Theory and applications of classical and quantum kinetic theory

Reinhard Illner (Victoria)
Irene Gamba (Austin)
Ansgar Jüngel (Vienna)
Jean Dolbeault (Paris)

June 21 – 26, 2009

1 Overview of the Field

Kinetic theory offers many deep problems of fundamental significance in physics and engineering, typically related to questions of global well-posedness or validation of the basic equations; at the same time, kinetic equations have proved to be powerful models for numerous phenomena beyond the traditional realms of rarefied gases, plasmas or stellar systems. At the time the proposal of the workshop was submitted, it was mentioned that kinetic models were being studied for such diverse applications as electron flows in semiconductors, gene-gene interactions in the cell (involving reaction-diffusion of the transcribed mRNA and the translated proteins), radiative transfer in molten glass, traffic flows on multi-lane freeways, turbulence models, and even kinetic models for trade exchange and distribution of wealth, with applications in finance. A few topics have been added since then, such as the study of animal behavior (swarming, flocking) and mean-field games. The full modelling cycle, from the derivation of a realistic set of equations, to well-posedness, qualitative behaviour of solutions, and numerical simulations, is being traversed by a growing number of researchers pursuing these applications. This variety of models, applications, and mathematical methods was reflected in this workshop.

2 Recent Developments and Open Problems

Kinetic equations and more specifically Boltzmann related issues involve a large community of mathematicians working on PDEs and multiscale analysis. Specific methods as well as numerical tools have been developed and are now shared by the researchers working in this area. However, this community is very diverse in its interests, motivations and areas of application. The common background among participants is mostly a kinetic "toolbox", both theoretical and numerical, and an interest in modeling issues at scales which are intermediate between a full microscopic description (for instance in quantum mechanics) and a rough macroscopic description of the models based on diffusion techniques or fluid equations. It is therefore somewhat difficult to detect emerging topics or major trends. Let us try anyway to list a few of them:

1. As has always been the case in kinetic theory, new areas of application have emerged, for instance in biology and the study of animal behavior (swarming, flocking, milling), or in social sciences (mean field games, large tail distribution functions).

2. The study of large time behavior of solutions and of the rates of convergence to equilibrium for homogeneous and inhomogeneous models is a program that goes back by more than a century. It has achieved a certain level of maturity, with precise conjectures, for instance in the case of the Kac model, or in the development of specific tools like hypocoercivity.

3. The revival of interest for the interplay of linear or linearized problems with their nonlinear counterparts is noticeable. The experience gained from compactness methods based on *a priori* estimates like entropy or free energy is now largely taken into account, for instance in designing the appropriate weights.

4. Similarly, numerical studies are now in position to mix various points of view to achieve a better efficiency. Hence deterministic approaches are not any more systematically opposed to Monte-Carlo type methods, but rather used to attack simultaneously problems at various scales or regimes.

3 Presentation Highlights

The presentations were concerned with mathematical results on kinetic equations arising from various applications. Novel and "exotic" applications and new mathematical methods were of particular interest. The subjects of the talks naturally fall into several thematic groups.

Boltzmann and related models: Boltzmann and Vlasov-type models have been investigated by mathematicians for many years. They play an important role in the modeling of rarefied gases, ionized plasmas, granular media, relativistic stellar systems etc. The analysis of the corresponding equations (Vlasov-Poisson, relativistic Boltzmann, Einstein-Vlasov, gravitational Vlasov-Poisson) is extremely involved, and refined mathematical theories are needed. In recent years, quite some progress has been made concerning global existence of solutions and nonlinear stability. Some of these new exciting developments were presented during the workshop. In particular, we highlight the contribution of F. Méhats presented a new variational approach for the three-dimensional gravitational Vlasov-Poisson system which describes the mechanical state of a stellar system subject to its own gravity [5]. He has proved the nonlinear stability of steady-state solutions under radially symmetric perturbations.

New applications of kinetic and related models: Recently, kinetic models have proved to be very useful for the modeling of sprays, the swarming or flocking of animals, fragmentation of polymers, and economic models. As an example, let us detail the contribution of the young scientist V. Panferov. He presented a kinetic approach (BBGKY hierarchy) to describe flocking of individuals [4]. Generally, flocking means the motion of self-propelling and self-organizing species, such as birds or fish. The objective is to determine how fast and under which conditions flocking can occur. Since typically a large number of individuals is involved, one may try to derive limiting equations when the number of entities becomes infinite. In the mean-field limit, Panferov obtained a Vlasov-type equation and discussed its relation to a hydrodynamic model. He demonstrated that these models allow for flocking and milling solutions, showing that kinetic theory is able to describe the observed phenomena at least qualitatively.

Analysis of kinetic and related equations: Kinetic equations are often used to derive mean-field limit or diffusive models. The talk of C. Schmeiser was concerned with the proof of hypocoercivity for kinetic equations involving a confining potential and a linear collision operator. The method is based on a combination of entropy (Lyapunov functional) methods and a spectral gap inequality for the diffusive limit equation. The presentation provided new connections between spectral theory, kinetic models, and entropy methods. Other presentations were concerned with, for instance, the global regularity of solutions to nonlinear reaction-diffusion systems by extending the De Giorgi technique (A. Vasseur [1, 2]); the total energy decay of the damped semilinear Klein-Gordon equation (S. Ibrahim); and the nonlinear stability of rarefaction waves for the Boltzmann equation using techniques for viscous conservation laws and the micro-macro decomposition of the Boltzmann equation (T.-P. Liu, S.-H. Yu [6, 7]).

Entropy in the Kac model: E. Carlen presented a profound study of the Kac model and the issues of large-time behavior of solutions [3]. In particular, the three-dimensional Kac model, which is related to the spatially homogeneous Boltzmann equation for Maxwellian molecules, is analyzed. The main result is the exact computation of the spectral gap of the associated generator, as well as the corresponding eigenfunctions,

for any number of particles. This is quite satisfactory since previous works have treated the asymptotics of the spectral gap (for large values of the number of particles) only.

Macroscopic limit models: Macroscopic limit models are commonly derived from kinetic equations by using moment methods. Various macroscopic model hierarchies were studied along these lines during the workhsop. A. Tzavaras proved the passage from particle systems to nonlinear partial differential equations in the context of deterministic crystal surface relaxation by applying methods of kinetic theories. By these means, he obtained kinetic hierarchies and macroscopic limits for crystalline steps. H. Struchtrup derived macroscopic transport equations for rarefied gases beyond hydrodynamics by employing refined moment methods. Moreover, A. Jüngel reviewed analytical results for macroscopic quantum models originally derived by P. Degond and F. Méhats (who were also participants of the workshop) from a quantum Boltzmann-BGK equation.

Numerical techniques: The numerical approximation of kinetic equations is a very involved topic, and a few new developments in this direction were presented. Here, we mention only the interesting approach of L. Pareschi. He derived a reduced-variance Monte-Carlo method by guiding (via the moment equations) the particle positions and velocities in such a way that the parallel solution of the fluid and the kinetic models give the same macroscopic quantities (namely, the first three moments). This is achieved by decomposing the Boltzmann distribution function into the Maxwellian and the non-equilibrium part. One of the advantages of this method is its enormous flexibility and its remarkable reduction of fluctuations in all regimes compared to a direct Monte-Carlo solver. Other contributions on numerical methods were concerned with asymptotically correct finite-difference schems for Schrödinger-type problems (A. Arnold) and discontinuous Galerkin methods for the Vlasov-Poisson system, which are mass conservative and positivity preserving (P. Morrison).

References

[1] L. Caffarelli and A. Vasseur. Drift diffusion equations with fractional diffusion and the quasi-geostrophic equation. *To appear in Annals of Maths.*, 2009.

[2] M.C. Caputo and A. Vasseur. Global regularity of solutions to systems of reaction-diffusion with Sub-Quadratic Growth in any dimension. *To appear in Comm. Partial Differential Equations*, 2009.

[3] E.A. Carlen, M.C.C.J. LaRoux, M. Loss, and C. Villani. Entropy and chaos in the Kac model. *Preprint*, 2006.

[4] J.A. Carrillo, M.R. d'Orsogna, and V. Panferov. Double milling in self-propelled swarms from kinetic theory. *Kinetic and Related Models*, 2:363–378, 2009.

[5] M. Lemou, F. Méhats, and P. Raphaël. A new variational approach to the stability of gravitational systems. *Preprint arXiv:0904.2443*, 2009.

[6] T.-P. Liu and S.-H. Yu. Initial-boundary value problem for one-dimensional wave solutions of the Boltzmann equation. *Comm. Pure Appl. Math.*, 60(3):295–356, 2007.

[7] T.-P. Liu and S.-H. Yu. Boltzmann equation, boundary effects. *Discrete Contin. Dyn. Syst.*, 24(1):145–157, 2009.

4 Outcome of the Meeting

The workshop has brought leading exponents of the various research directions together in the unique and stimulating research environment that BIRS offers. Participants were exposed to a plethora of new techniques and ideas which might become important in their own research. Mutual future collaborative research visits and workshops between European, Asian and American participants were envisioned and discussed. Young researchers had the opportunity to meet experienced participants and to expand their knowledge in kinetic theory. A bunch of junior North American researchers were offered the opportunity to be exposed to this exciting area of research.

It is expected that the exchange of ideas coming from different backgrounds and applications, and recent developments of "exotic" applications, will lead to a better understanding of the mathematical foundations underlying the applications and to the development of new mathematical tools. Various subjects have been covered consisting in a combination of analytical and applied directions. It is furthermore to be expected that the cross-fertilization of ideas will have a significant impact on the scientific production of many participants to follow.

5 Titles and Abstracts of the Lectures

All speakers have accepted to post the pdf file of their presentation on-line or to display some related material. The corresponding files can be found at

`http://www.ceremade.dauphine.fr/~dolbeaul/BIRS2009/talks/`

Below are listed the titles and abstracts of all lectures (by alphabetic order of speaker). References are given in the pdf files listed above.

A variational approach to uniqueness of ground states for certain quasilinear PDEs

AGUEH, MARTIAL

We used a variational method based on optimal transport arguments to prove uniqueness of radial ground states for certain quasilinear elliptic equations, and we give the explicit expressions of the solutions. Our variational approach relies on a correspondence between the ground states of these equations and the equilibrium solutions of Fokker-Planck type equations.

Fluid dynamics for a vapor-gas mixture derived from kinetic theory

AOKI, KAZUO

When a vapor of a substance is in contact with its condensed phase, evaporation and condensation (or sublimation) take place on the interface between the vapor and the condensed phase. If we try to describe flows of the vapor with evaporation and/or condensation, we have to rely on kinetic theory even in the continuum limit, since the vapor is not in local equilibrium at the interface. In other words, even if the mean free path of the vapor molecules (or the Knudsen number based on it) is very small, we cannot derive correct fluid dynamics by macroscopic considerations. We can construct correct fluid-dynamic systems for small Knudsen numbers (including the continuum limit) only by considering the zero Knudsen number limit and its neighborhood on the basis of kinetic theory.

Although such systems had been derived some time ago for a single component system composed of a vapor and its condensed phase, their extension to multi-component systems was made rather recently. Here, we consider the fluid-dynamic system for a vapor in the presence of a noncondensable gas (another component that neither evaporates nor condenses on the interface). Starting from the Boltzmann equation for a binary mixture of gases and its kinetic boundary conditions, we derive a system consisting of fluid-dynamic-type equations and their boundary conditions by a systematic asymptotic analysis for small Knudsen numbers. The type of the fluid-dynamic system is different depending on the amount of the noncondensable gas contained in the system.

In the present talk, we will focus our attention on the case where the amount of the noncondensable gas is of the same order of magnitude as that of the vapor. In this case, the flow speed becomes slow, with Mach number being of the order of the Knudsen number, and the fluid-dynamic-type equations describing this flow contains non-Navier–Stokes terms originating from the thermal stress and concentration stress. The boundary conditions for the fluid-dynamic-type equations contain the velocity slip caused by the temperature gradient along the interface as well as that caused by the concentration gradient there. We show the outline of the derivation of the fluid-dynamic system, together with some numerical examples. The talk is based on the works in collaboration with S. Takata, S. Yasuda, and C.-J. T. Laneryd.

Asymptotically correct finite difference schemes for highly oscillatory ODEs

ARNOLD, ANTON

We are concerned with the numerical integration of ODEs of the form $\epsilon^2\psi_{xx} + a(x)\psi = 0$ for given $a(x) \geq \alpha > 0$ in the highly oscillatory regime $0 < \epsilon \ll 1$ (appearing as a stationary Schrödinger equation, e.g.). In two steps we derive an accurate finite difference scheme that does not need to resolve each oscillation: 1) With a WKB-ansatz the dominant oscillations are "transformed out", yielding a much smoother ODE. 2) For the resulting oscillatory integrals we devise an asymptotic expansion both in ε and h. In contrast to existing strategies, the presented method has (even for a large spatial step size h) the same weak limit (in the classical limit $\epsilon \to 0$) as the continuous solution. Moreover, it has an error bound of the order $O(\epsilon^2 h^2)$.

Ref.: A. Arnold, N. Ben Abadallah and C. Negulescu: WKB-based schemes for the Schrödinger equation in the semi-classical limit, preprint 2009.

Entropy and chaos in the Kăc model

CARLEN, ERIC

We investigate the behavior in N of the N-particle entropy functional for Kăc's stochastic model of Boltzmann dynamics, and its relation to the entropy function for solutions of Kăc's one dimensional nonlinear model Boltzmann equation. We prove a number of results that bring together the notion of propagation of chaos, which Kăc introduced in the context of this model, with the problem of estimating the rate of equilibration in the model in entropic terms. Joint work with Carvalho, Le Roux, Loss and Villani.

On strong convergence to equilibrium for the Boltzmann equation with soft potentials

CARVALHO, MARIA CONCEICAO

The paper concerns L^1- convergence to equilibrium for weak solutions of the spatially homogeneous Boltzmann Equation for soft potentials ($-4 \leq \gamma < 0$), with and without angular cutoff. We prove the time-averaged L^1-convergence to equilibrium for all weak solutions whose initial data have finite entropy and finite moments up to order greater than $2 + |\gamma|$. For the usual L^1-convergence we prove that the convergence rate can be controlled from below by the initial energy tails, and hence, for initial data with long energy tails, the convergence can be arbitrarily slow. We also show that under the integrable angular cutoff on the collision kernel with $-1 \leq \gamma < 0$, there are algebraic upper and lower bounds on the rate of L^1-convergence to equilibrium. Our methods of proof are based on entropy inequalities and moment estimates. This is joint work with E. A. Carlen and Xuguang Lee.

Collisions in the context of sprays

DESVILLETTES, LAURENT

Sprays are constituted of droplets dispersed in a surrounding gas. Those droplets collide in an inelastic way, and the lost kinetic energy is transformed in internal energy (temperature), which can also be exchanged during collisions. This leads to an original collision kernel which is reminiscent of kernels used in granular media. We describe some mathematical properties of this kernel, and some of the issues related to its numerical implementation.

Comment: The short talk that I propose describes works in common with Julien Mathiaud (CEA, Paris).

Generation and propagation of exponential weighted estimates to solutions of non-linear collisional equations

GAMBA, IRENE M.

We focus on non-linear non-conservative collisional models of Boltzmann type associated with the evolution of probability densities in the space homogeneous setting. We will described the connections between moment angular averaging estimates, conservation properties, moment summability properties, and interactions potentials in the study of generation and propagation of exponentially weighted estimates in L^1 and L^∞.

Global existence of a free boundary problem with non-standard sources

GUALDANI, MARIA

The talk is concerned with the analysis of a free-boundary problem with a distributional source, arising in economic market theory. The existence of global solutions and C^∞ regularity away from sources are shown, based on $L^1_{\rm loc}$ bounds on the flux and multiscale analysis.

Phase Transition in a Vlasov-Boltzmann model for binary fluid

GUO, YAN

In a joint project with Esposito and Marra, we study the phase transition phenomenon in a Vlasov-Boltzmann model for a binary fluid interacting with repulsive self consisitent potential as well as short-range collisions. We establish that below a critical temperature, a spatially non-homogeneous 'front' steady solution is dynamcially stable while the homogeneous steady state becomes dynamically unstable. On the other hand, above the critical temperature, the unique homogeneous steady state is stable. Our stability proof relies on a recent $L^p - L^\infty$ framework which controls the field with a large amplitude, and our instability proof relies on a continuity argument from the Penrose type criterion for the corresponding collisionless Vlasov-Boltzmann model.

A direct proof for the energy exponential decay of the damped nonlinear Klein-Gordon equation

IBRAHIM, SLIM

We study the total energy decay for the damped semilinear Klein-Gordon equation. The problem of stabilization consists in proving a uniform decay of the energy to zero. Global uniform stabilization and local stabilization are shown in the defocusing case. Furthermore, by using a global approximation of the damped Klein-Gordon equation by the undamped one and a global Strichartz bound for the undamped equation, the local stabilization in the focusing case can be proved.

Macroscopic quantum models with diffusion

JÜNGEL, ANSGAR

Macroscopic quantum models have been derived by Degond, Méhats, and Ringhofer from the Wigner-BGK equation by a moment method with a quantum Maxwellian closure. This leads to nonlocal quantum diffusion or quantum hydrodynamic equations. In this talk, we consider first two local approximations of these models, the fourth-order and the sixth-order quantum diffusion equations. New results on the global-in-time existence of solutions for the initial-boundary-value problems and the explicit decay rates to equilibrium will be given. The proofs are based on entropy dissipation methods, the algorithmic entropy technique by Juengel and Matthes, and exponential variable transformations. Furthermore, analytical results for quantum hydrodynamic equations with diffusion will be presented.

Short time behavior for Rayleigh problem

KUO, HUNG-WEN

Rayleigh's problem of an infinite flat plate set into uniform motion impulsively in its own plane is studied by using the BKW model, linearized Boltzmann equation and full Boltzmann equation, respectively. For a small impulsive velocity (small Mach number) and short time, the flow behaves like a free molecule flow. Our analysis is based on certain pointwise estimates for the solution of the problem and flow velocity.

Bifurcation phenomena for transonic condensation

LIU, TAI-PING

In this talk, we show that rarefaction waves for the Boltzmann equation are time-asymptotic stable and tend to the rarefaction waves for the Euler and Navier-Stokes equations. Our main tool is the combination of techniques for viscous conservation laws and the energy method based on micro-macro decomposition of the Boltzmann equation. The expansion nature of the rarefaction waves and the suitable microscopic version of the H-theorem are essential elements of our analysis.

Random kinetic models on the real axis

MATTHES, DANIEL

A class of one-dimensional homogeneous Boltzmann equations on R is considered. The (random) interaction coefficients inside the collision kernel are designed in such a way that the sum of the particle momenta in individual interactions is not conserved, but the total momentum of the system is. We prove that by adjusting the random coefficients suitably, one can achieve stationary solutions with arbitrarily fat high energy tails. Moreover, we investigate the propagation of regularity and estimate the rate of strong equilibration of transient solutions.

A new variational approach to the stability of gravitational systems

MEHATS, FLORIAN

We consider the three dimensional gravitational Vlasov Poisson system which describes the mechanical state of a stellar system subject to its own gravity. A well-known conjecture in astrophysics is that the steady state solutions which are nonincreasing functions of their microscopic energy are nonlinearly stable by the flow. This was proved at the linear level by Antonov in 1961. Since then, standard variational techniques based on concentration compactness methods as introduced by P.-L. Lions in 1983 have led to the nonlinear stability of subclasses of stationary solutions of ground state type.

We propose here a new variational approach based on the minimization of the Hamiltonian under equimeasurable constraints which are conserved by the nonlinear transport flow, and recognize any anisotropic steady state solution which is a decreasing function of its microscopic energy as a local minimizer. The outcome is the proof of its nonlinear stability under radially symmetric perturbations.

This work ishas been done in collaboration with Mohammed Lemou and Pierre Raphael.

A discontinuous Galerkin method for the Vlasov-Poisson system

MORRISON, PHILIP J.

A discontinuous Galerkin method for approximating the Vlasov-Poisson system of equations describing the time evolution of a Vlasov plasma is proposed. The method is mass conservative and, in the case where piecewise constant functions are used as a basis, the method preserves the positivity of the electron distribution function. The performance of the method is investigated by computing five example problems. In particular, computed results are benchmarked against established theoretical results for linear advection and the phenomenon of linear Landau damping for both the Maxwell and Lorentz distributions. Moreover, a nonlinear two-stream instability problem is computed. It is verified that the method conserves 'enstrophy', mass, momentum, and total energy. A final BGK state is obtained and investigated. Because of the high resolution, interesting details of the trapped particle population can be ascertained. The obtained results demonstrate that the discontinuous Galerkin method accurately approximates the Vlasov-Poisson system.

Stability for Rayleigh-Benard convective solutions of the Boltzmann equation

NOURI, ANNE

Nonlinear stability of a solution to the Boltzmann equation in a Rayleigh-Benard setting is proven. Emphasis is put on the control of the hydrodynamic part of the solution, which is one of the major difficulties of the problem.

On some kinetic models of flocking

PANFEROV, VLADISLAV

I will introduce a kinetic model for a dynamical system involving pairwise interactions and self-propulsion, that can be used to describe certain types of self-organization in biological systems (flocks of birds, schools of fish, swarms of insects...). The kinetic theory approach leads to the identification of macroscopic structures otherwise not recognized as solutions of the hydrodynamic equations, such as double rotating mills. Other macroscopic patterns, for instance spatially localized flocks and single rotating mills can also be obtained as particular steady solutions at the kinetic level.

Moment-guided Monte-Carlo methods

PARESCHI, LORENZO

The numerical simulation of the Boltzmann equation with deterministic techniques presents some severe drawbacks due to difficulties in treating the collision terms and to dimensionality problems. As a consequence, probabilistic techniques as Direct Simulation Monte Carlo (DSMC) methods are extensively used in real simulations for their great flexibility and low computational cost compared to finite volume, finite difference or spectral methods.

On the other hand DSMC solutions are affected by large fluctuations and, in non stationary situations, the impossibility to average leads to low accurate or computationally expensive simulations. Thus it is highly desirable to couple atomistic or molecular, and more generally microscopic stochastic models, to macroscopic deterministic models with the goal to obtain more accurate and efficient methods.

The basic idea of our method consists in obtaining reduced variance Monte Carlo by guiding through the moment equations the particles positions and velocities so that the parallel solution of the fluid and the kinetic models furnishes the same macroscopic quantities. These macroscopic models, in order to represent the correct physics for all range of Knudsen numbers include a kinetic correction term, which takes into account departures from thermodynamical equilibrium. We will focus here on a basic moment guided method based on a matching technique between the first three moments of the macroscopic and microscopic equations.

Gravitational collapse for the Einstein-Vlasov system

REIN, GERHARD

The Einstein-Vlasov system describes in the context of general relativity the time evolution of a large collisionless ensemble of particles which interact only through gravity. We give explicit conditions on regular, spherically symmetric initial data such that the corresponding solutions undergo a gravitational collapse and a black hole forms. In particular, the cosmic censorship conjecture holds for these solutions. Among the data there are such where the solution exists globally in Schwarzschild time.

A contribution to hypocoercivity

SCHMEISER, CHRISTIAN

A simple method will be presented for proving hypocoercivity for kinetic equations involving a confining potential and a linear collision operator with one conservation law. The method is based on the construction of a Lyapunov functional, using coercivity of the collision operator and a spectral gap inequality for the macroscopic limit equation. Examples will be presented, where previously known results are improved (joint work with J. Dolbeault and C. Mouhot).

Recent global results for the relativistic Boltzmann equation

STRAIN, ROBERT

We will discuss some recent stability results regarding global solutions of the relativistic Boltzmann equation coupled with their internally generated electric and magnetic forces. Despite its importance, no global in time solutions have been constructed so far for this Lorentz invariant model. We construct the first global in time classical solutions. This is joint work with Yan Guo.

Beyond hydrodynamics: macroscopic transport equations for rarefied gas flows

STRUCHTRUP, HENNING

Classical hydrodynamics – the laws of Navier-Stokes and Fourier – fail in the description of processes in rarefied gases. The Boltzmann equation, on the other hand, describes a gas on the microscopic level and gives a proper description for all gas processes; its numerical solution, however, is rather expensive. Macroscopic transport equations can be derived from the Boltzmann equation by averaging in velocity (moment method), and expansion in the Knudsen number (the ratio between the mean free path of a gas particle and a characteristic length of the process). Classical hydrodynamics result from expansion to first order, and higher order expansions promise to describe rarefied gases at lower cost than the Boltzmann equation. It will be shown that higher order Knudsen number expansions give meaningful equations sufficiently away from the wall, while the proper description of Knudsen boundary layers – which are dominant in slow rarefied flows – is not tied to the Knudsen number in a simple manner. Nevertheless, tests with moment systems show that a small number of moments can catch the most important Knudsen layer phenomena for Knudsen numbers below unity in sufficient accuracy. The regularized 13 moment equations are obtained by the order of magnitude method and are of third order accuracy in the bulk, but they also contain enough information to describe Knudsen layers. Analytical and numerical results for Couette, Poiseuille, and Transpiration flows and other processes will give evidence of the above statements.

Kinetic hierarchies and macroscopic limits for crystalline steps

TZAVARAS, ATHANASIOS

We apply methods of kinetic theory to study the passage from particle systems to nonlinear partial differential equations (PDEs) in the context of deterministic crystal surface relaxation. Starting with the near-equilibrium motion of N line defects ("steps") with atomic size a, we derive coupled evolution equations ("kinetic hierarchies") for correlation functions, F_n^a, which express correlations of n consecutive steps. We investigate separately the evaporation-condensation and the surface diffusion dynamics in 1+1 dimensions when each step interacts repulsively with its nearest neighbors. In the limit $a \to 0$ with $Na = \mathcal{O}(1)$, where a is appropriately nondimensional, the first equations of the hierarchies reduce to known evolution laws for the surface slope profile. The remaining PDEs take the form of simple continuity equations, which we solve exactly and thereby connect continuous limits of F_n^a with the slope profile.

Global regularity of solutions to systems of reaction-diffusion with sub-quadratic growth in any dimension

VASSEUR, ALEXIS F.

In this talk, we present the study of the regularity of solutions to some systems of reaction–diffusion equations, with reaction terms having a subquadratic growth. We show the global boundedness and regularity of solutions, without smallness assumptions, in any dimension N. The proof is based on blow-up techniques. The natural entropy of the system plays a crucial role in the analysis. It allows us to use of De Giorgi type methods introduced for elliptic regularity with rough coefficients. In spite these systems are entropy supercritical, it is possible to control the hypothetical blow-ups, in the critical scaling, via a very weak norm.

Fragmentation models

WAGNER, WOLFGANG

Fragmentation processes (breaking of various objects into pieces) occur in many sciences and applications such as polymer chemistry, nuclear physics, biology, or mining industry.

Kinetic equations are a common tool for describing the behavior of the size distribution of fragmenting particle systems. Solutions of those equations may be non-conservative (loosing mass), if fragmentation rate grows sufficiently fast at zero. This corresponds to a phase transition into dust ("zero size particles"). On the other hand, random fragmentation models can be used both for ana- lytical studies and for numerical purposes. The transformation into dust cor- responds to the explosion phenomenon of those models. In the talk we discuss explosion properties of two random fragmentation models based on a general criterion for explosion of jump processes. Some special cases and examples are presented.

A model for sympatric speciation

WENNBERG, BERNT

Sympatric speciation is the process where one species separates into two or more different species which then exist within one common geographical area. I will describe a Markov process that illustrates this phenomenon, and show the results of some computer simulations.

Pointwise time-asymptotic stability of a Boltzmann shock profile

YU, SHIH-HSIEN

In this talk a scheme to construct the wave propagation around a Boltzmann shock profile will be surveyed. This is a parallel processes to decompose a variable coefficient problems in constant coefficient problems around the far field of a shock wave and an essential scalar equation to analyze the global wave interactions. This reduction rely heavily on the GreenÕs function for the constant coefficient problem so that one can show the convergence of the scheme in a exponential pointwise estimate.