REPORT ON BIRS WORKSHOP 07W5052 "LOW-DIMENSIONAL TOPOLOGY AND NUMBER THEORY"

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1. Objectives of the workshop

The goal of the workshop was to bring together topologists and number theorists with the intent of exploring connections between low-dimensional topology and number theory, with the special focus on topics described in the section below. We hoped that the balance between the lectures and free time as well as the intimate setting of the Banff Research Station will stimulate many informal discussions and collaborations. We were delighted to see that these objectives were fulfilled.

We would like to thank BIRS staff for their hospitality and efficiency.

2. Topics of the workshop

2.1. **Arithmetic Topology.** Starting from the late 1960's, B. Mazur and others observed the existence of a curious analogy between knots and prime numbers and, more generally, between knots in 3-dimensional manifolds and prime ideals in algebraic number fields, [24, 25, 25, 30–32]. For example, the spectrum of the ring of algebraic integers in any number field has étale cohomological dimension 3 (modulo higher 2-torsion), [23]. Moreover, the étale cohomology groups of such spectra satisfy Artin-Verdier duality, which is reminiscent of the Poincare duality satisfied by closed, oriented, 3-dimensional manifolds. Furthermore, there are numerous analogies between algebraic class field theory and the theory of abelian covers of 3-manifolds. As an example we mention that the universal abelian cover surprisingly yields complete intersections on both the number-theoretic (de Smit–Lenstra) and topological/geometric (Neumann–Wahl) sides, [12, 26]. However, there is no good understanding of the underlying fundamental reasons for the above analogies, and of the full extent to which these analogies hold.

2.2. **Arithmetic of Hyperbolic Manifolds.** A hyperbolic 3-manifold M is an orientable 3-dimensional manifold that is locally modeled on hyperbolic three-space $\mathbf{H}^3$. Its fundamental group is a torsion-free discrete subgroup $\Gamma \subset PSL_2(\mathbf{C})$ defined uniquely (up to conjugation) by the requirement $M = \Gamma\backslash\mathbf{H}^3$. According to the geometrization program of Thurston [35], which has been the guiding force in 3-manifold topology research since the 1980's, hyperbolic 3-manifolds constitute the largest and least understood class of 3-manifolds. Such manifolds have been studied from a variety of perspectives, including geometric group theory, knot theory, analysis, and mathematical physics.

Recently, number theory has played a particularly crucial role in the study of hyperbolic 3-manifolds. The connection arises when one assumes that M has finite volume. In this case, Mostow rigidity asserts that the matrices representing $\Gamma \subset SL_2(\mathbf{C})$ can be taken to have entries in a number field. A consequence of this connection is that techniques from algebraic number theory now play a central role in the experimental investigation of hyperbolic 3-manifolds, to the extent that the premier software for performing computations with hyperbolic 3-manifolds (SNAP) incorporates the premier software for experimental algebraic number theory (GP-PARI) [10]. Furthermore, via this connection low-dimensional topology and number theory inform each other in surprising ways. For example, volumes of hyperbolic manifolds can be expressed in terms of special values of Dedekind zeta functions (Borel) [2]. Triangulations of hyperbolic 3-manifolds give rise to explicit elements in algebraic K-theory (Neumann–Yang) [27]. A very recent result of Calegari–Dunfield [5] shows that the Generalized Riemann Hypothesis together together with a natural assumption about certain Galois representations implies that the injectivity radii of hyperbolic 3-manifolds are unbounded. This provides strong evidence against a conjecture of Cooper related to the virtual Haken conjecture.

2.3. **Mahler measure and volumes of hyperbolic knots.** The Mahler measure of polynomials appears in surprisingly diverse areas of mathematics, for example, as the entropy of certain dynamical systems and as the value of higher regulators in K-theory (Beilinson, Boyd, Rodrigez-Villegas). It also appears in number theory (theory of heights and Lehmer's conjecture) and, more surprisingly, in the theory of hyperbolic 3-manifolds. More specifically, the Mahler measure of polynomials appearing naturally in 3-dimensional topology (such as A-polynomials, Alexander polynomials, and Jones polynomials) give interesting examples of Mahler measure from the perspective of arithmetic (Boyd-Rodriguez-Villegas, Silver-Williams, Champanerkar-Kofman), [3, 4, 6, 33, 34]. For example, the Mahler measure of the A-polynomial, which describes SL_2-representations of the fundamental group of a knot, is often equal to the hyperbolic volume of the knot, and is often related to special values of certain L-functions and the Bloch-Wigner dilogarithm.

2.4. **Quantum topology, modular forms, and multiple zeta functions.** Although originally defined analytically, Kontsevich's universal finite type invariant of links can be calculated algebraically, by an application of the Drinfeld associator, a certain formal infinite power series on two non-commuting variables, [1, 15]. It turns out that the coefficients of this series are given by special values of the Riemann zeta function and the multiple zeta functions of Euler and Zagier. By choosing different diagrams representing the same link L, one obtains different expressions for the Kontsevich invariant of L. Equating these expressions then yields non-trivial identities for the special values of these zeta functions. For example, the classical formula expressing $\zeta(2n)$ in terms of Bernoulli numbers can be proved in this way, [1].

Another connection between quantum topology and number theory appears in the context of quantum invariants of 3-manifolds and modular forms. These invariants are power series that are defined only at roots of unity, despite many efforts to extend their definition to an analytic function defined on the complex plane or, at least, on the unit disk. A partial success in this direction was achieved by R. Lawrence and D. Zagier who shown that for certain 3-manifolds (e.g. the Poincare homology sphere) the quantum invariants at roots of unity are limiting values of Eichler integrals of certain modular forms, [17].

3. Contents of the talks

Many talks at the workshop reported presented many new results related to the above list of topics. Other talks presented new connections between topology and number theory, such as analogies between Galois groups of infinite extensions and fundamental groups of 3-manifolds. In the following we summarize the contents of each talk and explain how they address or complement topics described above. We also indicate with a bullet (•) talks that presented an overview of a research area accessible to junior researchers at the conference.

3.1. **Arithmetic topology. Adam S. Sikora** (SUNY Buffalo), one of the organizers of the workshop, spoke on *Idele theory for* 3*-manifolds* (•). He discussed the analogies between number fields and 3-manifolds postulated by arithmetic topology and gave an overview of this field, emphasizing the role played by group actions. He then discussed the progress and the difficulties in constructing the theory of ideles, which has long been an essential part of the formulation of class field theory, for 3-manifolds.

Masanori Morishita (Kyushu) spoke on *Chern-Simons variation and Hida-Mazur theory*, which is joint work with Terashima [24,25]. This talk presented a new facet of arithmetic topology, namely a connection to the Hida-Mazur theory on the deformation of Galois representations and p-adic modular L-functions. Building on the analogy between knots and primes, Morishita discussed the variation of Chern-Simons invariants over the deformation space of hyperbolic structures on a knot complement from the view point of the analogy with One of the main results was to describe the variation of Chern-Simons invariants in terms of Deligne cohomology of the deformation curve.

3.2. **Arithmetic of hyperbolic** 3**-manifolds. Alan Reid** (U. of Texas) spoke on *The geometry and topology of arithmetic hyperbolic 3-manifolds* (•) [22]. This talk included a generous expository section that surveyed recent work on understanding the geometry and topology of arithmetic hyperbolic 3-manifolds. Much of the material was motivated by the virtual Haken and positive first Betti number conjectures, which concern the growth of invariants in towers of coverings.

Frank Calegari (Northwestern U.) gave a talk entitled *Rational Homology Growth in certain towers of hyperbolic 3-manifolds.* He discussed the question of obtaining upper and lower bounds for certain (exhaustive) covers of hyperbolic three manifolds using techniques from automorphic forms and non-commutative Iwasawa theory. Part of this included an exposition of his recent work with Dunfield [5].

Ted Chinburg (U. of Pennsylvania) spoke on *Length spectra and hyperbolic manifolds* (•), which surveyed some recent work on the question of when the length spectrum of a hyperbolic manifold determines and is determined by its commensurability class. Two particular cases are an affirmative result for compact arithmetic hyperbolic 3-manifolds [9] and a negative result for hyperbolic 5-manifolds by Prasad and Rapinchuk. After some general background he focused on quotients of products of hyperbolic upper half planes and half spaces. According to the speaker, these provide a test case for combining techniques from the theory of algebraic groups and algebraic number theory.

Melissa Macasieb (UCB) spoke on *Derived Arithmetic Fuchsian Groups of Genus Two* (•) [20]. She began with an overview of the construction of arithmetic 2- and 3-folds using Fuchsian and Kleinian groups, including explaining how arithmetic Fuchsian and Kleinian groups can be described in terms of quaternion algebras over number fields and discuss special properties of these groups. Then she outlined the classification of derived arithmetic Fuchsian groups of genus 2 and ended with some open questions for future work.

Thomas Mattman (CSU Chico) talked about *Trace fields of two families of knot complements.* The main point of his talk was to give an overview of some recent projects that aim to characterise the trace fields of two families of knot complements. The first project, which is joint work with Melissa Macasieb [21], follows the approach given by Reid and Walsh for two-bridge knots to investigate the trace fields of pretzel knots. The second, which is joint work with Indurskis, generalizes work of Hoste and Shanahan [14] on twist knots to other fillings of the Whitehead Link.

Mak Trifkovic (Fordham/Victoria) spoke on *Elliptic Curves over Imaginary Quadratic Fields, Homology of* 3*-Manifolds and p-adic Constructions of Rational Points.* This talk presented a conjectural p-adic construction of rational points on elliptic curves over imaginary quadratic fields, along with some numerical evidence [36]. The main ingredient in the construction are classes in (relative) homology of certain arithmetic quotients of the upper half-space. The hope is that hyperbolic geometry can to some extent mitigate the absence of the usual tools of algebraic geometry which makes the field mostly conjectural.

Finally, **Kathleen Petersen** (Queens) gave a talk entitled *Cusps and Congruence Subgroups.* One knows classically that the existence of non-congruence subgroups of $PSL(2, \mathbf{Z})$ can been seen topologically, by counting cusps. She discussed generalizations of this to $PSL(2, A)$ and $PGL(2, A)$, where A is an integer ring of a global field [28].

3.3. **Mahler measure and A-polynomials.** **Marc Culler** (UIC) gave the talk *A numerical method for computing A-polynomials.*, which presented a heuristic method for computing A-polynomials of hyperbolic knot manifolds that uses polynomial homotopy continuation and discrete Fourier transforms. Although he gave no rigorous proof of correctness, his method had the virtues of being almost completely automatic and of producing plausible results for a large class of examples. Examples presented included all 8 crossing knot complements and all of the hyperbolic knot complements that can be triangulated with no more than 7 tetrahedra, as tabulated by in databases by Callahan-Dean-Weeks and Champanerkar-Kofman-Patterson. Software implementing the method is available [11].

A pair of talks presenting joint work was given by **Abhijit Champanerkar** (U. South Alabama) and **Ilya Kofman** (CUNY Staten Island). Champanerkar spoke on *On links with cyclotomic Jones polynomials*, which showed that any infinite sequence of distinct prime alternating links with cyclotomic Jones polynomials must have unbounded hyperbolic volume [8]. Kofman presented *Mahler measure of Jones polynomials* [7]. In this talk, he proved that the Mahler measure of the Jones polynomial converges under twisting to that of a certain multivariable polynomial depending on the number of strands we twist at each site. This is consistent with the convergence of hyperbolic volume under Dehn surgery.

Matilde Lalin (Alberta) gave a talk entitled *Mahler measures under variations of the base group,* which was joint work with O. Dasbach [16]. This talk considered a generalization of the Mahler measure to elements in group rings, in terms of the Lück-Fuglede-Kadison determinant. She examined the variation of the Mahler measure when the base group changes, and in particular discussed the Mahler measure over infinite groups as the limit of Mahler measures over finite groups.

The next two talks concerned regulators and K-theory. **James Lewis** (Alberta) spoke on *Regulator currents on Milnor complexes* (cf. [19]). Let X be a projective algebraic manifold. For integers $k,\ m \geq 0$ he defined a cycle group $CH^k_M(X,m)$ in terms of the Zariski cohomology of the sheaf of Milnor K-groups on X, and a corresponding twisted variant $CH^k_{TM}(X,m)$. Then he constructed real logarithmic type maps ("real regulators") on $CH^k_{(TM)}(X,m)$ with values in Hodge cohomology, and investigated their properties.

Fernando Rodriguez Villegas (Texas) gave a talk entitled *Explicit examples of Beilinson's construction for elliptic curves* ($\bullet$). He discussed how one can make explicit the construction of elements of K_2 of an elliptic curve $E/\mathbf{Q}$ due to Beilinson, and used the curve $E = X_1(11)$ as the main example. As a byproduct he obtained infinitely many two-variable polynomials P with rational coefficients for which he could prove $m(P) = BL'(X_1(11), 0)$ for some $B \in \mathbf{Q}$, where $m(P)$ is the Mahler measure of P. The constant B can be intrinsically computed in advance, up to an integer factor that is a topological invariant. This invariant can be bounded a priori, but finding a closed expression remains an open problem.

Next, **Christopher Sinclair** (Boulder) spoke on *The geometry of polynomials with all roots on the unit circle,* which was joint work with Kathleen Petersen and Jeff Vaaler [29]. He demonstrated, using low dimensional examples, how the set of monic polynomials with all roots on the unit circle possesses a surprising amount of structure. He also drew a connection between these polynomials and random matrix theory.

Susan Williams (U. South Alabama) gave the report *Knot invariants from solenoid automorphisms.* The Pontryagin dual of a finitely generated $\mathbf{Z}[t, t^{-1}]$-module is a solenoid, with an automorphism dual to multiplication by t. She described classical and twisted Alexander invariants of knots in this dual universe, giving topological interpretations of periodic points and topological entropy of these algebraic dynamical systems. The leading coefficient of the (twisted) Alexander polynomial accounts for p-adic components of entropy. She was particularly interested in twisting by representations into $SL_2(\mathbf{Q})$ derived from faithful $SL_2(\mathbf{C})$ representations for hyperbolic knots.

Finally, **Christian Zickert** (Columbia U.) gave the last talk in this general area, entitled *Complex volume and the extended Bloch group.* A hyperbolic manifold defines a fundamental class in the extended Bloch group. Evaluating a suitable extension of Rogers dilogarithm on this class gives the complex volume (Vol+iCS) of the manifold. He described a new way of constructing the fundamental class, which gave rise to a very fast algorithm for computing complex volume. [37]. (It should be noted that Zickert was a graduate student at the time of this talk.)

3.4. **Quantum topology and knot invariants. Thang Le** (Georgia Tech) spoke on *On integrality of quantum invariants* ($\bullet$) [18]. This talk presented his joint work with Habiro on the strong integrality of quantum invariants (for general Lie algebras) of integral homology 3-spheres. He proved that the quantum invariants take values in an interesting ring namely the Habiro ring, which can be considered as a ring of "analytic functions" on the set of roots of unity. As a consequence, the quantum invariants of these spaces are always algebraic integers, and much more. An extension to rational homology spheres is also discussed.

A pair of talks presenting joint work was given by **S. Gukov** (Caltech) and **Don Zagier** (MPIM Bonn/College de France) Gukov spoke on *A State Sum Model for* $SL(2,\mathbf{C})$ *Chern-Simons Theory*, while Zagier spoke on *Modularity and three-manifolds* (•). The latter talk gave an overview of the relationships, both proven and conjectural, between 3-manifold invariants and modular forms, quasimodular forms, and mock modular forms.

3.5. **Complements. Nigel Boston** (U. of Wisconsin) spoke on *Random Pro-p Groups and Random Galois Groups.* He began by presenting work of Dunfield and Thurston [13] that studied how the distribution of finite quotients of a random g-generator g-relator abstract group compares with that of the fundamental group of a random 3-manifold obtained from a genus-g Heegard splitting. Then in his talk Boston considered analogous questions for random g-generator g-relator pro-p groups and for Galois groups of maximal pro-p extensions unramified away from a finite set S of primes with $|S| = g$.

Farshid Hajir (UMass Amherst) gave a talk entitled *The Fontaine–Mazur conjecture and infinite Galois groups* (•), which was intended to advertise some exciting conjectures in number theory to topologists to stimulate discussion and possible collaboration. Let p be a prime number, K a finite extension of $\mathbf{Q}$, and S a finite set of primes of K. Let K_S be the compositum of all finite Galois extensions of K of p-power order which are unramified outside S. The Galois group $G_{K,S}{=}Gal(K_S/K)$ is quite mysterious if S contains no primes above p. A deep conjecture of Fontaine and Mazur sheds some light on their structure. Hajir gave brief survey of recent advances in the study of $G_{K,S}$, most notably by Labute, and discussed possible analogous questions in the topology of hyperbolic manifolds.

4. Feedback and response from the conference

After the completion of the workshop, the organizers received many positive comments from both senior and junior participants. The junior participants in particular were happy about being able to present their work to more senior colleagues. All participants expressed excitement about being together in an environment that gave many opportunities to interact with experts from different fields, all with a common interest in the connections between number theory and topology. Based on these comments, we expect new collaborations to arise from the workshop.

At the completion of the workshop, it was agreed that a similar workshop should be held in the near future. Indeed, even as early as the Thursday of the workshop week, participants were remarking that a similar workshop should be proposed soon. As a result of this, three of the four organizers (Gunnells, Neumann, and Sikora), together with Don Zagier, proposed a followup workshop at Oberwolfach, which will be held in 2010. We hope having a workshop in Germany will encourage the participation of more European researchers, especially junior people. If this workshop receives the interest and enthusiasm we expect, we plan to apply again to BIRS for a North American workshop to be held in 2012.

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