REPORT ON THE WORKSHOP FOR WOMEN IN COMMUTATIVE ALGEBRA

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This is a report on the first-ever workshop for Women in Commutative Algebra. We organized and promoted the workshop around six working groups, each with two group leaders who are experts in very active research subareas. The group leaders and the topics of their groups were:

(1) Combinatorics and differential operators, leaders Christine Berkesch, University of Minnesota, and Laura Matusevich, Texas A&M Univesity.
(2) Methods in prime characteristic, leaders Karen Smith, University of Michigan, and Emily Witt, University of Kansas.
(3) Combinatorial commutative algebra, leaders Sara Faridi, Dalhousie University, and Susan Morey, Texas State University.
(4) Rees algebra, leaders Elisa Gorla, University of Neuchatel, Switzerland, and Claudia Polini, Notre Dame University.
(5) Finite resolutions and complexes, leaders Claudia Miller, Syracuse University, and Alexandra Seceleanu, University of Nebraska.
(6) Tropical commutative algebra, leaders Diane Maclagan, University of Warwick, United Kingdom, and Josephine Yu, Georgia Tech University.

We advertised the workshop on the AWM website and `commalg.org`. The application website asked for the year of Ph.D., current affiliation and position, ranked preferences for the groups, and any further relevant information. We had 98 candidates for the available 42 slots and we had to make some tough choices in selecting the participants. Depending on the preferences we divided the participants into the six groups so that each group would have about seven participants. We had a few cancellations, some of which we were able to replace, but due to late cancellations we were unable to replace two of the slots.

The main focus of the workshop was group work. On the morning of the first day the group leaders briefly presented the topics to all participants, and we met again in late afternoon to summarize the day's work. While the day's reports were interesting and informative, most groups felt that the afternoon meeting disturbed the work. Thus we did not have an all-participant meeting on Tuesday, but we did do brief summaries on Wednesday evening and longer summaries on Friday morning. On Thursday evening we had a group discussion/general panel long into the night in the social room on careers, professional climate, teaching advice, Ph.D. advising, and so on.

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We received a favorable report from the AWM ADVANCE evaluation team. Some excerpts from that report are:

Logistics: "Participants who responded to the survey were very happy with the workshop logistics. Most respondents (97%) agreed that the application process was convenient or agreed that accommodations were satisfactory (98%). Every participant (100%) thought the conference facilities were adequate,.."

Group Size & Effectiveness: "Most respondents (69%) thought their group size was just right, but a substantial minority (28%) thought their groups were too large and one person thought her group was too small. .... Most respondents (94% to 100%) expressed broad support for their workshop group, were favorably disposed toward their workshop project, and expected to continue collaborating with their group members..."

Conference Expectations and Future Intentions: "The majority of respondents (97%) would attend this conference again as a member and almost every participant (97%) would recommend this workshop to a friend. Three quarters (75%) of respondents indicated that they would attend this workshop again in the future as a workshop leader."

Productivity: "The women who attended the workshop and responded to the survey are very productive in terms of research, despite having so many junior scholars.... All participants (100%) had presented their research in the past two years and more than two-thirds (70%) had received external funding."

Collaborative Experience and Attitudes: "Most WICA participants (97%) have experience collaborating on a project for publication....As the research literature suggests, there are trade-offs to collaborating;....While most participants claimed that collaboration requires more communication (59%) and coordination (66%), no one (0%) thought that it slowed their career advancement."

Summary: "Based on this report, the WICA RCCW appears to have been very successful. As one participant wrote in her open-ended comments, 'It was so fantastic an environment. The people were so nice, so good to work with, so smart. It felt so different and so relaxing to work with and talk to women all week long.' "

A last comment regarding the AWM is that, through their ADVANCE grant NSF-HRD 1500481, they provided the WICA workshop with $4000 in travel funding for some of the participants. For those at US institutions, additional travel funding was obtained from the National Science Foundation, DMS-1934391. Organizer Spiroff was the PI on the grant, with organizers Smith, Swanson, and Witt providing support of the proposal as Senior Personnel.

All working groups reported substantial progress in research and concrete plans to continue working on the started projects. Below are the summaries of the mathematical content of the six groups.

1. Combinatorics and differential operators

Group leaders: Christine Berkesch and Laura Matusevich.

Group members: Jean Chan, Patricia Klein, Janet Page, Janet Vassilev.

Overview. The ring of $\mathbb{k}$-linear differential operators ($\mathbb{k}$ a field) over a commutative $\mathbb{k}$-algebra R is defined as $D(R) = \cup_{n=0}^{\infty} D_{\mathbb{k}}^n(R)$, where $D_{\mathbb{k}}^0(R) = R$ and $D_{\mathbb{k}}^n(R) = \{P \in \mathrm{End}_{\mathbb{k}}(R) \mid Pf - fP \in D_{\mathbb{k}}^{n-1}(R) \text{ for all } f \in R\}$. It is a fact that $D_{\mathbb{k}}^1(R) = R + \mathrm{Der}_{\mathbb{k}}(R)$, where $\mathrm{Der}_{\mathbb{k}}(R)$ is the space of $\mathbb{k}$-linear derivations on R, but we point out that in general, the derivations are not enough to generate $D(R)$ as a $\mathbb{k}$-algebra.

When R is the polynomial ring over a field of characteristic zero, $D(R)$ is the well-known *Weyl algebra*, the ring of linear differential operators with polynomial coefficients. When $\mathrm{char}(\mathbb{k}) > 0$, $D(R)$ is the *divided powers Weyl algebra*, which is also well understood.

Rings of differential operators were introduced by Grothendieck [18] and Sweedler [32]. It is known that $D(R)$ can shed light on $\mathrm{Spec}(R)$, and the study of differential operators is of wide interest in commutative algebra and algebraic geometry. For example, the theory of *Bernstein–Sato polynomials* [6, 29], which is of much relevance for studying singularities in algebraic geometry [37, 23, 22, 14, 11] relies on an explicit understanding of $D(R)$. Also, an explicit description of $D(R)$ in the case where R is a reduced monomial ring of characteristic $p > 0$, is used in [34, Corollary 5.5] to prove that tight closure commutes with localization in that setting.

While differential operators are both intrinsically interesting and useful once an explicit description is given, providing such characterizations for specific (classes of) varieties is not easy. Due to this inherent challenge, we aim to compute rings of differential operators in combinatorial settings, where the additional structure gives rise to specialized tools that are not available in general.

When R is a Stanley–Reisner ring, $D(R)$ has a combinatorial description [34] in terms of the associated simplicial complex. This description applies over any base field $\mathbb{k}$, regardless of the characteristic. Affine toric varieties and their coordinate rings (semigroup rings) form another fundamental class of examples. In this case, the ring of differential operators is known when the base field $\mathbb{k}$ is algebraically closed of characteristic zero [21, 27, 30, 31].

The Group Members. The members of this working group represent a broad cross-section of commutative algebra. In order to study rings of differential operators in combinatorial contexts, our participants bring expertise in combinatorial aspects of algebra and geometry, including toric geometry, convex geometry, Stanley–Reisner theory, D-modules, characteristic p methods, singularity theory, homological algebra, and affine semigroup rings.

Given our very different areas of specialization, this is a collaboration that could only have been developed during an event such as this. The group chemistry has been exceptional, and all participants are strongly committed to the success of this project. We are deeply grateful to the workshop organizers and BIRS, for making this possible for us.

Our Results and Future Directions. During the Workshop, we focused on computing rings of differential operators over quotients of semigroup rings by radical monomial ideals (the base field is assumed to be of characteristic zero). We have a working conjecture in this case, and are in the process of proving it. This result could be an article on its own, but we believe that our ideas can be applied in more general contexts. Future plans also involve considering the case when the base field has positive characteristic. In this case, even the differential operators over semigroup rings are not known.

2. Methods in prime characteristic

Group leaders: Karen Smith and Emily Witt.

Group members: Eloisa Grifo, Zhibek Kadyrsizova, Jenny Kenkel, Jyoti Singh, Adela Vraciu.

We began an investigation of the moduli space of Frobenius split (or globally F-regular) projective varieties.

For a ring of prime characteristic, F-purity and F-regularity are nice-ness assumptions defined using the p-th power (or Frobenius) map. Their study was pioneered by Hochster, in collaboration first with Roberts and later with Huneke. Important contributions were made in the early stages also by several Indian mathematicians, notably Mehta and Ramanathan (who used the term "Frobenius split" for what is usually known as "F-purity" among commutative algebraists).

The conditions of F-purity and F-regularity on R place strong algebraic conditions on R such as various local cohomology modules vanishing. They also place strong geometric restrictions on the projective variety whose homogeneous coordinate ring is R (called "log Calabi-Yau" and "log Fano", respectively) by a theorem of Schwede and Smith. These conditions are important in algebraic geometry. The study of the moduli spaces (parameter spaces) of such varieties is quite active.

We approached this from an algebraic point of view. Although moduli of smooth cubics has been studied in prime characteristic, and moduli of log Fanos have been studied in characteristic zero, no systematic classification, up to isomorphism, of the set of all finitely generated graded algebras (of fixed characteristic p) that are F-pure (or F-regular) has been completed.

At Banff, we focused on finding a moduli space for F-pure cubic surfaces of characteristic two. For various reasons, cubic surfaces of characteristic two are especially interesting. It is important to note that even for smooth cubics in this case, very little was understood until recently (for example, there is a 2018 preprint of Dolgachec and Duncan).

What we accomplished at WICA: Let f be a homogeneous polynomial of degree d in n variables. We assume f is non-degenerate in the sense that the projective scheme in defines in $\mathbb{P}^{n-1}$ is not contained in a hyperplane. This is what we accomplished:

(1) When $d = 2$ and n is arbitrary, we used the theory of quadratic forms to observe that there are only finitely many isomorphism types of quadric hypersurfaces in every charcteristic, though the story is a bit different in characteristic 2. Among these, we saw that all are F-pure, except the completely degenerate "double hyperplanes."
(2) When $d = 3$, we may have finished the classification of **non-F-pure** cubics in characteristic 2. It appears there are finitely many and we can enumerate representatives of each isomorphism class.

(3) When $d = 3$, we developed a program to prove that the **F-pure** cubics hypersurfaces in characteristic 2 are parameterized by the punctured four space $\mathbb{A}^4$ modulo a finite group action. Some details remain to be checked.

We intend to write the results of items (1) and (2) into a paper sometime soon and get together, hopefully this summer, to complete the work in (3).

Many open-ended areas of exploration remain and we hope to pursue them together in the future.

3. Combinatorial Commutative Algebra

Group leaders: Sara Faridi and Susan Morey.

Group members: Susan Cooper, Sabine El Khoury, Sarah Mayes-Tang, Liana Sega, and Sandra Spiroff.

None of the participants in this project had collaborated previously and the team involved participants at a variety of stages of their careers, resulting in beneficial new mentoring and research relationships.

The main question they worked on was: given an ideal I generated by monomials in a polynomial ring, if we know I has a free resolution supported on a simplicial complex Δ, can one find, from Δ, another simplicial complex supporting a free resolution of I^2? What about I^r for any r?

A *free resolution* of an ideal is a way to encode the relationships between the generators of the ideal into a sequence of free modules and maps between them. The smallest free resolution of I (in terms of the ranks of the modules involved) is unique up to isomorphism of complexes, and is called a *minimal free resolution* of I. If K is a field and I an ideal in the polynomial ring $S = K[x_1, \ldots, x_n]$, then the minimal free resolution is an exact sequence of free S-modules

$$0 \longrightarrow S^{\beta_p} \longrightarrow \ldots \longrightarrow S^{\beta_1} \longrightarrow S^{\beta_0} \longrightarrow I \longrightarrow 0,$$

where for each i, the non-negative integer β_i is called the *i-th betti number* of I. In the specific case of a monomial ideal, the betti numbers can be refined further into *multigraded betti numbers*, which are indexed by monomials in I.

The idea of supporting a resolution on a simplicial complex was initiated by Diane Taylor, who in her thesis [33] introduced a multigraded free resolution of a monomial ideal using the simplicial chain complex of a simplex whose vertices are labeled with the monomial generators of the ideal. Taylor's resolution always exists for any monomial ideal, but it is usually far from minimal. Taylor's work was extended in the decades to follow by discovering criteria for subcomplexes of the Taylor complex to support free resolutions of a monomial ideals [5, 4] and then further to cell complexes [5] and CW complexes [3, 20]. But it is shown by Velasco [36] that even CW complexes do no support *all* minimal resolutions.

The study of the behavior of powers of a fixed ideal is a classical problem. The powers of an ideal are used via the Rees algebra and related blowup algebras to understand resolutions of singularities, and are also of interest in their own right. For example, in [19], the depth function depth R/I^s is defined as a function of s. While this function is known to stabilize and the limiting behavior is known, finding the depths of the early powers of an ideal is an active area of research. Other invariants that are often studied for powers of ideals include regularity and associated primes. Having resolutions of the powers of an ideal that were close to minimal would provide a useful tool to advance all of these areas of study.

A second approach to studying powers of ideals is to understand their asymptotic behavior; often, there is uniformity in the limit that is not seen when studying individual members of the ideal. One direction of asymptotic relationships relates to the *graded betti numbers* of all powers of a monomial ideal, which are usually summarized in a two-dimensional *betti table* of the ideal. Recent research has demonstrated strong patterns in the sequence of betti tables of powers of a fixed ideal, both in cases where the tables eventually stabilize [25] and in cases where they continue to grow without bound [2].

The investigations of this group would tell us more about the graded betti numbers of powers of monomial rings, and could allow us to discern asymptotic patterns. It could also give new effective bounds for betti numbers of powers of monomial ideals.

For the purpose of a more concrete class of resolutions, the group began investigations at the recent Banff workshop by focusing on a special class of ideals whose resolutions are better-known, namely, those ideals that have resolution of length ("projective dimension") at most one [15], whose resolutions are supported on graphs. They were able to identify a structure that conjecturally supports a resolution of I^r for this class in general and prove that when the number of generators of the ideal is small, the conjecture holds for low powers, particularly $r = 2$. Along the way, they adapted an algorithm from [1] that prunes a Taylor resolution and produces a cellular one for a given monomial ideal to one that works in their setting.

The members of this research group came from varying areas of commutative algebra. Their differing mathematical backgrounds brought strength to the project, but the project also allowed each woman to expand her knowledge in the field. This area of research lies in the intersection of homological algebra and discrete topology. The field of combinatorial homotopy theory is rich with tools that, once found, any commutative algebraist could find indispensable (see [7] for a catalogue of some such tools). The expertise that the participants each brought (powers of ideals, combinatorial resolutions, homological methods, graph theory) allowed the group to produce a unique line of research and provides an opportunity of cross pollination by learning from the expertise of one another.

Since the conclusion of the Banff workshop, the group has agreed on how to advance the project given their disparate locations and the group's relatively large size. Their research is on-going. They have created a Dropbox for their project and have uploaded and shared various documents. Likewise, they have organized regular Skype meetings and intermittent visits of subgroups of the women and they have started applying for 1-2 week residential research opportunities, with an application to MSRI already submitted. In addition, face-to-face meetings are scheduled over the next several months at various locations.

4. Rees algebras

Group leaders: Elisa Gorla and Claudia Polini.

Group members: Ela Celikbas, Emilie Dufresne, Louiza Fouli, Kuei-Nuan Lin, Irena Swanson.

Let X be an $m \times n$ matrix whose entries are either zeros or distinct variables. A matrix X is *generic* if its entries are distinct variables, it is *sparse generic* if its entries are either zeros or distinct variables. Let $R = K[X]$ be the polynomial ring over a field K with variables the entries of X. Let I be the ideal of R generated by the maximal minors of X, I is called a *(sparse) determinantal* ideal. The study of rings and more generally of varieties that are defined by determinantal ideals of generic matrices has been a central topic of commutative algebra and algebraic geometry, see for example [9]. Sparse determinantal ideals were first studied by Giusti and Merle [17]. Recently, Boocher [8] determined their minimal free resolutions.

During the week at Banff, we studied the blowup algebras of sparse determinantal ideals, precisely their Rees algebras and special fiber rings. The Rees algebra of an ideal $J \subseteq R$ is the graded algebra $\mathcal{R}(J) = \oplus_{i=1}^{\infty} J^i t^i \subset R[t]$, where t is an indeterminate over R, and the *special fiber ring* $\mathcal{F}(J)$ is $\mathcal{R}(J)/\mathfrak{m}\mathcal{R}(J)$, where $\mathfrak{m}$ is the maximal (homogeneous) ideal of a local or graded ring R. The Rees algebra encodes many of the analytic properties of the variety defined by J and is the algebraic realization of the blowup of a variety along a subvariety. It plays a crucial role in the resolution of singularities of an algebraic variety. From the algebraic point of view the Rees algebra facilitates the study of the asymptotic behavior of the ideal and it is an essential tool for the computation of the integral closure of powers of ideals. Both the Rees algebra and the special fiber ring can be realized as quotients of polynomial rings. In particular, if J has n generators then $\mathcal{R}(J)$ is of the form $R[T_1, \ldots, T_n]/L$, where $L \subseteq R[T_1, \ldots, T_n]$ is the *defining ideal* of the algebra. The generators of L are the *defining equations* of $\mathcal{R}(J)$. Although blowing up is a fundamental operation, an explicit understanding of this process remains an open problem – for instance it is difficult in general to describe the generators of L. When J is generated by forms of the same degree, as in the case of sparse determinantal ideals, the problem of computing L explicitly is a classical problem in elimination theory. This question has been addressed in well over one hundred articles by commutative algebraists, algebraic geometers, and applied mathematicians. The problem is difficult and each class of ideals (or rational maps) seems to require different techniques. During the week at Banff, we studied this problem using techniques coming from SAGBI and Gröbner basis theory.

In the special case when I is the ideal generated by the maximal minors of a generic matrix, the Plücker relations among the minors are quadratic equations and they are the generators of the defining ideal of the special fiber ring. Together with the defining equations of the symmetric algebra, they are the defining equations of the Rees algebra.

We focused on the case when I is the ideal of maximal minors of a $2 \times n$ sparse matrix X. Our first goal was showing that $\text{in}(\mathcal{R}(I)) = \mathcal{R}(\text{in}(I))$, where $\text{in}(\mathcal{R}(I))$ denotes the initial algebra of the Rees algebra of I and $\mathcal{R}(\text{in}(I))$ denotes the Rees algebra of the initial ideal of I, with respect to suitable term orders. In situation when such a theorem holds, one can study the monomial algebra $\mathrm{in}(\mathcal{R}(I))$, then use standard deformation techniques to deduce properties for $\mathcal{R}(I)$ from those of $\mathrm{in}(\mathcal{R}(I))$.

By [12] the equality $\mathrm{in}(\mathcal{R}(I)) = \mathcal{R}(\mathrm{in}(I))$ would follow if we showed that $\mathrm{in}(I^k) = (\mathrm{in}(I))^k$ for all $k \leq \mathrm{rt}(\mathrm{in}(I))$, where $\mathrm{rt}(\mathrm{in}(I))$ denotes the relation type of $\mathrm{in}(I)$. By Conca, De Negri, and Gorla [**?**], the maximal minors of X are a universal Gröbner basis for I. By selecting a suitable term order and using results of Corso and Nagel [13] and Villarreal [36], we were able to determine the defining equations of $\mathcal{R}(\mathrm{in}(I))$, in particular we obtained $\mathrm{rt}(\mathrm{in}(I)) = 2$. By comparing Hilbert functions we were able to show that $\mathrm{in}(I^2) = (\mathrm{in}(I))^2$. Coupling this result with [12] we concluded that $\mathrm{in}(\mathcal{R}(I)) = \mathcal{R}(\mathrm{in}(I))$. Following the approach outlined, we were able to prove the following.

Theorem. (a) *$\mathcal{R}(I)$ is of fiber type and in particular, the defining ideal of $\mathcal{R}(I)$ is given by the relations of the symmetric algebra of I and by the Plücker relations on I.*
(b) *$\mathcal{R}(I)$ and $\mathcal{F}(I)$ have rational singularities (or they are F-rational in the positive characteristic case).*
(c) *$\mathcal{R}(I)$ and $\mathcal{F}(I)$ are Cohen-Macaulay normal domains.*
(d) *The Plücker relations of I form a Gröbner basis for the defining ideal of $\mathcal{F}(I)$.*
(e) *$\mathcal{R}(I)$ and $\mathcal{F}(I)$ are Koszul algebras.*

Next we plan to study the case of sparse generic matrices of arbitrary size. Furthermore, we would like to consider the case where the entries of the matrix are linear forms.

5. Canonical Resolutions using Koszul Algebras

Group leaders: Claudia Miller and Alexandra Seceleanu.

Group members: Faber Eleonore, Lindo Haydee, Martina Juhnke-Kubitzke, Rebecca R.G.

The project involves generalizing some canonical resolutions over polynomial rings to the setting of Koszul algebras. An algebra is Koszul if the residue field has a linear resolution, that is, has Castelnuovo-Mumford regularity equal to zero. A necessary condition is that its defining relations are quadratic, but not every quadratic algebra is Koszul.

Although the definition may seem specialized, in fact, Koszul algebras show up naturally in many places in algebra and topology. They were first introduced by the algebraic topologist Priddy in 1970 in [28] as algebras for which the bar resolution, which is far from minimal, can be reduced to a very small subcomplex. This explained ideas that had been floating around in the work of May on restricted Lie algebras and of Bousfield, Curtis, Kan, Quillen, Rector, and Schlesinger. Koszul algebras have since appeared naturally in many places, linked to fundamental concepts, and studied extensively in fields as diverse as topology, representation theory, commutative algebra, algebraic geometry, noncommutative geometry, and number theory.

Furthermore, the associated theory of Koszul duality is a generalization of the duality underlying the Bernstein-Gelfand-Gelfand correspondence describing coherent sheaves on projective space in terms of modules over the exterior algebra. This exemplifies the special relevance of the pair of Koszul dual algebras given by a polynomial ring and an exterior algebra, as well as the philosophy that facts relating these algebras often have Koszul duality counterparts.

Our project involves taking results that describe resolutions of various modules over a polynomial ring S constructed starting from the Koszul complex (exterior algebra) Λ and its generalizations and proving analogs of these results using any pair of Koszul dual algebras, A and $A^{\perp}$, instead of S and Λ.

The main first goal was to generalize the canonical resolutions over a polynomial ring for the powers of the homogeneous maximal ideal constructed by Buchsbaum and Eisenbud in [10] to obtain free resolutions for powers of the homogeneous maximal ideal of a Koszul algebra. As opposed to working over S, in general these would be infinite resolutions. We achieved this goal and wrote a complete proof over the first few days. We were aided in our endeavor by learning from another workshop participant (Şega) that these resolutions were linear. The next step will be to put a differential graded algebra structure on the resolutions we have constructed. Such a structure is enormously helpful in further homological explorations. For this, we have a clear method of building it based on a known technique if we can define an appropriate homotopy, for which we have some ideas.

Last, we wanted to extend the theory of Macaulay inverse systems from polynomial rings to Koszul algebras. For this extensive project, the first step would be to obtain some understanding of the injective hulls of the residue field over the Koszul algebra as well as its quotients. We made progress in understanding some of these, and realized that even an extension to one class of Koszul algebras would be a large step beyond the known theory over polynomial rings. For this we have a candidate that would give an important extension and have made progress in studying it.

If we successfully develop this theory, then we will be able to apply it to obtain a much large class of resolutions by expanding work of one of the co-leaders over polynomial rings [26] to this class of Koszul algebras.

The workshop was invaluable for us in forming research collaborations with women, when so many of us are in departments with almost all men. Indeed, most in our group had never worked with the others, and, even though we share common interests in homological commutative algebra, had never worked on the same topics before, and particularly not on Koszul algebras.

On the last half day of the workshop, we spent an hour of intensive discussions with another group (Berkesch and Matusevich) about a potential connection between our projects, more specifically about whether the same methods in our project could have some hope of providing an application to their topic of interest, and we hope to get in touch once we know better how our separate projects develop. This gives us even more connections for future collaborations with some of the best women in our field.

In the weeks since the workshop ended, our group has kept working regularly. We have started writing the result we obtained, and we have had some online meetings in subgroups to keep working and discuss issues issues arising from further explorations of the literature. We have a joint folder where we keep background literature and written summaries of the information from each meeting for those who could not make the meeting time.

6. Tropical commutative algebra

Group leaders: Diane Maclagan and Josephine Yu.

Group members: Francesca Gandini, Milena Hering, Fatemeh Mohammadi, Jenna Rajchgot, Ashley Wheeler.

One motivation of this group was to investigate whether the tropical Bertini theorem of [24] holds in characteristic p. This theorem states that when $X \subset (K^*)^n$ is an irreducible variety over an algebraically closed field K of characteristic zero, then the set of rational hyperplanes H for which $\mathrm{trop}(X) \cap H$ is the tropicalization of an irreducible variety is dense in $\mathbb{P}^n_{\mathbb{Q}}$. The characteristic assumption comes from the use in the proof of a toric Bertini theorem of Fuchs, Mantova, and Zannier [16], which has a characteristic zero requirement. While there are counterexamples in characteristic p to the method of proof of the toric Bertini theorem of [16], these do not immediately give a counterexample to the Bertini statement, and even such a counterexample would not necessarily give a counterexample to the tropical variant.

Some progress was made towards this goal, and the group has already begun follow-up meetings.

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