Stochastic Analysis and Mathematical Finance - A Fruitful Partnership

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1 An Overview of the Field

Mathematics of financial markets is one of the most active and exciting areas of contemporary applied mathematics. It provides the mathematical community with a rich source of challenges, which, in turn, enrich the society's understanding of the financial system and help both regulators and the financial industry make better and more informed choices. Its proximity to practice, in addition to its purely mathematical appeal, makes this field especially attractive to young mathematicians. Moreover, it provides for wider employment opportunities both within the academic world and outside of it.

One of the most challenging areas within mathematical finance, namely its foundations, aims to understand the basic structure shared by all financial markets. It uses probabilistic tools, together with a variety of methods from stochastic, functional and convex analysis and partial differential equations, and draws from a host of other mathematical disciplines to accomplish its goals. Prior advances in the foundational issues have not only made a huge impact on the financial practice, but have also inspired a number of breakthroughs in related areas of mathematics traditionally regarded as theoretical.

2 Scientific Progress Made

Arbitrage theory and model-independent finance. Several participants presented results in what can be broadly referred to as arbitrage theory. The early, conceptual, results in this area go back to the work of HARRISON, KREPS and PLISKA (see [5], [6] and [10]), a major (mathematical) breakthrough was accomplished by DELBAEN and SCHACHERMAYER, with their work culminating in the papers [3] and [2]. The intensity of the research output in the arbitrage theory only increased since, with new contributions being made even today.

Currently, the focus is on robust variants of the no-arbitrage condition and their relationships with deep probabilistic and measure-theoretic concepts such as Skorokhod embedding or optimal (martingale) transport. Presentations of BEATRICE ACCIAIO, SIGRID KÄLLBLAD, JAN OBŁÓJ and CLAUDIO FONTANA described important new contributions to this rich theory.

Fractional Brownian motion in finance. The important topic of volatility modeling has recently been reinvigorated by the re-introduction of fractional Brownian motion-based models. These seem to exhibit an extraordinarily good fit to the observed data, and provide a parsimonious explanation for a number of empirically observed features. The talks of MATHIEU ROSENBAUM and CHRISTIAN BAYER provided an excellent overview of the latest results both in the modelling, as well as option-pricing framework.

A different use of the fractional Brownian motion has been explored by CHRISTOPH CZICHOWSKY who talked about recent progress in the long-standing optimal-investment problem with transaction costs when the asset price is modeled by fractional Brownian motion.

Multi-agent approaches. Game-theoretic models, as well as problems involving the notion of a competitive equilibrium, traditionally tackled by economists, have found their way to the mainstream of mathematical research in quantitative finance thanks to the versatility and mathematical richness of the stochastic models they support.

The central fixture of the game-theoretic side is the notion of a mean-field game (introduced by LASRY and LIONS). The financial-mathematics community has recently recognized this model as especially relevant for applications, while, at the same time, sufficiently tractable. This has led to an explosion of activity; the talks of LUCIANO CAMPI, MARCEL NUTZ and DYLAN POSSAMAI all centered around mean-field games in various financial/economical settings.

The more mature subject of competitive equilibria received an equal amount of attention. With the early research by DUFFIE, HUANG, KARATZAS, LEHOCZKY and SHREVE (see [4] or [8]) setting the stage, new developments (including the treatment of “incomplete markets”) have been appearing regularly to this day. Recent connections with the theory of systems of parabolic PDE and backward SDE have been particularly visible in an array of talks on the subject (and some related fields), by KASPER LARSEN, JOHANNES MUHLE-KARBE, SERGIO PULIDO and HAO XING. The talk of SCOTT ROBERTSON introduced a competitive model into the important and practically relevant area of mortgage-backed securities.

Optimal investment, risk, and related topics. The roots of the optimal-investment (utility maximization) theory can be traced to the seminal work [11] of ROBERT MERTON, or, one may argue, even to DANIEL BERNOULLI (see [1]). The theory has been adopted by the financial-mathematics community through the work of KARATZAS, KRAMKOV, LEHOCZKY, SCHACHERMAYER, SHREVE, XU (see, e.g., [7] and [9]) and many others, during the 1990-ies. In the early 2000s it joined with the axiomatic theory of risk measures, introduced by ARTZNER, DELBAEN, EBER and HEATH, and further developed by these authors, as well as HANS FÖLLMER and many others. The theory quickly evolved to encompass a large number of loosely related topics, the common denominator of which is the rich interplay between convex and stochastic analysis. The talks by JAN KALLSEN, MICHAEL KUPPER and DANIEL LACKER can all be classified under this heading.

Another important area at the intersection of optimal investment and arbitrage theory is stochastic portfolio theory. Introduced by ROBERT FERNHOLZ and later developed by FERNHOLZ, KARATZAS and others, this theory studies the properties of financial markets and a class of simply-generated trading strategies in them from a descriptive—as opposed to normative—point of view. The presentations of CHRISTA CUCHIERO, SOUMIK PAL and JOHANNES RUF provided new results and insights in this direction.

Topics in Stochastic Analysis. Several talks dealt with mathematically and financially interesting issues that do not fit neatly into the categories singled out above, but use financial insights, or draw inspiration from problems originating in finance, to produce interesting results in stochastic analysis. The talks of MATHIAS BEIGLBOECK, BRUNO BOUCHARD, UMUT ÇETIN, RAMA CONT and KAVITA RAMANAN deal, respectively, with peacocks and the Skorokhod embedding, a semimartingale-type decomposition, an inverse problem for a diffusion, an extension of Itô-calculus and a class of reflected diffusions. JOSEF TEICHMAN and MARTIN LARSSON focus on affine and polynomial classes of Markov processes, while PETER TANKOV solves an optimal-tracking problem in the continuous semimartingale setting. Finally, the talk of NIZAR TOUZI provides a novel numerical scheme for semilinear PDE based on a Monte-Carlo simulation involving branching processes.

3 Presentation Highlights

Beatrice Acciaio

"*Model-independent pricing with additional information: a Skorokhod embedding approach*"

ABSTRACT. We analyze the pricing problem of an agent having additional (potentially insider) information on the market in a model-independent setup. Following Hobson's approach we reformulate this problem as a constrained Skorokhod embedding problem, and show a natural supperreplication result. Furthermore, we establish a monotonicity principle for the constrained SEP, giving a geometric characterization of the support of the optimizers (in the spirit of Beiglboeck, Cox and Huesmann (2014)) which allows to link the additional information with geometric properties of the optimizers to the constrained embedding problem. Surprisingly, for certain types of information the absence of arbitrage can be easily checked by considering only unconstrained solutions. (The talk is based on a joint work with Alex Cox and Martin Huesmann.)

Christian Bayer

"*Pricing under rough volatility*"

ABSTRACT. From an analysis of the time series of realized variance (RV) using recent high frequency data, Gatheral, Jaisson and Rosenbaum (2014) previously showed that log-RV behaves essentially as a fractional Brownian motion with Hurst exponent H of order 0.1, at any reasonable time scale. The resulting Rough Fractional Stochastic Volatility (RFSV) model is remarkably consistent with financial time series data. We now show how the RFSV model can be used to price claims on both the underlying and integrated variance. We analyze in detail a simple case of this model, the rBergomi model. In particular, we find that the rBergomi model fits the SPX volatility markedly better than conventional Markovian stochastic volatility models, and with fewer parameters. Finally, we show that actual SPX variance swap curves seem to be consistent with model forecasts, with particular dramatic examples from the weekend of the collapse of Lehman Brothers and the Flash Crash.

Mathias Beiglboeck

"*Shadow couplings and related extremal peacocks*"

ABSTRACT. A classical result of Strassen asserts that given probabilities on the real line which are in convex order, there exists a martingale coupling with these marginals. Remarkably, it is a non trivial problem to construct particular solutions to this problem. In this article, we introduce a family of such martingale couplings, each of which admits several characterizations in terms of optimality properties / geometry of the support set / representation through a Skorokhod embedding. As one particular element of this family we recover the (left) monotone martingale transport which can be viewed as a martingale analogue of the classical monotone rearrangement. As another canonical element of this family we identify a martingale coupling that resembles the usual product coupling and enjoys several curious properties related Lipschitz-kernels and general transport costs as recently introduced by Gozlan etal. Finally we shall consider the multi-period martingales / peacocks related to these couplings. (joint work with Nicolas Juillet)

Bruno Bouchard

"*A Doob-Meyer-Mertens decomposition for BSDEs, and general estimates*"

ABSTRACT. In this talk, I will present how two (well-known) results of the general semi-martingale theory can be extended to the context of BSDEs. One is the the Doob-Meyer-Mertens decomposition for super-martingales that are possibly not right continuous. The second is Meyers estimate of the bounded variation part of a super-martingale by the supermatingale itself, under general (not quasi-left continuous) filtrations. Examples of applications (in finance) will be discussed.

Luciano Campi

"*N-player games and mean-field games with absorption*"

ABSTRACT. We consider an N-player games with weakly interacting diffusions and an absorbing set. We study the existence of Nash equilibria of the corresponding mean-field game and establish, under a non-degeneracy condition on the diffusion coefficient, that the latter provide nearly optimal strategies for the N-player games. Moreover, we provide an example of a mean-field game with absorption whose Nash equilibrium is not a good approximation of the pre-limit game. This is based on a joint work with Markus Fisher.

Umut Çetin

"*Linear Inverse Problems for Diffusions*"

ABSTRACT. Let $(P_t)_{t>=0}$ be the semigroup corresponding to a one-dimensional diffusion, X, with speed measure m. After giving a brief motivation from market microstructure theory I will discuss L^2 solutions of the integral equation $g = P_T f$ for a given deterministic $T > 0$ and g in $L^2(m)$. If g is a probability density, the solution of this problem amounts to finding an initial distribution for X so that its law at time-T is defined by gdm. In such a setting this inverse problem can also be viewed as an alternative to finding a martingale that vanishes at time-0 and has the required distribution at time-T. The crucial difference between the two is that the dynamics of the martingale is already given in the formulation of the inverse problem and one looks for an appropriate initial distribution. Although fixing the martingale might be desirable for the modeller for numerical or empirical reasons, the catch is that the inverse problem does not always have a solution since the inverse of P_T is an unbounded operator. In this talk I will give a necessary and sufficient condition for the existence of an L^2 solution to the inverse problem described above and present a formula for the inversion.

Rama Cont

"*Functional calculus and pathwise integration for paths of finite quadratic variation*"

ABSTRACT. We construct a pathwise integral, defined as the limit of non-anticipative Riemann sums, with respect to paths of finite quadratic variation, for a class of path-dependent integrands which includes all 'delta-hedging' strategies. We show that this pathwise integrals enjoys an isometry property, which may be viewed as a pathwise analog of the well-known Ito isometry for stochastic integrals. This property is then used to represent the integral as a continuous map on an appropriately defined vector space of integrands and obtain

a pathwise 'signal plus noise' decomposition for functionals of irregular paths with non-vanishing quadratic variation. The proofs are based on the Functional Ito calculus. Relations with rough path integration are discussed. These results provide a framework for pathwise construction and analysis of gain processes for continuous-time hedging strategies for path-dependent derivatives. (Joint work with Anna ANANOVA, Imperial College)

Christa Cuchiero

"*Polynomial processes in Stochastic Portfolio Theory with extensions to large markets*"

ABSTRACT. Inspired by volatility stabilized market models introduced by Robert Fernholz and Ioannis Karatzas [2], we introduce a class of processes which are polynomial in the sense of [1] to model both asset prices (or market capitalizations of companies in an equity market) and their corresponding market weights. These models substantially extend volatility stabilized models, while preserving tractability, since the geometry of SPT, more precisely the transformation between asset prices and market weights, perfectly fits the structural properties of polynomial processes. In view of large financial market modeling, for potentially a continuum of assets and their corresponding market weights, we also consider measure valued analogues of these processes. [1] C. Cuchiero, M. Keller-Ressel, and J. Teichmann. Polynomial processes and their applications to mathematical finance. Finance and Stochastics, 16(4):711740, 2012. [2] R. Fernholz and I. Karatzas. Relative arbitrage in volatility-stabilized markets. Annals of Finance, 1(2):149177, 2005.

Christoph Czichowsky

"*The log-optimal portfolio and fractional Brownian motion*"

ABSTRACT. While absence of arbitrage in frictionless financial markets (i.e. without transaction costs) requires price processes to be semimartingales, non-semimartingales can be used to model prices in an arbitrage-free way, if proportional transaction costs are taken into account. In this talk, we show how to obtain the existence of the log-optimal portfolio under proportional transaction costs in the fractional Black-Scholes model by establishing a so-called shadow price. This is a semimartingale price process, taking values in the bid ask spread, such that frictionless trading for that price process leads to the same optimal strategy and utility as the original problem under transaction costs. The relation to frictionless financial markets can then be used to describe the behaviour of the optimal trading strategy under transaction costs. (The talk is based on joint work with Remi Peyre and Walter Schachermayer.)

Claudio Fontana

"*General dynamic term structures under default risk*"

ABSTRACT. We consider the problem of modelling the term structure of bonds subject to default risk, under minimal assumptions on the default time. In particular, we do not assume the existence of a default intensity and we therefore allow for the possibility of default at predictable times. We extend the Heath, Jarrow and Morton (1992) framework by introducing an additional term driven by a general random measure, which encodes information about those times where default can happen with positive probability. In this framework, we derive necessary and sufficient conditions for a reference probability measure to be a local martingale measure for the large financial market of credit risky bonds, also considering general recovery schemes. To

this end, we establish a new Fubini theorem with respect to a random measure by means of enlargement of filtrations techniques. (Joint work with Thorsten Schmidt.)

Sigrid Källblad

"*Model-independent bounds for Asian options - a dynamic programming approach*"

ABSTRACT. We consider the problem of finding model-independent bounds on the price of an Asian option, when the call prices at the maturity date of the option are known. Our method di ffer from most approaches to model-independent pricing in that we consider the problem as a dynamic programming problem, where the controlled process is the conditional distribution of the asset at the maturity date. By formulating the problem in this manner, we are able to determine the model-independent price through a PDE formulation. Notably, this approach does not require specific constraints on the payoff function (e.g. convexity), and would appear to be generalisable to many related problems. This is joint work with A.M.G. Cox.

Jan Kallsen

"*On portfolio optimization under small fixed transaction costs*"

ABSTRACT. While optimal investment under proportional transaction costs is quite well understood by now, less has been done in the presence of fixed fees for any single transaction. In this talk we consider the asymptotics of the no-trade region for small fixed costs. More specifically, we sketch the rigorous verification for a general univariate Ito process market under exponential utility.

Michael Kupper

"*Robust exponential hedging in discrete time*"

ABSTRACT. We focus on the robust exponential utility maximization problem with random endowment in discrete time. An investor is allowed to invest dynamically in the market and maximizes his/her worst case expected exponential utility of the endowment plus terminal wealth with respect to a family of non-dominated probabilistic models. Under two assumptions regarding the tightness of this family and the existence of certain martingale measures we provide the existence of an optimal trading strategy defined simultaneously under all models. Further, we characterize the dual problem and provide duality for measurable endowments. The talk is based on joint work with Daniel Bartl and Patrick Cheridito.

Daniel Lacker

"*Convex risk measures and non-exponential large deviations*"

ABSTRACT. The convex duality between relative entropy and the entropic risk measure (a.k.a. cumulant generating functional) underlies several arguments in large deviations (especially the weak convergence approach of Dupuis and Ellis) and concentration inequalities (particularly their formulations in terms of transportation

inequalities). In fact, essentially only the basic convex duality relations and the chain rule for relative entropy are needed to derive Sanov's theorem as well as various tensorization properties of concentration inequalities. We use the rich duality theory for convex risk measures along with a suitable substitute for the chain rule to derive a vast generalization of Sanov's theorem in which the entropic risk measure appearing in the Laplace principle is replaced by an arbitrary convex risk measure. Some of the many applications include non-exponential large deviations for i.i.d. samples, uniform large deviation principles, and asymptotics for constrained superhedging problems and variational problems involving optimal transport costs.

Kasper Larsen

"*Radner equilibrium in incomplete Levy models*"

ABSTRACT. We construct continuous-time equilibrium models based on a finite number of exponential utility investors. The investors income rates as well as the stocks dividend rate are governed by discontinuous Levy processes. Our main result provides the equilibrium (i.e., bond and stock price dynamics) in closed-form. As an application, we show that the equilibrium Sharpe ratio can be increased and the equilibrium interest rate can be decreased (simultaneously) when the investors income streams cannot be traded. (Joint work with Tanawit Sae-Sue)

Martin Larsson

"*Polynomial jump-diffusion models*"

ABSTRACT. A jump-diffusion process is called polynomial if its extended generator maps polynomials to polynomials of the same or lower degree. Many fundamental stochastic processes, for instance affine processes, are polynomial, and their tractable structure have made them useful in a wide range of areas such as interest rates, credit risk, variance swaps, stochastic portfolio theory, etc. While their general behavior is relatively well understood in the diffusion case, much less is known in the presence of jumps. I will illustrate the remarkably diverse behavior that arises beyond the diffusion case, including phenomena such as non-uniqueness of solutions to the martingale problem and jump intensities with countably many poles. Nonetheless, a number of interesting structural properties can be established, and useful parametric subclasses can be identified.

Johannes Muhle-Karbe

"*Equilibrium Models with Small Frictions*"

ABSTRACT. How does the introduction of a small trading friction such as a transaction tax affect financial markets? To answer questions of this kind, one needs to consider equilibrium models, where prices are determined endogenously. Indeed, taxes change agents' individual decision making, which in turn affects the market prices determined by their interactions. The new market environment then again alters the agents' behavior, leading to a notoriously intractable fixed point problem. In this talk we report on recent progress using asymptotic techniques for small trading frictions. In this practically relevant limiting regime, explicit solutions become available for the arising singular control problems, bringing analytical results for the equilibrium problem within reach. We also discuss how this allows to endogenize the trading friction, and study the arising link between liquidity and fundamental volatility. (Joint work with Martin Herdegen)

Marcel Nutz

"A Mean Field Game of Optimal Stopping"

ABSTRACT. We formulate a stochastic game of mean field type where the agents solve optimal stopping problems and interact through the proportion of players that have already stopped. Working with a continuum of agents, typical equilibria become functions of the common noise that all agents are exposed to, whereas idiosyncratic randomness can be eliminated by an Exact Law of Large Numbers. Under a structural monotonicity assumption, we can identify equilibria with solutions of a simple equation involving the distribution function of the idiosyncratic noise. Solvable examples allow us to gain insight into the uniqueness of equilibria and the dynamics in the population.

Jan Obłój

"Robust FTAP and superhedging in discrete time."

ABSTRACT. We study robust pricing and hedging in a general discrete time setup with dynamic trading in risky assets and static trading in finitely many options with given initial prices. We allow to express modelling beliefs by specifying a (universally measurable) subset of feasible paths. We establish a robust FTAP: absence of robust (model-independent) arbitrage is equivalent to existence of a martingale measure calibrated to the given option prices. The arbitrage here corresponds to the situation when all agents agree that there is an arbitrage albeit they might disagree as to how to realise it. The arbitrage strategy is measurable with respect to a larger filtration which aggregates these views but, at the same time, does not perturb the structure of martingale measures. Our proof is iterative and uses a robust pricing-hedging duality which we also establish. This work builds on earlier contributions by Burzoni, Frittelli and Maggis (2015) and is a joint work with Burzoni, Frittelli, Hou and Maggis.

Soumik Pal

"Exponentially concave functions and a new information geometry"

ABSTRACT. Exponentially concave functions are concave functions whose exponentials are also concave. The gradient map of such functions over the unit simplex appear as functionally generated portfolios introduced by Fernholz. These gradient maps are also solutions of a remarkable Monge-Kantorovich optimal transport problem. Suppose we are trading a functionally generated portfolio in discrete time, in the absence of transaction costs. What is the optimal frequency of trading? Contrary to popular beliefs, more frequent trading is not necessarily better. The answer lies in a new information geometry which is an exponential version of the celebrated classical information geometry of Bregman divergence. This new geometry is not flat, and yet, the geodesics satisfy a Pythagoras theorem. The answer to the optimal frequency of trading lies in studying this Pythagoras theorem. Based on joint work with Leonard Wong.

Dylan Possamai

"A tale of a Principal and many Agents"

ABSTRACT. In this talk, I will present ongoing work on a problem of moral hazard involving a Principal and a system of interacting Agents whose actions are summed up in a mean-field game. This is a new problem

in the literature, and our main current result is that the problem of the Principal can always be rewritten as stochastic control problem of a system of McKean-Vlasov SDEs, for which we can derive explicit solutions in specific instances. This a joint work with Romuald Elie and Thibaut Mastrolia.

Sergio Pulido

"*Stability and analytic expansions of local solutions of systems of quadratic BSDEs with applications to a price impact model*"

ABSTRACT. We obtain stability estimates and derive analytic expansions for local solutions of multi-dimensional quadratic BSDEs. We apply these results to a financial model where the prices of risky assets are quoted by a representative dealer in such a way that it is optimal to meet an exogenous demand. We show that the prices are stable under the demand process and derive their analytic expansions for small risk aversion coefficients of the dealer. This is joint work with Dmitry Kramkov.

Kavita Ramanan

"*Sensitivity analysis for reflected diffusions in convex polyhedral domains*"

ABSTRACT. Differentiability of flows and sensitivity analysis are classical topics in dynamical systems. However, the analysis of these properties for constrained processes, which arise in a variety of applications, is challenging due to the discontinuities in the dynamics at the boundary of the domain, and is further complicated when the boundary is non-smooth. We establish pathwise differentiability of a large class of obliquely reflected diffusions in convex polyhedron domains and characterize the derivative process as a (degenerate) Markov process that satisfies a stochastic differential equation with time-varying domain and directions of reflection. We also provide conditions under which this Markov process has a unique invariant distribution, and discuss its relevance for calculating sensitivities of performance measures in the so-called Atlas Model in mathematical finance. (This is based on joint work with David Lipshutz.)

Scott Robertson

"*Endogenous Mortgage Current Coupons*"

ABSTRACT. We consider the problem of identifying endogenous current coupons for To-Be-Announced (TBA) agency mortgage pass through securities. Current coupons play a crucial role in the mortgage industry for pricing and determining the relative value of mortgage backed securities. The current coupon is said to be endogenous if it gives rise to a fairly, or par valued, TBA. Since prepayments both affect the value of the mortgage and depend heavily upon the coupon, the identification of current coupons involves solving a highly non-trivial fixed point problem. In a doubly stochastic reduced form model which allows for prepayment intensities to depend upon both current and origination mortgage rates, as well as underlying investment factors, we identify the current coupon with solutions to a degenerate elliptic, non-linear fixed point problem. Using Schaefer's theorem we prove existence of current coupons. We also provide an explicit approximation to the fixed point, valid for compact perturbations off a baseline factor-based intensity model. Numerical examples are provided which show the approximation performs well in estimating the current coupon.

Mathieu Rosenbaum

"*Rough Volatility: From Microstructural Foundations to Smile*"

ABSTRACT. It has been recently shown that rough volatility models reproduce very well the statistical properties of low frequency financial data. In such models, the volatility process is driven by a fractional Brownian motion with Hurst parameter of order 0.1. The goal of this talk is first to explain how such fractional dynamics can be obtained from the behaviour of market participants at the microstructural scales. Using Hawkes processes, we show that a rough volatility naturally arises in the presence of high frequency trading combined with metaorders splitting. Then we will demonstrate that such result enables us to derive an efficient method to compute the smile in rough volatility models. This is joint work with Omar El Euch, Masaaki Fukasawa, Jim Gatheral and Thibault Jaisson.

Johannes Ruf

"*Some remarks on functionally generated portfolios*"

ABSTRACT. In the first part of the talk I will review Bob Fernholz' theory of functionally generated portfolios. In the second part I will discuss questions related to the existence of short-term arbitrage opportunities. This is joint work with Bob Fernholz and Ioannis Karatzas

Peter Tankov

"*Asymptotic optimal tracking: lower bounds and feedback strategies*"

ABSTRACT. We consider the problem of tracking a target whose dynamics is modeled by a continuous It semimartingale. The aim is to minimize both deviation from the target and tracking efforts. We establish the existence of asymptotic lower bounds for this problem, depending on the cost structure. These lower bounds can be related to the time-average control of Brownian motion, which is characterized as a deterministic linear programming problem. Furthermore, we provide a comprehensive list of examples for which the lower bound is sharp and is attained by an explicit feedback strategy. This talk is based on the following papers: * Jiatu Cai, Mathieu Rosenbaum and Peter Tankov. Asymptotic Lower Bounds for Optimal Tracking: a Linear Programming Approach, arxiv preprint arXiv:1510.04295 * Jiatu Cai, Mathieu Rosenbaum and Peter Tankov. Asymptotic Optimal Tracking: Feedback Strategies, arxiv preprint arXiv:1603.09472

Josef Teichman

"*Affine processes and non-linear (partial) differential equations*"

ABSTRACT. Affine processes have been used extensively to model financial phenomena since their marginal distributions are very tractable from an analytic point of view (up to the solution of a non-linear differential equation). It is well known by works of Dynkin-McKean-LeJan-Sznitman that one can turn this around and represent solutions of non-linear PDEs by affine processes. Recent advances in mathematical Finance in this direction have been contributed by Henry-Labordere and Touzi. We shall provide some general theory in this direction from the affine point of view. (Joint work with Georg Grafendorfer and Christa Cuchiero)

Nizar Touzi

"*Branching diffusion representation of semilinear PDEs*"

Hao Xing

"*Quadratic BSDE systems and applications*"

ABSTRACT. In this talk, we will establish existence and uniqueness for a wide class of Markovian systems of backward stochastic differential equations (BSDE) with quadratic nonlinearities. This class is characterized by an myabstract structural assumption on the generator, an a-priori local-boundedness property, and a locally-H older-continuous terminal condition. We present easily verifiable sufficient conditions for these assumptions and treat several applications, including stochastic equilibria in incomplete financial markets, stochastic differential games, and martingales on Riemannian manifolds. This is a joint work with Gordan Zitkovic.

Other invited participants: **Jinhyuk Choi, Jakša Cvitanić, Daniel Hernndez-Hernndez, Dmitry Kramkov, Martin Schweizer, Mihai Sîrbu** and **Thaleia Zariphopolou.**

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