High Dimensional Probability

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1 Overview of the Field

High Dimensional Probability has its roots in the investigation of limit theorems for random vectors and regularity of stochastic processes. It was initially motivated by the study of necessary and sufficient conditions for the boundedness and continuity of trajectories of Gaussian processes and extension of classical limit theorems, such as laws of large numbers, laws of the iterated logarithm and central limit theorems, to Hilbert and Banach space-valued random variables and empirical processes [9].

It resulted in the creation of powerful new tools - the methods of high dimensional probability and especially its offshoots, the concentration of measure phenomenon [1, 8] and generic chaining techniques [11], were found to have a number of applications in various areas of mathematics, as well as statistics and computer science . These include random matrix theory, convex geometry , asymptotic geometric analysis, non-parametric statistics , empirical process theory, statistical learning theory, compressed sensing, strong and weak approximations, distribution function estimation in high dimensions, combinatorial optimization, random graph theory, stochastic analysis in infinite dimensions and information and coding theory - cf. recent monographs [2, 3, 5, 6].

In recent years there has been a substantial progress in the area. In particular numerous important results were obtained concerning the connections between various functional inequalities related to the concentration of measure phenomenon, application of generic chaining methods to the study suprema of stochastic processes and norms of random matrices, Malliavin-Stein theory of Gaussian approximation, various stochastic inequalities and their applications in high-dimensional statistics and computer science.

2 Recent Developments and Open Problems

Functional inequalities and concentration of measure:

A new direction which has been recently explored in this field is the study of concentration properties of non-Lipschitz functions over a metric probability space. F. Götze presented new deviation inequalities for polynomials obtained, in joint works with S. Bobkov and H. Sambale, using Logarithmic Sobolev inequalities for continuous or discrete random variables.

Another natural development of the notion of concentration of measure is to consider dependent random variables. In this direction, M. Maïda presented new transport inequalities, obtained with D. Chafaï and A. Hardy, satisfied by some particle systems with repulsive Coulombian interactions. The particles of these systems are exchangeable but not independent. These new transport inequalities enable to obtain tail estimates for the Wasserstein distance between the empirical measure of the system and the equilibrium measure (which is the limit of the empirical measure when the number of particles goes to ∞). Interestingly the spectrum of the Ginibre ensemble enters this framework. This result motivates the following challenging open question (also related to the first item) : describe the concentration properties of the empirical measure of the spectrum of a random matrix with i.i.d entries. The case of symmetric matrices is well known and easy because the spectrum is in this case a Lipschitz image of the entries of the matrix due to the Hoffmann-Wielandt inequality. In the non symmetric case, the mapping between the entries and the spectrum is continuous but no longer Lipschitz, making the question highly non trivial. Nevertheless, the fact that the concentration properties of the Ginibre ensemble (corresponding to i.i.d Gaussian entries) have been obtained gives some hope for a possible general result in this direction.

Another natural example exhibiting concentration without independence is the symmetric group of permutations. P.-M. Samson presented new transport inequalities for the uniform distribution on the symmetric group. These new transport inequalities give back a deep result by Talagrand for random permutations. The interest of the proof of Samson is that is flexible enough to include other models of random permutations like for instance the Ewens distributions.

From a statistical point of view, the concentration of measure phenomenon also plays a very important role because it enables a non asymptotic analysis of different non-parametric tests. In this direction, M. Albert presented a new application of Talagrand's concentration on the symmetric group to a non-asymptotic test of independence.

Another direction where nice developments have been recently obtained is the study of the so-called superconcentration phenomenon (named after a recent monograph by S. Chatterjee [4] on the subject). This phenomenon can already be observed for instance by considering the maximum of i.i.d standard normal random variables. Simple calculations show that the variance of the maximum is then asymptotically much smaller than what can be obtained using the classical tools from concentration of measure. It is a major open problem in the field to give an accurate description of this superconcentration phenomenon using functional inequalities. The celebrated L_1/L_2 Poincaré inequality by Talagrand is one of the few known examples of functional inequalities implying this type of improved concentration. Talagrand's inequality is satisfied either by the uniform distribution on the discrete hypercube or by continuous models as for instance the standard Gaussian measures. In the Gaussian framework, Talagrand's inequality gives the optimal asymptotic for the variance of independent standard Gaussian random variables. Motivated by these questions, K. Tanguy presented a new higher order version of Talagrand's inequality both in discrete and in continuous settings. This new functional inequality has also potential applications to the analysis of Boolean functions on the hypercube (higher order KKL theorem).

Entropy and related inequalities:

T. Tkocz presented new results, obtained in a recent work with A. Eskenazis and P. Nayar, on the maximum and minimum values that the entropy and absolute moments of weighted sums of independent identically Gaussian mixture random variables, subject on the variance of the weighted sum to be fixed. In the case of entropy, the minimum value is known to be reached when exactly one of the weights is 1, while the rest are 0. He then proceeded to valid the natural conjecture that the maximum is reached when all the weights are the same.

R. Adamczak obtained with Latała, Puchała and Życzkowski new results on uncertainty principles in quantum information theory. For two unitary matrices, such an uncertainty principle goes back to Maassen and Uffink, whereas the new results treat any number of unitary matrices. While most of the results are probabilistic (i.e., for random unitaries drawn from the Haar measure), the uncertainty is quantified not just using entropy (or equivalently, relative entropy from a uniform distribution), but also using Hellinger distance and (following Fawzi-Hayden-Sen) total variation distance.

J. Melbourne discussed new results, obtained with M. Madiman and P. Xu, on maximizing the L^∞-norm of the convolution of a collection of probability densities. These may equivalently be thought of as lower bounds on the Rényi entropy (of order ∞) of convolutions, and are thus Rényi analogs of the well known Shannon-Stam entropy power inequality. General results were given for general spaces, and quantitative results expressed in terms of the Rényi entropy of summands were given for Euclidean spaces.

M. Fradelizi overviewed new results, obtained with M. Madiman, A. Marsiglietti and A. Zvavitch, on the existence and nonexistence of volume analogs of the leave-one-out entropy power inequality of Artstein-Ball-Barthe-Naor. Motivated by the connection of such inequalities to the convexification phenomenon for Minkowski sums of compact sets, they explored monotonicity of various non-convexity measures of Minkowski self-averages (which capture how such self-averages converge to the convex hull).

Limit theorems, Gaussian approximation:

The celebrated strong approximation result by Komlos-Major-Tusnady [7] from the seventies states that it possible to approximate almost surely the partial sums of size n of i.i.d. centered random variables by a Wiener process with an asymptotically controlled error term of order whose magnitude depends on the integrability of the variables. F. Merlevède presented a new extension of the KMT approximation result, obtained in a recent work with C. Cuny and J. Dedecker, to the case of dependent random variables. Their conditions are well adapted to a large variety of models, including contracting iterated random functions, autoregressive Lipschitz processes, and some ergodic Markov chains.

I. Nourdin discussed new achievements in the Malliavin-Stein theory of Gaussian approximation [10], obtained in collaboration with G. Peccati and M. Rossi. Using chaos decomposition and the Nualart-Peccati fourth moment theorem, they recently validated long standing conjectures for the limit distribution of the length of nodal lines in the so-called Berry random wave model.

C. Döbler presented an extension, obtained in a recent joint work with G. Peccati, to the Poisson space of the famous fourth moment theorem for Gaussian approximation of chaos.

R. Meller established two-sided bounds for moments of random chaoses of order two (quadratic forms) generated by symmetric random variables with comparable L_{2p} and L_p-norms. Previously such bounds were known only for chaoses generated by regular random variables (with log-concave or log-convex tails). It is an open problem to get similar estimates for chaoses of higher order (only the Gaussian case seems to be well understood).

P. Hitczenko investigated sequences of polynomials with nonnegative coefficients that satisfy differential-difference recurrences. After a normalization such polynomials are generating polynomials of integer-valued random variables. Hitczenko presented several examples where the law of the corresponding random variables converges to various probability distributions (Poisson, normal or Raleigh).

C. Houdré presented an overview of the state of the art of asymptotic results for the length of the longest common subsequence of two sequences of i.i.d. random variables taken values on a finite alphabet, including a new central limit theorem for the length of the longest common subsequence.

Probability in Banach spaces:

M. Veraar recalled the definition of γ-radonifying operators and presented a number of recent developments in stochastic analysis of vector-valued processes, where such operators play a crucial role and allow to extend results from finite to infinite dimensions.

I. Yaroslavtsev showed that the UMD spaces may be characterized via Meyer-Yoeurp and Yoerpl decompositions of vector-valued martingales. Such decompositions turn out to be crucial tools in stochastic integration in UMD spaces, one mey use them in particular to obtain sharp bounds for moments of L^q-valued stochastic integrals.

W. Bednorz discussed an old open question of Kwapień concerning the stability of convex domination of measures under convolution. He showed how the chaining techniques may be applied to treat this question if the dominating measure is logconcave.

K. Oleszkiewicz explored a question of A. Naor about a type of reverse triangle inequality for Banach space valued random variables. Specifically, while $\|X-z\|+\|z-Y\| \geq \|X-Y\|$ for any random variables X, Y and constant z, the question is whether the infimum over z of the mean of the left side can be bounded by the mean of the right side up to a constant term (a similar question can also be asked for powers of the norm). In the setting of separable Banach spaces, the optimal constant 3 was identified by explicit construction, and it was also shown that the constant can be improved if the space embeds isometrically into $L^p([0,1])$ for some $p \in (0,2]$.

E. Werner presented recent work with several collaborators that explores the volume ratio and cotype of triple tensor products of the ℓ_p^n spaces. Because of the fact that cotype properties do not behave in a transparent manner with respect to tensor products, the results constitute an advance in the understanding of this class of Banach spaces.

High-Dimensional Statistics:

V. Koltchinskii presented new results on the efficient estimation of a Besov function of the covariance operator of a Gaussian r.v. in a Hilbert space based on a random sample. It is shown that the natural estimator, in which the Besov function is applied to the sample covariance, admits a normal approximation with the best possible asymptotic variance but centered at its mean and, hence, biased in general. He then proceeded to construct an efficient estimator, which is also a function of the sample covariance, but in general different from the original Besov function, by an iterative bias reduction technique.

M. Peligrad discussed new estimation results, jointly obtained with N. Zhang, for the spectral density of a stationary random field. The method of proof is based on new probabilistic methods based on martingale approximations and nontrivial extensions of the Fejér-Lebesgue and Carleson theorem from harmonic analysis to stationary random fields.

S. Minsker presented a joint work with Xiaohan Wei on a new estimator of the mean of a random matrix that satisfies a nonasymptotic concentration inequality that in turn allows to construct confidence interval for the mean with some desired coverage probability. This results requires only finite second moments and includes the important problem of covariance estimation as a special case.

A. Olivier investigated the problem of finding nonparametric estimates of the division rate in growth-fragmentation models.

Special classes of stochastic processes:

J. Rosiński presented new results in the theory of infinitely divisible processes. New type of isomorphism identities for infinitely divisible Poissonian processes were first developed and then showed their long reaching applications. These included: (i) new results about the short-time behavior of Lévy processes; (ii) new methods to determine whether a process is infinitely divisible and to find their associated Lévy measure; and (iii) the celebrated Dynkin's isomorphism theorem that links Gaussian and Markov processes. In passing, J. Rosiński raised the open problem of whether a Rosenblatt process is infinitely divisible and suggest a general Cameron-Martin formula for Poissonian infinitely divisible processes by approximating a general translation or perturbation process by the superposition of square-shaped translations.

S. Chatterjee discussed the recent work with E. Bates that proves a version of the longstanding conjecture that the endpoint distribution of directed polymers is asymptotically localized in a region of stochastically bounded diameter in the low temperature phase.

V. Pérez-Abreu presented a panoramic overview of free-probability and new connection and asymptotic results related to Hermitian Brownian motion, Dyson-Brownian process, and free Brownian motion.

3 Presentation Highlights

Radosław Adamczak: *Uncertainty relations for high dimensional unitary matrices.*
I will present various types of uncertainty relations satisfied by Haar distributed random unitary matrices in high dimensions. If time permits I will also discuss their applications to quantum information theory (in particular to the problems of information locking and data hiding) and to special cases of the Dvoretzky theorem.

Melisande Albert: *Concentration inequalities for randomly permuted sums.*
Initially motivated by the study of the non-asymptotic performance of non-parametric tests based on permutation methods, some concentration inequalities for uniformly permuted sums are derived from the fundamental inequalities for random permutations of Talagrand. The idea is to first obtain a rough inequality for the square root of the permuted sum, and then, iterate the previous analysis and plug this first inequality to obtain a general concentration of permuted sums around their median. Then, concentration inequalities around the mean are deduced. This method allows us to obtain a Bernstein-type inequality. In particular, one recovers the Gaussian behavior of such permuted sums under classical conditions encountered in the literature.

Witold Bednorz: *The chaining approach to comparison.*
Abstract: I would like to explain the role of chaining method in the comparison of certain functionals of processes - expectations of suprema or tails of suprema. The basic setting for the problem is that we have two processes and we can compare their increments in the sense of moments or tails. Now the question is does it imply that the suprema of these two processes can also be compared and what are the consequences when such a comparison is possible?

Sourav Chatterjee, *The endpoint distribution of directed polymers.*
Probabilistic models of directed polymers in random environment have received considerable attention in recent years. Much of this attention has focused on integrable models. In this talk, I will describe some new computational tools that do not require integrability. As an application, I will sketch the proof of a subsequential version of the longstanding conjecture that in the low temperature phase, the endpoint distribution

is asymptotically localized in a region of stochastically bounded diameter. This is based on joint work with Erik Bates.

Christian Döbler: *The fourth moment theorem on the Poisson space.*
By combining Stein's method and Malliavin calculus on the Poisson space with an adaption of the spectral viewpoint highlighted by Ledoux (2012) we provide exact fourth moment bounds for the normal approximation of multiple Wiener-Ito integrals on the Poisson space. In particular, we show that the fourth moment phenomenon first discovered by Nualart and Peccati (2005) also holds true in this discrete framework. This is joint work with Giovanni Peccati.

Matthieu Fradelizi: *On the convergence of Minkowski sums to the convex hull.*
Let us define, for a compact set $A \subset \mathbb{R}^n$, the Minkowski averages of A:

$$A(k) = \left\{ \frac{a_1 + \cdots + a_k}{k} : a_1, \ldots, a_k \in A \right\} = \frac{1}{k}\Big(\underbrace{A + \cdots + A}_{k \text{ times}} \Big).$$

I shall show some monotonicity properties of $A(k)$ towards convexity when considering, the Hausdorff distance, the volume deficit and a non-convexity index of Schneider. For the volume deficit, the monotonicity holds in dimension 1 but fails for $n \geq 12$, thus disproving a conjecture of Bobkov, Madiman and Wang. For Schneider's non-convexity index, a strong form of monotonicity holds. And for the Hausdorff distance, some monotonicity holds for k large enough, depending of the ambient dimension and not on the set A. Based on a work in collaboration with Mokshay Madiman, Arnaud Marsiglietti and Artem Zvavitch.

Friedrich Goetze: *Second and higher order concentration of measure.*
Higher order concentration results are proved for differentiable functions on Euclidean spaces with LSI type measures provided that they are Lipschitz bounded of order $d \geq 2$ and orthogonal to polynomials of order $d1$. This is recent joint work with S. Bobkov and H. Sambale. It extends to concentration of measure for functions on discrete spaces subject to higher order L_2 type differences uniformly bounded and an appropriate Hoeffding expansion structure. The results yield uniform exponential bounds for $|f|^{2/d}$ extending previous second order results for functions on these spaces. Some applications of these bounds are given. In particular applications of second order concentrations for polynomials on the sphere and spherical averages of second order uncorrelated isotropic vector are discussed, illustrating previous joint results with S.Bobkov and G. Chistyakov.

Christian Houdre: *Asymptotics in Sequences Comparisons.*
Both for random words and random permutations, I will present a panoramic view of recent results on the asymptotic law of the, centered and normalized, length of their longest common (and increasing) subsequences. Tools and results involve concentration inequalities for geodesics of LCSs paths, Stein' s method as well as maximal eigenvalues of some Gaussian random matrices.

Pawel Hitczenko: *Recurrences for generating polynomials.*
In this talk we consider sequences of polynomials that satisfy differential–difference recurrences. Our interest is motivated by the fact that polynomials satisfying such recurrences frequently appear as generating polynomials of integer valued random variables that are of interest in discrete mathematics. It is, therefore, of interest to understand the properties of such polynomials and their probabilistic consequences. We will be primarily interested in the limiting distribution of the corresponding random variables and we give a few examples, leading to a Poisson, normal, and Rayleigh distributions.

Vladimir Koltchinskii: *Efficient Estimation of Smooth Functionals of Covariance Operators*
A problem of efficient estimation of a smooth functional of unknown covariance operator Σ of a mean zero Gaussian random variable in a Hilbert space based on a sample of its i.i.d. observations will be discussed. The goal is to find an estimator whose distribution is approximately normal with a minimax optimal variance in a setting when either the dimension of the space, or so called effective rank of the covariance operator are allowed to be large (although much smaller than the sample size). This problem has been recently solved in our joint paper with Loeffler and Nickl in the case of estimation of a linear functional of unknown eigenvector of Σ corresponding to its largest eigenvalue (the top principal component). The efficient estimator developed in this paper does not coincide with the naive estimator based on the top principal component of sample

covariance which is not efficient due to its large bias. An approach to a more general problem of efficient estimation of a functional $\langle f(\Sigma), B\rangle$ for a given sufficiently smooth function $f : \mathbb{R} \mapsto \mathbb{R}$ and given operator B will be also discussed.

Mylène Maïda: *Concentration of measure for Coulomb gases.*
A Coulomb gas is the canonical Gibbs measure associated with a system of particles in electrostatic interaction. As the number of particles grows to infinity, the empirical measure of a Coulomb gas converges weakly towards an equilibrium measure, characterized by a variational principle. We obtain sub-gaussian concentration inequalities around this equilibrium measure, in the weak and Wasserstein topologies. This yields for instance a concentration inequality at the correct rate for the Ginibre ensemble. The proof relies on new functional inequalities, which are counterparts of Talagrand's transport inequality in the Coulomb interaction setting. Joint work with Djalil Chafaï and Adrien Hardy.

James Melbourne: *Bounds on the maximum of the density for certain linear images of independent random variables.*
We will present a generalization of a theorem of Rogozin that identifies uniform distributions as extremizers of a class of inequalities, and show how the result can reduce specific random variables questions to geometric ones. In particular, by extending "cube slicing" results of K. Ball, we achieve a unification and sharpening of recent bounds on densities achieved as projections of product measures due to Rudelson and Vershynin, and the bounds on sums of independent random variable due to Bobkov and Chistyakov. Time permitting we will also discuss connections with generalizations of the entropy power inequality.

Rafał Meller: *Two sided moment estimates for random chaoses.*
Let $X_1, ..., X_n$ be the random variables such that there exists a constant $C > 1$ satisfying $||X_i||_{2p} \leq C||X_i||_p$ for every $p \geq 1$. We define random chaos $S = \sum a_{i_1,...,i_d} X_{i_1}...X_{i_d}$. We will show two sided deterministic bounds on $\|S\|_p = (\mathbb{E}|S|^p)^{1/p}$, with constant depending only on C and d in two cases:
1) $X_1, ..., X_n$ are a.s nonnegative, and $a_{i_1,...,i_d} \geq 0$.
2) $X_1, ..., X_n$ are symmetric, $d = 2$.

Florence Merlevède: *On strong approximations for some classes of random iterates.*
This talk is devoted to strong approximations in the dependent setting. The famous results of Komlós, Major and Tusnády (1975-1976) states that it is possible to approximate almost surely the partial sums of size n of i.i.d. centered random variables in L^p ($p > 2$) by a Wiener process with an error term of order $o(n^{1/p})$. In the case of functions of random iterates generated by an iid sequence, we shall give new dependent conditions, expressed in terms of a natural coupling (in L^∞ or in L^1), under which the strong approximation result holds with rate $o(n^{1/p})$. The proof is an adaptation of the recent construction given in Berkes, Liu and Wu (2014). As we shall see our conditions are well adapted to a large variety of models, including left random walks on $GL_d(R)$, contracting iterated random functions, autoregressive Lipschitz processes, and some ergodic Markov chains. We shall also provide some examples showing that our L^1-coupling condition is in some sense optimal. This talk is based on a joint work with J. Dedecker and C. Cuny.

Stanislav Minsker: *Random Matrices with Heavy-Tailed Entries: Tight Mean Estimators and Applications to Statistics.*
Estimation of the covariance matrix has attracted significant attention of the statistical research community over the years, partially due to important applications such as Principal Component Analysis. However, frequently used empirical covariance estimator (and its modifications) is very sensitive to outliers, or "atypical points in the sample. As P. Huber wrote in 1964, ...This raises a question which could have been asked already by Gauss, but which was, as far as I know, only raised a few years ago (notably by Tukey): what happens if the true distribution deviates slightly from the assumed normal one? As is now well known, the sample mean then may have a catastrophically bad performance Motivated by Tukey's question, we develop a new estimator of the (element-wise) mean of a random matrix, which includes covariance estimation problem as a special case. Assuming that the entries of a matrix possess only finite second moment, this new estimator admits sub-Gaussian or sub-exponential concentration around the unknown mean in the operator norm. Our arguments rely on generic chaining techniques applied to operator-valued stochastic processes, as well as bounds on the trace moment-generating function. We will discuss extensions of our approach to matrix-valued U-statistics and examples such as matrix completion problem. Part of the talk will be based on a joint work with Xiaohan Wei.

Ivan Nourdin: *Asymptotic behavior of Berry random wave model.*
I will explain how to use chaos decomposition technique to analyze the asymptotic behavior of geometric quantities related to the so-called Berry random wave model.

Krzysztof Oleszkiewicz: *On a question of Assaf Naor.*
For any separable Banach space $(F, \|\cdot\|)$ and independent F-valued random vectors X and Y such that $\mathbf{E}\|X\|, \mathbf{E}\|Y\| < \infty$, we have

$$\inf_{z \in F} (\mathbf{E}\|X - z\| + \mathbf{E}\|Y - z\|) \leq 3 \cdot \mathbf{E}\|X - Y\|.$$

Indeed, it suffices to consider $z = (\mathbf{E}X + \mathbf{E}Y)/2$ and use Jensen's inequality. Assaf Naor asked whether the constant 3 in the inequality is optimal. We will discuss this and related problems.

Adélaïde Olivier: *Estimation of the division rate in growth-fragmentation models.*
This talk is concerned with growth-fragmentation models, implemented for investigating the growth of a population of cells. From a stochastic point of view, we are dealing with the evolution of a system of particles. The evolution of the system is then driven by two phenomenons. First, particles evolve on a deterministic basis: they age, they grow. Secondly, particles split randomly: a particle of age a or size x splits into two particles (of age 0, of size at birth $x/2$), at a rate B depending on the age or on the size of the splitting particle. The division rate B is an unknown function, on which we focus our attention.

The main goal of this statistical work is to get a nonparametric estimate of the division rate. To do so, various observation schemes may be considered: **(i)** Observation *up to a given generation* in the genealogical tree of the population. **(ii)** Continuous time observation *up to time* T. This scheme, which differs radically from those previously used, entails specific difficulties – mainly a bias selection.

As different as these two schemes may be, I will try to highlight their common features. Mainly, a competition occurs between the growth of the tree (measured by the Malthus parameter) and the convergence to equilibrium of the branching process (measured by some ρ_B say). We still have to deal with open questions in these toy models: adaptativity with respect to the smothness of B - which requires some deviation inequalities (availaible to treat the observation scheme (i) but not (ii)), but also adaptivity with respect to the parameter ρ_B measuring the convergence to equilibrium.

Magda Peligrad: *Central limit theorem for Fourier transform and periodogram of random fields.*
The talk is motivated by the properties surrounding the spectral density of a stationary process and of a random field. We start by presenting a characterization of the spectral density in function of projection operators on sub-sigma fields. We also point out that the limiting distribution of the real and the imaginary part of the Fourier transform of a stationary random field is almost surely an independent vector with Gaussian marginal distributions, whose variance is, up to a constant, the fields spectral density. The dependence structure of the random field is general and we do not impose any conditions on the speed of convergence to zero of the covariances, or smoothness of the spectral density. The only condition required is that the variables are adapted to a commuting filtration and are regular in some sense. The results go beyond the Bernoulli fields and apply to both short range and long range dependence. The method of proof is based on new probabilistic methods based on martingale approximations and also on borrowed tools from harmonic analysis. Several examples to linear, Volterra and Gaussian random fields will be presented. This is a joint work with Na Zhang.

Victor Pérez-Abreu: *Some extensions of the chain: From Hermitian Brownian motion to Dyson-Brownian process to free Brownian motion.*
We overview connections between several classical and noncommutative stochastic processes.

Jan Rosinski: *Isomorphism identities for perturbed infinitely divisible random fields.*
We consider infinitely divisible random fields perturbed by an additive independent noise. We investigate admissible perturbations under which the perturbed field, which need not be infinitely divisible, is absolutely continuous with respect to the unperturbed one, and establish the related isomorphism identities. The celebrated Dynkin's isomorphism theorem is an example of such phenomenon, where the local time of a Markov process is the perturbation of a permanental field.

Paul Marie Samson: *Transport-entropy inequalities related to Talagrand's concentration results, for a class of probability measures on groups of permutations.*

Following Talagrand's concentration results for permutations picked uniformly at random from a symmetric group, Luczak and McDiarmid have generalized it to more general groups of permutations which act suitably 'locally'. Here we extend their results by setting transport-entropy inequalities on these permutations groups. Talagrand and Luczak-Mc-Diarmid concentration properties are consequences of these inequalities. The results are also generalised to a larger class of measures including Ewens distributions of arbitrary parameter on the symmetric group. By projection, we derive transport-entropy inequalities for the uniform law on the slice of the discrete hypercube and more generally for the multinomial law. One typical application is deviation bounds for the so-called configuration functions, such as the number of cycles of given lenght in a random permutation.

Kevin Tanguy: *Talagrand's inequality of higher order, application to boolean analysis*
We present a new representation formula of the variance of a function f under the standard gaussian measure along the Ornstein-Uhlenbeck semi-group. This representation can bee seen as Taylor formula with remainder term. The proof rests on elementary arguments as interpolation, integration by parts and fundamental theorem of analysis. From this representation we can deduce new bound of the variance, in term of norms L^1 and L^2 of partial derivatives, in the same spirit as Talagrand's inequality. The scheme of proof can be also used in the framework of the discrete cube $C_n = \{-1, 1\}^n$ and permit us to obtain a theorem for the influence of boolean function which can be seen as an extension of the theorem of Kahn-Kalai-Linail at the order 2.

Tomasz Tkocz: *The entropy and moments of sums of certain iid random variables.*
We shall discuss extrema of the entropy and moments of weighted sums of iid random variables with densities proportional to $\exp(-|t|^q)$, subject to the variance being fixed. Based on joint work with A. Eskenazis and P. Nayar.

Mark Veraar: *Applications of γ-radonifying operators in (stochastic) analysis.*
Since the 60's γ-radonifying operators have been used in connection to Gaussian measures on Banach spaces. In recent years γ-radonifying operators play an important role in several subfields of (stochastic) analysis, usually when extending results from finite dimensions to infinite dimensions. For example this has played a major role in: harmonic analysis, functional calculus, control theory, stochastic integration theory, Malliavin calculus. In the talk I will give an overview on this and mention several recent results.

Elisabeth Werner: *On the geometry of projective tensor products.*
We study the volume ratio of the projective tensor products $\ell_p^n \otimes_\pi \ell_q^n \otimes_\pi \ell_r^n$ with $1 \leq p \leq q \leq r \leq \infty$. We obtain asymptotic formulas that are sharp in almost all cases. From the Bourgain-Milman bound on the volume ratio of Banach spaces in terms of their cotype 2 constant, we obtain, as a consequence of our estimates, information on the cotype of these 3-fold projective tensor products. Our results naturally generalize to k-fold products. Based on joint work with O. Giladi, J. Prochno, C. Schütt and N. Tomczak-Jaegermann.

Ivan Yaroslavtsev: *Martingale decompositions in UMD Banach spaces.*
In this talk we present the Meyer-Yoeurp decomposition for UMD Banach space-valued martingales. Namely, we prove that X is a UMD Banach space if and only if for any fixed $p \in (1, \infty)$, any X-valued martingale M has a unique decomposition $M = M^d + M^c$ such that M^d is a purely discontinuous martingale, M^c is a continuous martingale, $M_0^c = 0$, and $\mathbb{E}\|M_\infty^d\|^p + \mathbb{E}\|M_\infty^c\|^p \leq c_p \mathbb{E}\|M_\infty\|^p$. An analogous assertion is shown for the Yoeurp decomposition of a purely discontinuous martingale into a sum of a quasi-left continuous martingale and a martingale with accessible jumps.

Meyer-Yoeurp and Yoeurp decompositions play a significant role in stochastic integration theory for cdlg martingales, For instance one can show sharp estimates for an L^p-norm of an L^q-valued stochastic integral with respect to a general local martingale. An important tool for obtaining these estimates are the recently proven Burkholder-Rosenthal-type inequalities for discrete L^q-valued martingales.

This talk is partially based on joint work with Sjoerd Dirksen (RWTH Aachen University).

4 Outcome of the Meeting

Participants of the conference supported the idea of preparing the proceedings volume. In particular, motivated by the talks by Albert and Samson, a group of participants (Adamczak, Albert, Chatterjee, Chafaï, Samson) have decided to prepare for the proceedings a survey devoted to recent developments in the study of concentration properties of random permutations models. It is expected that other such surveys and original research papers will also be submitted to the proceedings volume.

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