about the random field Ising model on sparse random graphs, and also about some correlation inequalities ruling out earlier conjectured phases.

Guilhem Semerjian explained how to construct quantum spin models on sparse random graphs using the cavity method in a non-cummutative setting. The results are of interest e.g. for quantum computing and quantum annealing.

• Disordered systems

Various topics in the realm of disordered systems were addressed.

Michael Aizenman spoke about "Imry-Ma" (volume/surface) versus "Parisi-Sourlas" (dimensional reduction) for the random field Ising model, showing that in two dimensions disorder destroys the phase transition no matter how weak. He subsequently argued that for quantum systems the same type of result holds, requiring however different techniques.

Gerard Ben Arous spoke about complexity of the spherical p-spin Hamiltonian, showing that the number of critical points of a given degree (minima, saddle points, etc.) at a given height can be counted in terms of the spectrum of large random matrices drawn from the Gaussian Orthogonal Ensemble. This provides an interesting link between two major research areas.

Silvio Franz provided a renormalization scheme for p-spin models on a Dyson hierarchical lattice, where the interaction between the spins depends on their hierarchical distance and decays with a given exponent. This model interpolates between the mean-field model (infinite range) and the random energy model (zero range). The renormalization scheme is hard to control mathematically, but numerical computations suggest the presence of a phase transition similar to that in the random energy model.

Balint Virag reviewed a number of topics from the theory of random Schrödinger operators, in particular, the decay of the probability of large gaps in the spectrum of associated large random matrices, indicating a strong form of repulsion between the eigenvalues.

• Mean-field spin glasses and neural networks

Various results for mean-field spin glasses and neural networks were presented.

Shannon Starr explained the theory of thinning of random partition structures, which serve as a caricature for the description of the organization of phases in mean-field spin glasses. He outlined the central role that is played by the Poisson-Kingman partition structure.

Anton Klimovsky considered the SK-model with vector-valued spins. Examples are the Potts spin glass and the Heisenberg spin glass. A Parisi-type solution was found for the case where the a priori distribution of the spins is Gaussian. For an alternative model, based on isotropic Gaussian processes, a full description of the replica symmetric solution and the solution with partial and full replica symmetry breaking can be obtained. The behavior is similar to that of the random energy model.

Alessandra Bianchi derived sharp estimates on metastable exit times for the Curie-Weiss random field Ising model subject to Glauber spin-flip dynamics. For a particular starting distribution called the last-exit biased distribution, the exit time can be computed with the help of potential theory. By using coupling methods, it can subsequently be shown that the exit time is largely independent of the starting configuration, allowing for a proof that the normalized exit time has an exponential distribution.

Christof Külske spoke about metastates for mean-field disordered systems, such as the Curie-Weiss random field Ising or Potts model with binary disorder. He showed that the metastates are convex combinations of a restricted set of pure states, called the ”visible” phases, while other pure states never occur, called the ”invisible” phases. He also explained how the weights in the convex combination can be computed.

Adriano Barra spoke about the Hopfield model with Gaussian random patterns. With the help of a new interpolation method he was able to find the replica symmetric solution of the model. The model can be applied to describe the phenomenon of migration integration in social networks.

• Graphs, random graphs and probabilistic structures

A number of related topics on graphs were presented.

Allon Percus addressed the problem of bisection of the Erdös-Renyi random graph. Of particular interest is the regime where the giant component takes up more than half of the vertices. He showed that, just past the critical threshold for this regime, the bisection solutions occur in clusters. This result requires a detailed understanding of the geometry of the giant component.

Malwina Luczak considered the problem of generating a countably infinite partially ordered random set with an order-invariance property, i.e., its probability distribution is exchangeable. It turns out the generation can be done with the help of a Markovian growth process that adds on elements sequentially. She showed how these growth processes can be classified.

Raffaela Burioni spoke about statistical mechanics on “physical” graphs, i.e., infinite graphs with certain large scale regularity properties that are common in applications. She explained why the spectral dimension of such graphs plays a central role in the behavior of a wide range of random processes on the graph, including random walks, random polymers and Ising models.

• Applications

Several applications of spin-glass theory were presented.

Matteo Marsili told the participants about models from statistical mechanics aimed at describing the recent financial crisis. These are based on dynamic random networks where the sites represent banks, the edges represent trading between banks, and parameters are chosen in accordance with a priori insights. Such models may exhibit sharp transitions between "non-crisis" and "crisis" states, corresponding to phase transitions of the network. This allows for some control of the network.

Nicolas Macris presented a theory for error correcting codes based on sparse random graphs, such as the "low density parity check" code. The code words to be transmitted along the noisy channel are the ground states of a spin Hamiltonian with random interactions. The spin glass phase of the Hamiltonian allows for many ground states and therefore for an efficient transmission. The theory uses cluster expansion techniques.

Nicolas Kistler talked about Branching Brownian Motion and its relation to Derrida's REM and GREM models. This process is relevant in application of spin glasses to evolution and population models, where a recent class of models having mutation and selection are described by the FKPP-equation.

Elena Agliari discussed the use of statistical mechanics on random structures to model the immune system. She introduced a random bond Ising ferromagnet embedded in a random diluted network, which provides good accuracy in explaining the autopoietic nature of the immune system.

David Sherrington closed the workshop with a talk that presented many possible new applications of spin-glass models to range-free networks. For instance, he described a model having a phase transition from a regime of Poisson-degree distribution to a regime of power-law degree distribution.

4 Abstracts of the talks

(in alphabetic order by speaker surname)

- *Speaker*: Elena Agliari (University of Parma).

  Title: The autopoietic immune network: a statistical physics perspective.

  Abstract: A systematic approach to the immune system has been argued already three decades ago, yet, due to lack of a paved mathematical backbone, this was not investigated exhaustively. Here we develop a minimal model, which takes into account the reciprocal affinities among immunoglobulins, giving rise to a random-bond Ising ferromagnet embedded in a diluted network. We first discuss the topology of the emerging underlying graph and the statistical mechanics approach for its study. Then, we derive its thermodynamics and analyse both the equilibrium and the linear response regimes by means of mathematical modeling and extensive numerical simulations. Our results are consistent with experimental data and strongly support the autopoietic nature of the immune system.

- *Speaker*: Michael Aizenman (Princeton University).

  Title: Effects of quenched disorder on 1st-order quantum phase transitions (QPT_1).

  Abstract: The talk will present the recent proof that the addition of an arbitrarily small random perturbation to a quantum spin system rounds a first-order phase transition in the conjugate order parameter in $d > 2$ dimensions, or for cases involving the breaking of a continuous symmetry in $d > 4$. This establishes rigorously for quantum systems the existence of the Imry-Ma phenomenon, which for classical systems was proven by Aizenman and Wehr. The extension was enabled by an improvement in the argument, in which the role of the classical metastate (covariant Gibbs equilibrium measure) was de-emphasized and replaced by a more direct analysis of the free energy. This is joint work with R.L. Greenblatt and J.L. Lebowitz.

- *Speaker*: Louis-Pierre Arguin (New York University).

  Title: Uniqueness of the ground state for the EA model in the half-plane II.

  Abstract: We consider the EA model on the half-plane with Gaussian (or other) couplings, zero external field and periodic boundary conditions in the horizontal coordinate and free boundary conditions in the vertical coordinate. We show that, for almost every realization of the couplings, the distribution on Ground State Pairs (in the metastate sense) is supported on a single pair. This talk, the third of a four-part series, presents results that are joint work of L.-P. Arguin, M. Damron, C. Newman and D. Stein.

- *Speaker*: Gerard Ben Arous (New York University).

  Title: Critical points of random Morse functions on the sphere.

  Abstract: How many critical points does a random (Gaussian) smooth Morse function have on a large dimensional sphere? How many below a given level and of a given index? In physics this question comes under the name of Complexity of Spherical Spin Glasses. In a joint work with J. Cerny and A. Auffinger we solve it using Random Matrix Theory. I will show that the answer presents a surprising structure for the low-lying critical poins of Spherical Spin Glasses.

- *Speaker*: Adriano Barra (Roma University).

  Title: The replica symmetric scenario in the analogical neural network.

  Abstract: We present some recent progress in our understanding of neural networks whose patterns are stored continuously (Gaussian weighted) on the real line. Mapping this model onto an equivalent bipartite spin-glass, we use a new interpolating scheme to obtain the free energy explicitly in the replica symmetric Ansatz. Then we study the rescaled order parameter fluctuations to identify, via their divergences, the critical line defining ergodicity breaking, which indeed matches earlier results by Amit and coworkers in the dichotomic counterpart. This is joint work with Francesco Guerra.

- *Speaker*: Alessandra Bianchi (Bologna University).

  Title: Coupling in potential wells: pointwise estimates and exponential laws in metastable systems.

  Abstract: In many situations of interest, the potential-theoretic approach to metastability allows to derive sharp estimates for quantities characterizing the metastable behavior of a given system. In this framework, the average metastable times can be expressed through the capacity of corresponding metastable sets, and capacities can be estimated with the application of two different variational principles. After recalling these basic concepts and techniques, we will describe a new method to couple the dynamics inside potentials wells. Under some general hypothesis, we will show that this yields pointwise estimates and exponential laws on metastable times. Our key example will be the random field Curie-Weiss model.

- *Speaker*: Raffaella Burioni (Parma University).

  Title: Statistical mechanics on physical graphs and spectral properties.

  Abstract: The spectral dimension of an infinite physical graph, defined according to the asymptotic behavior of the infrared singularities of the Gaussian model, appears to be the right generalization of the Euclidean dimension d of lattices to non-translational invariant networks when dealing with dynamical and thermodynamical properties. The spectral dimension exactly replaces d in most physical laws where dimensional dependence explicitly appears: the spectrum of harmonic oscillations, the average autocorrelation function of random walks, the generalized Mermin-Wagner theorem and the generalization of the Fröhlich-Simon-Spencer bound to graphs. We present the proof of the invariance properties of the spectral dimension under quasi-isometries (including coarse-graining and addition of finite-range couplings) and we discuss its relevance in phase transitions on graphs.

- *Speaker*: Michael Damron (Princeton University).

  Title: Uniqueness of the ground state for the EA model in the half-plane I.

  Abstract: We consider the EA model on the half-plane with Gaussian (or other) couplings, zero external field and periodic boundary conditions in the horizontal coordinate and free boundary conditions in the vertical coordinate. We show that, for almost every realization of the couplings, the distribution on Ground State Pairs (in the metastate sense) is supported on a single pair. This talk, the fourth of a four-part series, present results that are joint work of L.-P. Arguin, M. Damron, C. Newman and D. Stein.

- *Speaker*: Amir Dembo (Stanford University).

  Title: Unimodularity, random cluster models and Bethe states on sparse random graphs.

  Abstract: Theoretical models of disordered materials lead to challenging mathematical problems with applications to random combinatorial problems and coding theory. The underlying mathematical structure is that of many discrete variables that are strongly interacting according to a mean-field model determined by a random sparse graph. Focusing on random cluster measures on graphs that converge locally to trees, we review recent progress in validating the 'cavity' prediction for the limiting free energy per spin. This talk is based on collaborations with Andrea Montanari and with Nike Sun.

- *Speaker*: Sander Dommers (Eindhoven University)

  Title: Ising models on power-law random graphs.

  Abstract: In many real-world networks, such as the Internet and social networks, power-law degree sequences have been observed. This means that, when the graph is large, the proportion of vertices with degree k is asymptotically proportional to k^a, for some $a > 1$. Often, these networks have a degree distribution with finite mean, but infinite variance $(2 < a < 3)$. We will study a ferromagnetic Ising model on random graphs with a power-law degree distribution and compute the thermodynamic limit of the pressure when the mean degree is finite $(a > 2)$.

- *Speaker*: Silvio Franz (Universite Paris-Sud 11).

  Title: Hierarchical random energy models.

  Abstract: A long-standing problem in statistical physics of disordered systems is the possible existence of ideal glassy phases beyond mean-field theory. In this talk I will discuss convergent evidences for a finite-temperature condensation in a hierarchical REM coming from: (1) exact numerical solution; (2) high temperature series expansion; (3) stability arguments.

- *Speaker*: Claudio Giberti (Modena and Reggio Emilia University).

  Title: Rigorous and numerical results for the Edwards-Anderson model.

  Abstract: The first part of the talk will be devoted to a numerical study of conditional quenched measures in the Edwards-Anderson model. In particular two issues are discussed in the restricted ensemble: the relative fluctuations of different overlaps (related to the property of overlap equivalence) and the structure of overlap correlation (clustering). In the second part of the talk I will discuss the properties of fluctuations of free and internal energies of two spin-glass systems that differ by having some of the interactions flipped. From a bound on fluctuations new overlap identities for the equilibrium state are obtained.

- *Speaker*: Nicolas Kistler (Bonn University).

  Title: REM, GREM and Branching Brownian Motion.

  Abstract: Derrida's Random Energy Models (REM and GREM) have played a crucial role in our understanding of the Parisi Theory. It has however become clear that this class of models cannot fully encode the large-time properties of more realistic spin glasses of mean-field type. A natural extension of the GREM is the so-called hierarchical field, the continuous counterpart being Branching Brownian Motion (BBM): both models have an in-built hierarchical structure where the number of levels in the underlying tree grows with the size of the system. Contrary to the GREM, for which we have a remarkably accurate and rigorous understanding, the microscopic properties of BBM still remain rather mysterious, even at a heuristic level. I will try to give an account of what is known, and report on some modest progress from an ongoing project with L.-P. Arguin and A. Bovier.

- *Speaker*: Anton Klimovsky (Erlangen-Nurnberg University).

  Title: Around one-and-a-half Parisi-type formulae for the free energy.

  Abstract: We start by describing our results on the Sherrington-Kirkpatrick model with multidimensional spins. We identify the candidates for the order parameters and for the Parisi-type functional. These candidates are related to the free energy through a saddle-point variational formula obtained by means of Guerra's interpolation. So far, we have not been able to prove the Parisi-type formula for the general a priori distributions of multidimensional spins, though we can do so in the case of the Gaussian distribution. The proof boils down to showing that Guerra's remainder term vanishes on the optimiser of the Parisi-type functional. In the second part of the talk, motivated by recent work of Fyodorov and Sommers concerning the Gibbsian random landscapes generated by isotropic Gaussian random processes indexed by high-dimensional Euclidean balls, we prove the Parisi-type formula for the free energy of a single particle in the random landscape. One of the main messages that can be extracted from our analysis is that the overlap-like order parameters (familiar in the context of disordered spin systems) are fundamental also in the context of continuous parameter Gaussian processes with isotropic stationary increments (at least, if the parameter space is high-dimensional). The Gaussian processes are allowed to have short- and long-range correlations (e.g., the multiparameter fractional Brownian motion). Depending on whether the Gaussian process has short- or long-range correlations, the order parameter is either a step distribution function with two jumps (one step of replica symmetry breaking) or a continuous distribution function (full replica symmetry breaking), respectively. Both proofs of the Parisi-type formulae are based on the techniques of the remainder term estimates due to Talagrand and exploit the abundantly available rotational symmetries.

- *Speaker*: Christof Külske (Bochum University).

  Title: The metastate approach to random statistical mechanics systems.

  Abstract: The metastate is a probabilistic concept to describe random symmetry breaking. It was introduced by Chuck Newman and Dan Stein to describe the behavior of random systems in situations when the Gibbs measure is not unique, by assigning probability weights to each Gibbs measure according to how frequently it appears in sequences of large volumes. We will discuss lattice and mean-field systems, including a new geometric characterization of visible and invisible phases in the mean-field setup. This is joint work with Giulio Iacobelli.

- *Speaker*: Malwina Luczak (London School of Economics).

  Title: Order-invariant measures on causal sets.

  Abstract: A causal set is a partially ordered set on a countably infinite ground-set such that each element is above finitely many others. A natural extension of a causal set is an enumeration of its elements that respects the order. We bring together two different classes of random processes. In one class, we are given a fixed causal set, and we consider random natural extensions of this causal set: we think of the random enumeration as being generated one point at a time. In the other class of processes, we generate a random causal set, again working from the bottom up, adding one new maximal element at each stage. Processes of both types can exhibit a property called order-invariance: if we stop the process after some fixed number of steps, then, conditioned on the structure of the causal set, every possible order of generation of its elements is equally likely. We develop a framework for the study of order-invariance that includes both types of example: order-invariance is then a property of probability measures on a certain space. Our main result is a description of the extremal order-invariant measures. This is joint work with Graham Brightwell.

- *Speaker*: Nicolas Macris (Ecole Polytechnique Lausanne).

  Title: Correlations in sparse graph error correcting codes.

  Abstract: The subject of the talk will be transmission over noisy channels using error correcting codes based on sparse graphs. The optimal decoder based on the posterior measure over the code bits, and its relationship to the sub-optimal belief propagation decoder, will be discussed. A proof will be outlined of the exponential decay of correlations between code bits in suitable noise regimes. A consequence is the equality of performance curves for the optimal and belief propagation decoders. These systems can be interpreted as a special class of spin glasses and the analysis proves that the replica predictions are exact.

- *Speaker*: Matteo Marsili (ICTP Trieste).

  Title: Stability and complexity in financial markets (spin-glass techniques for understanding economic equilibria).

Abstract: Trust lies at the foundation of market economies, as starkly remarked by the recent financial crisis. Important progress has been made in understanding, from the game-theoretic perspective, the mechanisms by which trust can break down, relating it to strategic uncertainty. The aim of this talk is to scale these insights to the system level by analyzing a simple model of a large population of individuals engaged in credit relationships. This economy can converge to a "good" equilibrium, where a dense network of credit relations exists and the risk of a run, and subsequent default, is negligible. However, a "bad" equilibrium is also possible: here, the credit network is sparse because investors are more nervous and are prone to prematurely foreclose their credit relationships, thereby precipitating counterparty default and contagion. The transition between the two equilibria is sharp and both states exhibit a degree of resilience; once a credit crisis tips the system into the sparse state state, the restoration of a dense credit network requires a shift of the parameters well beyond the turning point. At the same time, when the system reverts to the good state, this is robust even to deteriorating conditions.

- *Speaker*: Andrea Montanari (Stanford University).

  Title: A local limit theorem for Ising models on locally tree-like graphs.

  Abstract: Sequences of graphs that converge locally to trees are of interest for a number of reasons. Among others, sequences of sparse random graphs fall in this class for several graph distributions. In this talk we consider Ising models on locally tree-like graphs and prove a complete characterization of the limiting measure when the graphs are regular. In particular, we establish a coexistence phenomenon that was understood so far only in the case of finite-dimensional lattices. This talk is based on joint work with Elchanan Mossel and Allan Sly.

- *Speaker*: Chuck Newman (New York University).

  Title: Introduction to metastates for Edwards-Anderson models.

  Abstract: We introduce metastates as probability measures on the space of infinite-volume Gibbs states that may be used to study disordered systems such as the Edwards-Anderson (EA) model with potentially many 'competing' states. Extensions to metastates and excitation metastates for competing ground states are also discussed. This talk will be the first of a four-part series, which gives some of the background needed for the second part by Stein, concerning domain walls for the two-dimensional EA model (in the full plane), and for the third and fourth parts by Arguin and Damron, presenting a new result on uniqueness of ground states in the half plane.

- *Speaker*: Hidetoshi Nishimori (Tokyo Institute of Technology).

  Title: Absence of a spin glass transition in two dimensions.

  Abstract: Although numerical evidence is overwhelming for the absence of spin glass transitions in two dimensions, analytical studies are still rare. We have developed a theory to show the absence of finite-temperature spin glass transitions for the Ising spin glass on self-dual lattices. The analysis is performed by an application of duality relations, the replica method and gauge symmetry. I will discuss how and when the predictions of this theory can be exact.

- *Speaker*: Allon Percus (Claremont Graduate University).

  Title: The peculiar phase structure of random graph bisection.

  Abstract: The phase structure of mincut graph bisection displays certain familiar properties when considered over sparse random graphs, but also some surprises. It is known that when the mean degree is below the critical value of $2 \log 2$, the cutsize is zero with high probability. We study how the minimum cutsize increases with mean degree above this critical threshold, finding a new analytical upper bound that improves considerably upon previous bounds. Combined with recent results on expander graphs, our bound suggests the unusual scenario that random graph bisection is replica symmetric up to and beyond the critical threshold, with a replica symmetry breaking transition possibly taking place above the threshold. An intriguing algorithmic consequence is that although the problem is NP-hard, we can conceivably find near-optimal cutsizes (whose ratio to the optimal value approaches 1 asymptotically) in polynomial time for typical instances near the phase transition. This is joint work with Gabriel Istrate, Bruno Goncalves, Robert Sumi and Stefan Boettcher.

- *Speaker*: Guilhem Semerjian (Ecole Normale Paris).

  Title: Quantum spin models on sparse random graphs.

  Abstract: Classical spin models defined on random graphs have been the object of an intense research activity motivated, among others, by their relationship to random combinatorial optimization problems. The heuristic cavity method allowed to make several qualitative and quantitative predictions about the behaviour of such random systems in their large size limit, some of these predictions having been confirmed rigorously. In this talk I will discuss a more recent development of the heuristic cavity method towards quantum models defined on random graphs. These models can be constructed, for instance, by turning a classical energy function of N Ising spins into an operator acting on the Hilbert space spanned by the 2^N configurations, and adding to it a non-commutative operator as a transverse field. Such models can be represented through path integrals of imaginary time configurations. The cavity method can then be implemented at the quantum level by devising a sampling procedure of such spin trajectories. The case of frustrated classical spin models (for which finding the minimal energy states can be a difficult task) in a transverse field is of particular interest in view of application to quantum computing.

- *Speaker*: David Sherrington (University of Oxford).

  Title: Dynamics of information-driven and range-free networks: some results, some thoughts and some questions.

  Abstract: Range-free (or infinite-range) many-body problems are independent of spatial dimensionality and offer the opportunity for exact solution in the large N limit, through mapping to descriptions in terms of macroscopic variables. These mappings and their subsequent analysis are non-trivial when there is frustration between fast variables and either other control parameters or rules are microscopically quench-disordered or are themselves dynamical with slower characteristic timescales. In quenched cases with equilibrating fast-variable dynamics conventional Boltzmann statistical mechanics can be utilised, but still with subtle challenges for rigorous analysis. Without such equilibration often some progress can be made at the level of theoretical physics, but many challenges remain in complete analysis as well as in rigorous justification. Such range-free problems are first-pass reasonable models for a number of situations. They are also a natural effective mapping for problems driven by distance-independent information such as occurs for the internet, stock-market indices, news, etc. Hence, in many man-made social and economic scenarios there are important effective correlation effects with no separation or spatial dimension-dependence. Coupled with conflicting aims and global sum rules/constraints, one has frustration, typically also with a distribution of individual inclinations. Network growth is also range-free in certain cases, again typically those driven by internet interaction, simplifying analyses, but still leaving interesting issues of optimal algorithms, topological phase transitions, consequences of both uncontrolled and rule-controlled evolution and possible effects of frustration. In this talk I shall discuss some of these issues, giving some examples, but also posing questions for discussion and future study.

- *Speaker*: Shannon Starr (Rochester University).

  Title: Thinning partition structures.

  Abstract: A random partition structure is a random point process on $[0, 1]$ such that the points add up to 1. We consider a thinning dynamics: independently removing points, keeping each one with probability $p > 0$, and then rescaling all remaining points to make the sum 1. We assume infinitely many points initially. We prove that all partition structures that are infinitely-divisible with respect to this process are mixtures of Poisson-Kingman processes. The ones that are invariant for all p include the Poisson-Dirichlet structures, which are also the invariant measures for the uncorrelated cavity step dynamics, as proved by Aizenman and Ruzmaikina, and Arguin. But there are also others. This is joint work with Brigitta Vermesi.

- *Speaker*: Dan Stein (New York University).

  Title: Domain wall structure in the two-dimensional Edwards-Anderson model.

Abstract: We review a result due to Newman and Stein, for the 2D EA model with Gaussian (or other) couplings and in zero field, which shows that if a coupling-independent boundary condition metastate is supported on incongruent (i.e., statistically dissimilar) ground states, then the symmetric difference between two such states must consist of a single positive-density simply-connected domain wall. This talk will be the second of a four-part series, using results on metastates presented by Newman in the first part and laying the groundwork for a new result, presented by Arguin and Damron in the third and fourth parts, on uniqueness of ground states in the half-plane.

- *Speaker*: Balint Virag (Toronto University).

  Title: One-dimensional random Schrödinger operators and random matrices.

  Abstract: Random Schrödinger operators can be thought of as Markov kernels for random walks in a random environment of obstacles. In the critical regime, the probability decay for large gaps between eigenvalues of these operators resembles those for random matrices. However, the eigenvalue repulsion is much stronger.

- *Speaker*: Lenka Zdeborova (Los Alamos National Laboratory).

  Title: Revisiting the random field Ising model.

  Abstract: Since the dynamical behavior of the random field Ising model (RFIM) has some glassy features many authors have discussed the existence of a putative spin-glass phase in this model. In this talk, I will first show how an elementary, yet rigorous, bound can be derived for the spin-glass susceptibility that allows to essentially answer the question. In the second part I will present an exact solution of the model on a random graph. I will discuss a method to compute the phase diagram at fixed values of the magnetization and its consequences.