Volume Inequalities
Mar 28 – Apr 2, 2010

MEALS

*Breakfast (Buffet): 7:00–9:30 am, Sally Borden Building, Monday–Friday
*Lunch (Buffet): 11:30 am–1:30 pm, Sally Borden Building, Monday–Friday
*Dinner (Buffet): 5:30–7:30 pm, Sally Borden Building, Sunday–Thursday
Coffee Breaks: As per daily schedule, 2nd floor lounge, Corbett Hall
***Please remember to scan your meal card at the host/hostess station in the dining room for each meal.**

MEETING ROOMS

All lectures will be held in Max Bell 159 (Max Bell Building accessible by walkway on 2nd floor of Corbett Hall). LCD projector, overhead projectors and blackboards are available for presentations. Note that the meeting space designated for BIRS is the lower level of Max Bell, Rooms 155–159. Please respect that all other space has been contracted to other Banff Centre guests, including any Food and Beverage in those areas.

SCHEDULE

Sunday

16:00	Check-in begins (Front Desk - Professional Development Centre - open 24 hours)
	Lecture rooms available after 16:00
17:30–19:30	Buffet Dinner, Sally Borden Building
20:00	Informal gathering in 2nd floor lounge, Corbett Hall
	Beverages and a small assortment of snacks are available on a cash honor system.

Monday

7:00–8:45	Breakfast
8:45–9:00	Introduction and Welcome by BIRS Station Manager, Max Bell 159
9:00–9:30	John M. Sullivan, *"Ropelength and related packing problems"*
9:35–10:05	Emanuel Milman, *"Correlation Inequalities for non-Gaussian measures"*
10:05–10:45	Coffee Break, 2nd floor lounge, Corbett Hall
10:45–11:15	Mohammad Ghomi, *"Relative isoperimetric inequality outside convex bodies"*
11:20–11:50	Gary Lawlor, *"An elementary proof of the isoperimetric property of spheres in all dimensions"*
11:50–13:00	Lunch
13:00–14:00	Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall
14:15–14:45	Wlodzimierz Kuperberg, *"The set of packing and covering densities of convex disks"*
14:45–15:25	Coffee Break, 2nd floor lounge, Corbett Hall
15:25–15:55	Steven Taschuk, *"The Banach–Mazur distance to the cube in low dimensions"*
16:00–16:30	Balazs Csikos, *"On the volume of the intersection of geodesic balls"*
17:30–19:30	Dinner

Tuesday

7:00–9:00	Breakfast
9:00–9:30	Igors Gorbovickis, *"Kneser-Poulsen conjecture for low density configurations"*
9:35–10:05	Boaz Slomka, *"Characterizing order-isomorphisms in* $\mathbb{R}^n$*"*
10:05–10:45	Coffee Break, 2nd floor lounge, Corbett Hall
10:45–11:15	Vitali Milman, *"Polarity and Stability in Convex Geometry"*
11:20–11:50	Rolf Schneider, *"A Volume Inequality and Coverings of the Sphere"*
11:50–13:30	Lunch
13:45–14:15	Satyan L. Devadoss, *"Combinatorics and Topology of Particle Collisions"*
14:15–14:55	Coffee Break, 2nd floor lounge, Corbett Hall
14:55–15:25	Dmitry Slutsky, *"An infinitesimally nonrigid polyhedron with nonstationary volume in the Lobachevsky 3-space"*
15:30–16:00	Simon Cox, *"The minimal perimeter for N confined deformable bubbles of equal area"*
17:30–19:30	Dinner

Wednesday

7:00–9:00	Breakfast
9:00–9:30	Artem Zvavitch, *"The intersection body operator"*
9:35–10:05	Vlad Yaskin, *"On unique determination of convex polytopes"*
10:05–10:45	Coffee Break, 2nd floor lounge, Corbett Hall
10:45–11:15	Elisabeth Werner, *Relative entropy of cone measures and* L_p *centroid bodies (joint work with G. Paouris)*
11:20–11:50	Peter Pivovarov, *"On the volume of caps and bounding the mean-width of an isotropic convex body"*
11:50–13:30	Lunch
17:30–19:30	Dinner

Thursday

7:00–9:00	Breakfast
9:00–9:30	Oleg R. Musin, *"The Tammes problem for N=13"*
9:35–10:05	Long Yu, *"A characterization of Euclidean ball associated to the volume ratio of packing cone of a convex body."*
10:05–10:45	Coffee Break, 2nd floor lounge, Corbett Hall
10:45–11:15	Gabor Fejes Toth *"Partial covering of a convex domain with translates of a centrally symmetric convex disc"*
11:20–11:50	Max Engelstein, *"The Least-Perimeter Partition of the Sphere into Four Equal Areas"*
11:50–13:30	Lunch
13:45–14:15	Veit Elser, *"Some volume inequalities, at least two of which are silly"*
14:15–14:55	Coffee Break, 2nd floor lounge, Corbett Hall
14:55–15:25	Frank Morgan, *"The Isoperimetric Problem in Spaces with Density"*
15:30–16:00	Karoly Bezdek, *"Illuminating Ball-Polyhedra"*
17:30–19:30	Dinner

Friday

7:00–9:30	Breakfast
9:30–11:30	Informal Discussions
11:30–13:30	Lunch
Checkout by 12 noon.	

** 5-day workshops are welcome to use BIRS facilities (2nd Floor Lounge, Max Bell Meeting Rooms, Reading Room) until 3 pm on Friday, although participants are still required to checkout of the guest rooms by 12 noon. **

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ABSTRACTS
(in alphabetic order by speaker surname)

Speaker: **Karoly Bezdek** (University of Calgary)
Title: *Illuminating Ball-Polyhedra*
Abstract: Ball-polyhedra are intersections of finitely many congruent balls in Euclidean space. The ball-polyhedron is called a fat ball-polyhedron, if it contains the centers of its generating balls. Extending Schramm's theorem (1988) on illuminating convex bodies of constant width, I give an outline of a proof of the Boltyanski-Hadwiger conjecture (1960) for fat ball-polyhedra. The probabilistic method of the proof is centered around estimating the volume of convex bodies of constant width in spherical space.

Speaker: **Simon Cox** (Aberystwyth University)
Title: *The minimal perimeter for N confined deformable bubbles of equal area*
Abstract: Candidates to the least perimeter partition of various polygonal shapes into N planar connected equal-area regions are calculated for $N \leq 42$, compared to partitions of the disc, and discussed in the context of the energetic groundstate of a two-dimensional monodisperse foam. The total perimeter and the number of peripheral regions are presented, and the patterns classified according to the number and position of the topological defects, that is non-hexagonal regions (bubbles). The optimal partitions of an equilateral triangle are found to follow a pattern based on the position of no more than one defect pair, and this pattern is repeated for many of the candidate partitions of a hexagon. Partitions of a square and a pentagon show greater disorder.

Candidates to the least perimeter partition of the surface of the sphere into N connected equal-area regions are also calculated. For small N these can be related to simple polyhedra and for $N \geq 14$ they consist of 12 pentagons and $N - 12$ hexagons.

Speaker: **Balazs Csikos** (Eotvos University, Institute of Mathematics)
Title: *On the volume of the intersection of geodesic balls*
Abstract: We study complete Riemannian manifolds, having the property that the volume of the intersection of k geodesic balls depends only on the radii of the balls and the distances between their centers. For $k \geq 3$, this property implies that the space is a simply connected space of constant curvature. It is conjectured that the property for $k = 2$ is equivalent to the 2-point homogeneity of the space. We present some results supporting this conjecture.

Speaker: **Satyan L. Devadoss** (Williams College)
Title: *Combinatorics and Topology of Particle Collisions*
Abstract: My current interests are in the intersection of discrete and computational geometry with algebraic topology, especially in the world of convex polytopes. In particular, I am curious about the topology of particle collisions and their relationship to algebraic structures such as the associahedron polytope. These ideas can be examined from several viewpoints: from the geometry of moduli spaces, to blowups of hyperplane arrangements, to underlying operad structures, and to spaces of phylogenetic trees.

Speaker: **Veit Elser** (Cornell University)
Title: *Some volume inequalities, at least two of which are silly*
Abstract: I will discuss three topics: (1) Bounds on the maximum sublattice index, constrained by the property that translates of the lattice Voronoi cell by the sublattice vectors is a connected set. (2) Bounds on the maximum packing density of regular simplices in three and four dimensions. (3) A conjecture about

disk packings. At least two of my inequalities are so poor that it will take a real act of courage to present them.

Speaker: **Max Engelstein** (Yale University)
Title: *The Least-Perimeter Partition of the Sphere into Four Equal Areas*
Abstract: We prove that the least-perimeter partition of the sphere into four equal areas is the tetrahedral partition.

Speaker: **Gabor Fejes Tóth** (Alfréd Rényi Institute of Mathematics)
Title: *Partial covering of a convex domain with translates of a centrally symmetric convex disc*
Abstract: Let D be a convex domain in the plain and let $\mathcal{S}$ be a family of n translates of a centrally symmetric convex disc C. We give an upper bound for the area of the part of D covered by the discs of $\mathcal{S}$. The bound is best possible in the sense that it is asymptotically tight when n and the area of D approach infinity so that the density of the discs relative to D is fixed. The result generalizes some old theorems of L. Fejes Tóth and C.A. Rogers. The main tool to the proof is a generalization of an area formula of L. Fejes Tóth which he derived for coverings by circles.

Speaker: **Mohammad Ghomi** (Georgia Institute of Technology)
Title: *Relative isoperimetric inequality outside convex bodies (joint work with J. Choe and M. Ritore)*
Abstract: We prove that the area of a hypersurface which traps a given volume outside of a convex body in Euclidean n-space must be greater than or equal to the area of a hemisphere trapping the given volume on one side of a hyperplane. This result generalizes the classical isoperimteric inequality. The proof is based on a sharp estimate for total positive curvature of hyperurfaces whose boundary lies on a convex body and meets that body orthogonally from the outside. We will also discuss some possible generalizations.

Speaker: **Igors Gorbovickis** (Cornell University)
Title: *Kneser-Poulsen conjecture for low density configurations*
Abstract: The Kneser-Poulsen conjecture says that if a finite set of disks in E^d is rearranged so that the distance between each pair of centers does not decrease, then the volume of the union does not decrease, and the volume of the intersection does not increase. In full generality the conjecture is proved only on the plane. In this talk I will give an outline of the proof of the following statement: if before the rearrangement each disk intersects with no more than $d + 2$ other disks, then the conjecture for the volume of the union holds.

Speaker: **Wlodzimierz Kuperberg** (Auburn University)
Title: *The set of packing and covering densities of convex disks*
Abstract: For each convex disk K (i.e., a convex compact subset of the plane, with a non-void interior), its packing density $\delta(K)$ and covering density $\vartheta(K)$ form a pair of real numbers, i.e., a point on the coordinate plane. The set Ω, consisting of points assigned this way to all convex disks, is the subject of this talk. A few known inequalities on $\delta(K)$ and $\vartheta(K)$ jointly outline a relatively small convex polygon that contains Ω, but its exact shape remains a mystery. We present this polygonal region, and then we explicitly exhibit a certain convex region contained in Ω and occupying a good portion of it.

Speaker: **Gary Lawlor** (Brigham Young University)
Title: *An elementary proof of the isoperimetric property of spheres in all dimensions*
Abstract: The proof in this talk is distinguished by its elementary nature and its straightforward calculation. Unlike most isoperimetric proofs, we do not rely on the complicated theory of existence of a minimizer. Unlike the calibration-style proofs of E. Schmidt and H. Knothe/ M. Gromov, we do not require a form of Stokes @ Y theorem nor any multivariable calculus. In fact, the proof in all dimensions could be adapted into a culminating project for Arst year single-variable calculus, using induction, the intermediate value theorem, the Cauchy-Schwarz and triangle inequalities, the chain rule, integration by

parts, the fundamental theorem of calculus, minimization of a function of one variable, inequalities, the chain rule, integration by parts, the fundamental theorem of calculus, minimization of a function of one variable, and a surface area - slicing lemma.

Speaker: **Emanuel Milman** (University of Toronto)
Title: *Correlation Inequalities for non-Gaussian measures (joint work with Young-Heon Kim)*
Abstract: The Gaussian Correlation Conjecture asks whether $\gamma_n(A \cap B) \geq \gamma_n(A)\gamma_n(B)$ for any centrally-symmetric convex Borel sets A, B in $\mathbb{R}^n$, where γ_n denotes the standard Gaussian probability measure on $\mathbb{R}^n$. It was shown by Hargé that the conjecture is true provided either A or B are centered ellipsoids, and a different proof was given by Cordero–Erausquin by using Caffarelli's Contraction Theorem. The latter states that the optimal-transport Brenier map between γ_n and $\gamma_n \exp(-V)$, when V is convex, always contracts Euclidean distance.

We generalize Caffarelli's Theorem to handle more general non-Gaussian source measures, by using a *different* map, constructed as a flow along an advection field associated to an appropriate heat-diffusion process. The contraction property is then reduced to showing that log-concavity is preserved along the corresponding diffusion semi-group, under some symmetry assumptions. As an application, we obtain new correlation inequalities for these non-Gaussian measures, for convex sets with some additional symmetries.

Speaker: **Vitali Milman** (Tel Aviv University)
Title: *Polarity and Stability in Convex Geometry (joint work with S. Artstein-Avidan)*

Speaker: **Frank Morgan** (Williams College)
Title: *The Isoperimetric Problem in Spaces with Density*
Abstract: The solution to the isoperimetric problem in Euclidean space with Gaussian density e^{-r^2} is famously a half-space; with density e^{+r^2} a ball about the origin. It is conjectured to be such a ball whenever the log of the density is convex.

Speaker: **Oleg R. Musin** (Univ. of Texas at Brownsville)
Title: *The Tammes problem for N=13 (joint work with Alexey Tarasov)*
Abstract: The thirteen spheres problem is asking if 13 equal size nonoverlapping spheres in three dimensions can touch another sphere of the same size. This problem was the subject of the famous discussion between Isaac Newton and David Gregory in 1694. The problem was solved by Schütte and van der Waerden only in 1953. A natural extension of this problem is the strong thirteen spheres problem (or the Tammes problem for 13 points) which asks to find an arrangement and the maximum radius of 13 equal size nonoverlapping spheres touching the unit sphere. In the paper we give a solution of this long-standing open problem in geometry. Our computer-assisted proof is based on a enumeration of the so-called irreducible graphs.

Speaker: **Peter Pivovarov** (University of Alberta)
Title: *On the volume of caps and bounding the mean-width of an isotropic convex body*
Abstract: Let K be a convex body in $\mathbb{R}^n$ with volume $\mathrm{vol}(K) = 1$. Let θ be a unit vector and consider the linear functional $\langle x, \theta \rangle := x_1\theta_1 + \ldots + x_n\theta_n$, $(x \in K)$. The search for sub-Gaussian upper bounds for the volume of caps of the form

$$C(\theta, t) := \{x \in K : \langle x, \theta \rangle > t\}, \quad (t > 0)$$

is connected to various open problems. I will discuss the reverse *super-Gaussian* lower bounds for $\mathrm{vol}(C(\theta, t))$; in particular, when K is assumed to be (i) isotropic and (ii) symmetric with respect to each of the coordinate hyperplanes. I will also discuss similar cap estimates and how they relate to the problem of bounding the mean-width of an arbitrary isotropic convex body.

Speaker: **Rolf Schneider** (University of Freiburg)
Title: *A Volume Inequality and Coverings of the Sphere (joint work with K. Bezdek)*

Abstract: We use a volume inequality in the spherical space $\mathbb{S}^n$ to prove a Tarski type theorem for coverings of the sphere. If $\mathbb{S}^n$ is covered by the spherically convex bodies $K_1, \ldots, K_m$, then the inradii $r(K_i)$ of these bodies satisfy $\sum_{i=1}^m r(K_i) \geq \pi$. Similarly, if a hemisphere of $\mathbb{S}^n$ is covered by $K_1, \ldots, K_m$, then $\sum_{i=1}^m r(K_i) \geq \pi/2$. Both inequalities are sharp. They follow from an inequality between spherical volume σ and inradius r of spherically convex bodies, namely $\sigma(K)/r(K) \leq \sigma(\mathbb{S}^n)/\pi$. Here equality holds if and only if K is a lune. The proof is achieved after dualizing the assertion and transferring it into Euclidean space.

Speaker: **Boaz Slomka** (Tel Aviv University)
Title: *Characterizing order-isomorphisms in $\mathbb{R}^n$ (joint work with S. Artstein-Avidan)*
Abstract: We characterize order-isomorphisms in $\mathbb{R}^n$, for vector orderings induced by non-degenerate cones.

Speaker: **Dmitry Slutsky** (Russian Academy of Sciences)
Title: *An infinitesimally nonrigid polyhedron with nonstationary volume in the Lobachevsky 3-space*
Abstract: We give an example of an infinitesimally nonrigid polyhedron in the Lobachevsky 3-space and construct an infinitesimal flex of that polyhedron such that the volume of the polyhedron isn't stationary under the flex.

Speaker: **John M. Sullivan** (TU Berlin)
Title: *Ropelength and related packing problems*
Abstract: The thickness of a space curve is the diameter of the largest embedded normal tube around it. The ropelength problem considers curves of thickness at least one, and asks to minimize the tube volume (or equivalently length) within a given knot type. Related problems include finding the maximum thickness for a knot fitting into a given box (or into a compact manifold like the three-sphere).

Speaker: **Steven Taschuk** (University of Alberta)
Title: *The Banach–Mazur distance to the cube in low dimensions*
Abstract: The Banach–Mazur distance from any centrally symmetric convex body in $\mathbb{R}^n$ to the n-dimensional cube is at most

$$\sqrt{n^2 - 2n + 2 + \frac{2}{\sqrt{n+2} - 1}}\ ,$$

which improves previously known estimates for "small" $n \geq 3$. (For large n, asymptotically better bounds are known; in the asymmetric case, exact bounds are known.) The proof of this estimate combines an idea of Lassak with a new lemma asserting that every decomposition of the identity as in John's theorem contains two contact points which are nearly orthogonal, in the sense that their absolute inner product is at most $1/\sqrt{n+2}$.

This lemma on contact points is closely related to an open problem on equiangular systems of lines. A result of Gerzon asserts that such a system contains at most $\frac{1}{2}n(n+1)$ lines, and gives necessary conditions on n for this bound to be achieved. Systems achieving this bound are known to exist only for the first four values of n satisfying Gerzon's conditions. John's decompositions for which our lemma is sharp are exactly the spanning vectors for equiangular systems of lines achieving Gerzon's bound.

Speaker: **Elisabeth Werner** (Case Western Reserve University)
Title: *Relative entropy of cone measures and L_p centroid bodies (joint work with G. Paouris)*
Abstract: We introduce a new affine invariant, which we call Ω_K, that can be found in three different ways: as a limit of normalized L_p-affine surface areas, as the relative entropy of the cone measure of K and the cone measure of K°, as the limit of the volume difference of K and L_p-centroid bodies. We investigate properties of Ω_K and of related new invariant quantities. In particular, we show new affine isoperimetric inequalities and we show a "information inequality" for convex bodies.

Speaker: **Vlad Yaskin** (University of Alberta)
Title: *On unique determination of convex polytopes*

Abstract: Abstract. We will discuss the following two open problems. 1) A question of Barker and Larman asks whether convex bodies that contain a sphere of radius t in their interiors are uniquely determined by the volumes of sections by hyperplanes tangent to the sphere. 2) In his book "Geometric Tomography" Gardner asks whether origin-symmetric convex bodies in $\mathbb{R}^3$ are uniquely determined by the perimeters of sections through the origin. One can also formulate an n-dimensional version of the problem.

We solve these problems in the class of convex polytopes.

Speaker: **Long Yu** (University of Alberta)
Title: *A characterization of Euclidean ball associated to the volume ratio of packing cone of a convex body*
Abstract: Given a convex body $K \subset \mathbb{R}^n$ and $\mathbf{u} \in S^{n-1}$, we introduce a new volume ratio $r(K, \mathbf{u})$ of the packing cone associated to K. We prove that if K is an **o**-symmetric convex body in $\mathbb{R}^n$ and $r(K, \mathbf{u})$ is a constant function of $\mathbf{u}$, then K is a Euclidean ball.

Speaker: **Artem Zvavitch** (Kent State University)
Title: *The intersection body operator (joint work with A. Fish, F. Nazarov and D. Ryabogin)*
Abstract: The notion turned out to be quite interesting and useful in Convex Geometry and Geometric tomography. It is easy to see that the intersection body of a ball is again a ball. E. Lutwak asked if there is any other star-shaped body that satisfy this property. We will present a solution to a local version of this problem: if a convex body K is closed to a unit ball and intersection body of K is equal to K, then K is a unit ball. We will also discuss a harmonic analysis version of this question which studies the Radon transform of powers of a given function.