Banff International Research Station
for Mathematical Innovation and Discovery

BIRS Workshop
Evolutionary Games
Sunday, June 13, 2010 to Friday, June 18, 2010

MEALS

*Breakfast (Buffet): 7:00–9:30 am, Sally Borden Building, Monday–Friday
*Lunch (Buffet): 11:30 am–1:30 pm, Sally Borden Building, Monday–Friday
*Dinner (Buffet): 5:30–7:30 pm, Sally Borden Building, Sunday–Thursday
Coffee Breaks: As per daily schedule, 2nd floor lounge, Corbett Hall
***Please remember to scan your meal card at the host/hostess station in the dining room for each meal.**

MEETING ROOMS

All lectures will be held in Max Bell 159 (Max Bell Building accessible by walkway on 2nd floor of Corbett Hall). LCD projector, overhead projectors and blackboards are available for presentations. Note that the meeting space designated for BIRS is the lower level of Max Bell, Rooms 155–159. Please respect that all other space has been contracted to other Banff Centre guests, including any Food and Beverage in those areas.

SCHEDULE

<table>
<tr><th></th><th>Monday</th><th>Tuesday</th><th>Wednesday</th><th>Thursday</th><th>Friday</th></tr>
<tr><td>7:00-9:30</td><td colspan="5">Buffet Breakfast, Sally Borden Building</td></tr>
<tr><td>8:45-10:15</td><td>IF[1]</td><td>IF</td><td>IF</td><td>IF</td><td>IF</td></tr>
<tr><td>10:15-10:45</td><td colspan="5">Coffee Break, 2nd floor lounge, Corbett Hall</td></tr>
<tr><td>10:30-11:00</td><td></td><td></td><td>Taylor</td><td></td><td></td></tr>
<tr><td>11:00-11:30</td><td>Sigmund[2]</td><td>Lessard</td><td>Apaloo</td><td>Krivan</td><td rowspan="2">Discussion</td></tr>
<tr><td>11:30-12:00</td><td>Antal</td><td>Ohtsuki</td><td>Ochea</td><td>Hauert</td></tr>
<tr><td>12:00-13:30</td><td colspan="5">Buffet Lunch, Sally Borden Building</td></tr>
<tr><td>13:30-14:30</td><td>Guided Tour[3]</td><td>Fletcher</td><td rowspan="5">free afternoon for excursions</td><td>Berger</td><td rowspan="6">12PM checkout</td></tr>
<tr><td>14:30-15:00</td><td>Garay[4]</td><td>Doebeli</td><td>Sandholm</td></tr>
<tr><td>15:00-15:30</td><td colspan="2">Coffee Break</td><td>Coffee</td></tr>
<tr><td>15:30-16:00</td><td>Traulsen</td><td>Fu</td><td>Miękisz</td></tr>
<tr><td>16:00-16:30</td><td>Szabo</td><td>Cressman</td><td>Li</td></tr>
<tr><td>17:30-19:30</td><td colspan="4">Buffet Dinner, Sally Borden Building</td></tr>
</table>

[0]Check-in begins on Sunday June 13 at 16:00(Front Desk - Professional Development Centre - open 24 hours). Buffet Dinner (17:30-19:30), Sally Borden Building. Lecture rooms available after 16:00. Informal gathering in 2nd floor lounge after 20:00, Corbett Hall, where beverages and a small assortment of snacks are available on a cash honor system.

[1]*IF*: Inclusive fitness in evolutionary modeling workshop presentations in Max Bell 159, subsequent discussions in Max Bell 156. Max Bell 156 is available to EG workshop participants between 9AM and 11AM.

[2]On Monday June 14th at 10:45, the BIRS Station Manager will make introduction and welcome remarks in Max Bell 159.

[3]A free guided tour of The Banff Centre is offered to all participants and their guests on Monday starting at 1:00 pm. The tour takes approximately 1 hour.

[4]A group photo will be taken on Monday at 2:20 pm, just before the first lecture of the afternoon. Please meet on the front steps of Corbett Hall.

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ABSTRACTS
(in alphabetic order by speaker surname)

Speaker: **Tibor Antal** (Program for Evolutionary Dynamics, Harvard University, 1 Brattle Square, Suite 6, Cambridge, MA 02138, USA. E-mail: antal@fas.harvard.edu)
Title: *Who laughs last? Perturbation theory for games*
Abstract: Evolutionary game theory studies frequency dependent selection. The fitness of a strategy is not constant, but depends on the relative abundances (=frequencies) of strategies in the population. The main question is which strategy is the most abundant in the long run. When the effect of selection is sufficiently weak, we can answer this question by developing a perturbation theory. The method is quite general, and easily applicable to a large class of models. First we consider the simplest possible example of a well mixed population, then we turn to more general cases, including games with multiple strategies and games in structured populations.

Speaker: **Joe Apaloo** (Department of Mathematics, Statistics and Computer Science, St. Francis Xavier University, Antigonish, Nova Scotia, B2G 2W5, Canada. E-mail: japaloo@stfx.ca)
Title: *Multi-dimensional evolutionary game theory: a current state*
Abstract: The mathematical theory for describing the eventual outcomes of evolutionary games involving single species and one-dimensional strategies drawn from a continuous strategy set is well established. One aspect of this theory is the connection between static evolutionary stability criteria and the outcome of dynamic evolutionary models. There is limited work in the literature on extending the connections between static stability criteria and the outcomes of dynamic evolutionary models to the multi-dimensional case. We review the current state of these connections and generate discussion on possible ways of extending research work in this area of multi-dimensional evolutionary models.

Speaker: **Ulrich Berger** (Department of Economics, Vienna University of Economics Business Administration, Augasse 2-6, 1090 Wien, Austria. E-mail: ulrich.berger@wu.ac.at)
Title: *Learning to cooperate via indirect reciprocity*
Abstract: Cooperating in the Prisoners Dilemma is irrational and some supporting mechanism is needed to stabilize cooperation. Indirect reciprocity based on reputation is one such mechanism. Assessing an individuals reputation requires first-order information, i.e. knowledge about its previous behavior, as it is utilized under image scoring. But there seems to be an agreement that in order to successfully stabilize cooperation higher-order information is necessary, i.e. knowledge of others previous reputations. We show here that such a conclusion might have been premature. "Tolerant scoring", a first-order assessment rule with built-in tolerance against single defections, can lead a society to stable cooperation.

Speaker: **Ross Cressman** (Department of Mathematics, Bricker Academic Building, BA534 Wilfrid Laurier University, 75 University Ave. W. Waterloo, ON, N2L 3C5, Canada. E-mail: rcressman@wlu.ca)
Title: *Game experiments on cooperation through punishment and/or reward*

Abstract: We report results from two different experiments that test the effects of punishment and/or reward schemes on the cooperative behavior of players in repeated Prisoner*s Dilemma (PD) and Public Goods (PGG) games. In the two-player PD experiment, each player has the third option of punishing his opponent at a cost to himself (i.e. the player chooses between cooperation (C), defection (D) and costly punishment (P)) in each round based on payoffs observed and strategies used in previous rounds. In the four-player PGG experiment, an outside agency rewards, punishes, or rewards and punishes players between rounds based on their contributions to the public good. Subjects for both game experiments were university students in Beijing.

For our PD experiment, costly punishment does not increase the average level of cooperation compared to the control experiment where this option is not available. This result contrasts with several similar experiments conducted in western societies. On the other hand, in our PGG control experiment (i.e. the standard repeated PGG without reward or punishment), the average contribution levels to the public good match closely those found in the same control conducted in Boston. Our PGG experiment shows that combined reward and punishment schemes are most effective in increasing contributions, followed by punishment on its own and that reward on its own has no significant effect on contributions. These results differ from those in Boston that exhibited little difference in contribution levels when players reward, punish, or reward and punish each other between rounds. The experiments are discussed in relation to cultural differences in attitudes to a player*s reputation and to institutional incentive schemes to increase the cooperative behavior of its members.

Speaker: **Michael Doebeli** (Department of Zoology and Department of Mathematics, 6270 University Boulevard, Vancouver BC V6T 1Z4, Canada. E-mail: doebeli@zoology.ubc.ca)
Title: *Diversification in high-dimensional phenotype spaces*
Abstract: The mechanisms for the origin and maintenance of biological diversity are not fully understood. It is known that frequency-dependent selection, generating advantages for rare types, can maintain genetic variation and lead to speciation, but in models with simple phenotypes (i.e., low dimensional phenotype spaces), frequency dependence needs to be strong to generate diversity. In this talk I show that if the ecological properties of an organism are determined by multiple traits with complex interactions, the conditions for frequency dependent selection to generate diversity are relaxed to the point where they are easily satisfied in high-dimensional phenotype spaces. Mathematically, this phenomenon is reflected in properties of eigenvalues of quadratic forms. Since all living organisms have many phenotypes, this casts the potential importance of frequency dependence for the origin and maintenance of diversity in a new ligh

Speaker: **Jeff Fletcher** (Systems Science and University Studies, Portland State University, Portland, OR 97215-0751, USA . E-mail: jeff@pdx.edu)
Title: *Theories and Models of the Evolution of Altruism: Unification vs. Unique Explanations*
Abstract: Recently the debate over how altruistic traits evolve has intensified. Part of this debate centers on whether alternative theories (e.g. inclusive fitness, multilevel selection, and reciprocal altruism) represent different perspectives on the same underlying phenomena, and can therefore be unified, or whether each different theory represents a unique empirical mechanism. Towards unification, I present a simple and general framework that highlights an underlying necessary condition for the evolution of altruism: assortment of carriers of the altruistic trait with the helping behaviors of others. I also critically examine the claim that inclusive fitness theory uniquely explains the evolution of altruism. While many see the controversy around this claim as merely semanticarising out of different definitions for common terms (e.g. altruism and cooperation)I argue that it also represents different views about the proper hierarchical level at which to assign causation.

Speaker: **Feng Fu** (Program for Evolutionary Dynamics, Harvard University, 1 Brattle Square, Suite 6, Cambridge, MA 02138, USA. E-mail: fufeng@gmail.com)
Title: *Coevolution of cooperation and favoritism*
Abstract: Green beard, or similarity-based cooperation, has been suggested as a simple yet effective

cooperation-promoter in recent years. Most previous studies are based on the predisposition of individual cooperation with like-minded ones and defection otherwise. This sort of contingent cooperation can be termed as in-group favoritism if group memberships depend on an individuals beard color or phenotype in general. It is of significant interest to study how in-group favoritism arises in the first place.

In this talk, I will first present a minimal model of in-group favoritism in homogeneous populations. The population is divided in groups according to each individuals randomly assigned tag. Each individual has the same level of in/out group helping tendency. Individuals tags and behavioral strategies both are heritable traits that are subject to mutation and selection. Using coalescent theory, I will derive an analytical condition for cooperation to evolve under weak selection. I will show that the critical benefit-to-cost ratio reaches minimum when individuals only help in-group members and restrain from helping out-group members. Further, I will present the case where the tendency of in/out group severing behaviors coevolves with cooperation dynamics. In other words, there is no priori heuristic clue of cooperating with similar ones. I will show that under natural selection pressure, the whole population evolves towards the biased world, where each individual, if being a cooperator, has much high tendency to help an in-group member, and yet has very low declination to help out-group members.

Speaker: **Christoph Hauert** (Department of Mathematics, The University of British Columbia, 1984 Mathematics Road, Vancouver, B.C., V6T 1Z2, Canada. E-mail: christoph.hauert@math.ubc.ca)
Title: *Reward & reputation in public goods games*
Abstract: Public goods games have become the mathematical metaphor for game theoretical investigations of cooperative behavior in groups of interacting individuals. Cooperation is a conundrum because cooperators make a sacrifice to benefit others at some cost to themselves. Exploiters or defectors reap the benefits and forgo costs. Despite the fact that groups of cooperators outperform groups of defectors, Darwinian selection or utilitarian principles based on rational choice should favor defectors. This social dilemma can be overcome by introducing incentives to cooperate. Negative incentives based on the punishment of shirkers are efficient in stabilizing cooperation once established but fail to initiate cooperation. Here we focus on the complementary case of positive incentives created by rewarding those that did contribute to the public good. Indeed, handing out rewards is capable of stimulating cooperation in interaction groups of arbitrary size but, in contrast to punishment, fails to stabilize it. In fact, the dynamics of reward is complex and dominated by unpredictable oscillations.

Speaker: **József Garay** (Research Group of Theoretical Biology and Ecology, Hungarian Academy of Science; Department of Plant Taxonomy and Ecology, L. Eötvös University, Pázmány Péter sétány1/c, H-1117 Budapest, Hungary. E-mail: garayj@ludens.elte.hu)
Title: *Envy, charity and selfishness*
Abstract: We build up a family of classical evolutionary games in which we consider envy (reducing the fitness of more successful individuals at ones own cost) and charity (increasing the fitness of less successful individuals at ones own cost). If the damage is additive and its cost is low envy defeats selfish strategy (which neither decreases nor increases others' fitness). Moreover, envy and selfish strategist can replace the spiteful one (which unconditionally decreases others' fitness at ones own cost) if the cost of damage is relatively high. However, envy can be selected out in a mixed population of envious, selfish and spiteful. If damage is multiplicative, then coexistence of selfish and envious strategists is possible. Envy is a conditional spiteful strategy, so in envious groups there is less damage than in spiteful groups, so envy decreases the total cost of the spiteful competition.

Moreover, the envious-donator strategy (who gives to the poorest and damages the richer individuals) can spread in the envious group. Charity is a "buy-off" behaviour in the sense that donator can decrease the damage of envy, and charity further decreases the total cost of the spiteful competition. In the light of our results, egalitarianism motives (envy and charity) can be considered as a consequence of Darwinian competition, i.e. individuals aim at increasing their own relative proportion rather than their absolute contribution to the next generation and other-regarding-preference is an "instrument" for this aim.

Finally, in a simple kin-selection scenario the envious-spiteful strategists (envious within its kin and spiteful outside its kin) outperform selfish and spiteful ones as well.

Speaker: **Vlastimil Krivan** (Department of Theoretical Ecology, Biological Research Center, Academy of Sciences of the Czech Republic, and Department of Mathematics and Biomathematics, Faculty of Science, Ceske Budejovice, Czech Republic. E-mail: vlastimil.krivan@gmail.com)
Title: *On evolutionary stability of optimal foraging games*
Abstract: In my talk I will re-analyze models of the optimal foraging theory with respect to evolutionarily stability of optimal strategies. I will focus on the prey and patch models that were introduced by E. Charnov. In their original setting these fundamental models assume frequency independent fitness functions that are proportional to energy intake rate by a single consumer. Such a fitness function does not allow us to measure fitness of a mutant in a resident population in the case where two or more strategies have the same payoffs. In fact, this often happens when population dynamics are considered. In my talk I derive frequency dependent fitness functions and corresponding evolutionarily stable strategies. I will show how in this dynamic setting partial preferences for food types arise.

Speaker: **Sabin Lessard** (Department of Mathematics and Statistics, Montréal University, C.P. 6128 Succursale Centre-ville, Montréal, Québec, H3C 3J7, CANADA. E-mail: lessards@dms.umontreal.ca)
Title: *Kin selection in evolutionary game theory*
Abstract: IInteractions in finite group-structured populations will be considered in order to study the effects of relatedness and population subdivision on long-term evolution.

Speaker: **Cong Li** (Key Lab of Animal Ecology and Conservation Biology, Centre for Computational Biology and Evolution, Institute of Zoology, Chinese Academy of Science, Beijing, China. E-mail: Neilli@163.com)
Title: *Evolution of cooperation and network structure*
Abstract: A simple rule for the evolution of cooperation on graphs and social networks developed by it Ohtsuki et al. (2006, Nature, 44, 502-505) shows that for all graphs, natural selection favors cooperation if the altruistic act, b, divided by the cost, c , exceeds the average number of neighbors, k, which means $b/c > k$. On the other hand, *Santos et al.* (2008, Nature, 454, 213-216) investigated the emergence of cooperation in public goods games. They introduce the concept of social diversity and show that cooperation is promoted by the diversity. When social ties follow a scale-free distribution, cooperation is enhanced whenever all individuals are expected to contribute a fixed amount. In this research, we focused our attention on how the evolution of cooperation is affected by the network structure. Our main goal is to show whether the evolution of cooperation sensitively depends on the network structure. Many biological, technological and social networks lie somewhere between two extremes which are regular and random networks, respectively. *Watts and Strogatz* (1998, Nature, 393, 440-442) found these systems can be highly clustered, like regular lattices, yet have small characteristic path lengths, like random graphs, and they called these networks the small-world networks. *Barabasi and Albert* (1999, Science, 286, 509-512) also noticed that a common property of many large networks is that the vertex connectivities follow a scale-free power distribution, i.e., $P(k) \sim k^{-\gamma}$, where $P(k)$ is the probability that a vertex in the network interacts with other vertices. Here, we first consider whether the small-world networks can promote more effectively cooperation than the regular and random networks (since we notice that the effects of regular and random networks on the evolution of cooperation seems to be the same). Secondly, the effect of the scale-free network is also considered, where we focus our attention on the relationship between the parameter γ and the evolution of cooperation.

Speaker: **Jacek Miękisz** (Institute of Appplied Mathematics and Mechanics, Faculty of Mathematics, Informatics, and Mechanics, Warsaw University, Banacha 2, 02-097 Warsaw, Poland. E-mail: miekisz@mimuw.edu.pl)
Title: *Equilibrium transitions in stochastic models of finite populations*
Abstract: We will address the question how a method of matching players affects the level of cooperation in stochastic evolutionary games.

For the stag hunt game, the long run behavior of populations depends on the relation between its size and the noise level. We observe multiple transitions between the effective strategy phase and the risk-dominant one as the size of the population increases (for the review see Evolutionary Game Theory and Population Dynamics, Lecture Notes in Mathematics 1940: 269-316, 2008, http://www.mimuw.edu.pl/ miekisz/cime.pdf).

In the random matching model, players are randomly matched with a finite number of opponents - equal to the number of neighbors in the corresponding spatial model. We will show for the snow drift game that the spatial structure promotes cooperation much better than the random matching of players.

Finally, we will discuss the Prisoner's Dilemma game on the Barabasi-Albert scale-free network with costs of maintaining links. It will be shown that a population of players undergoes a sharp transition from the cooperation phase to a mixed cooperation/defection phase as the cost passes through its critical value.

Speaker: **Marius Ochea** (Faculty of Economics and Business Administration, Department of Economics, Tilburg University, Room K 424, PO Box 90153, 5000 LE Tilburg, Netherlands. E-mail: M.I.Ochea@uvt.nl)
Title: *Evolution in iterated Prisoner's Dilemma games under logit dynamics*
Abstract: In an evolutionary set-up, we append an ecology of iterated Prisoner's Dilemma (IPD) game strategies, consisting of unconditional cooperators (AllC), unconditional defectors (AllD) and reactive players (TFT) with two repeated strategies that have received less attention in the evolutionary IPD game literature: the error-proof, "generous" tit-for-tat (GTFT) which, with a certain probability, re-establishes cooperation after a (possibly by mistake) defection of the opponent and the penitent, "stimulus-response" (WSLS) strategy that resets cooperation after the opponent punished for defection. An abundance of Rock-Paper-Scissors(RPS) like patterns is discovered in the 3x3 ecologies comprising Pavlovian and "generous" (GTFT) players whereas in one 4x4 ecology the co-existence of such RPS cycle along with an chaotic attractor is observed. Interestingly, the evolutionary success of Pavlov seems to depend on the absence of unconditional cooperators (AllC) in the ecologies investigated.

Speaker: **Hisashi Ohtsuki** (Department of Value and Decision Science, Graduate School of Decision Science and Technology, Tokyo Institute of Technology, 2-12-1 Ookayama Meguro-ku, Tokyo, 152-8552 JAPAN. E-mail: ohtsuki.h.aa@m.titech.ac.jp)
Title: *Evolutionary games in island model*
Abstract: Recently there has been considerable interest in studying evolutionary game dynamics in structured populations. Here I investigate the evolutionary dynamics of the game played in a subdivided population which follows the Wright's island model. Limited dispersal produces positive association among neighbors' strategies, hence coefficients of relatedness appear in our main equation. We interpret our results in terms of inclusive fitness. We demonstrate how several previous results follow from our result. Relationship with group selection is also discussed.

Speaker: **Bill Sandholm** (Department of Economics, University of Wisconsin, 1180 Observatory Drive, Madison, WI 53706, USA. E-mail: whs@ssc.wisc.edu)
Title: *Sampling best response dynamics and deterministic equilibrium selection*
Abstract: We consider a model of evolution in games in which revising agents observe the strategies of k randomly sampled opponents and then choose a best response to the distribution of strategies in the sample. We prove that under the resulting deterministic evolutionary dynamics, which we call k-sampling best response dynamics, any iterated $(1/k)$- dominant equilibrium is almost globally asymptotically stable. We show as well that this sufficient condition for stability is also necessary in supermodular games. Since the selection occurs by way of a deterministic dynamic, the selected equilibrium is reached quickly; in particular, the long waiting times associated with equilibrium selection in stochastic stability models are absent.

Speaker: **Karl Sigmund** (Faculty for Mathematics, University of Vienna, Nordbergstrasse 15, A-1090 Vienna, Austria. E-mail: Karl.Sigmund@univie.ac.at)

Title: *Peer versus pool punishment: the emergence of rudimentary sanctioning institutions*
Abstract: Sanctions promote collaboration in public goods games, but since they are themselves a public good, second-order free riders, who do not punish defectors, can exploit this and subvert cooperation. The punishment of second-order defectors is called second-order punishment. Most experiments use peer punishment, which is ill suited for second-order punishment. But another form of punishment, called pool punishment, is better suited to the task. In an open competition of peer with pool punishment, the latter prevails if and only if second order punishment is included. Pool punishment trades efficiency for stability. It is an implementation of the self-financed contract enforcement mechanisms which are frequently found in real-life institutions implementing the 'governance of the commons'.

Speaker: **György Szabó** (Research Institute for Technical Physics and Materials Science, H-1525 Budapest, POB. 49, Hungary. E-mail: szabo@mfa.kfki.hu)
Title: *Social dilemmas in spatial systems with collective strategy updates*
Abstract: Social dilemmas are studied with players following unconditional cooperative or defective strategies. The players are located on a square lattice and each player's income is collected from 2×2 games (including Prisoner's Dilemma, Stag Hunt and Hawk-Dove games) with the four nearest neighbors. The evolution of strategy distribution is governed by random sequential strategy updates. During an elementary process several players choose new strategies at random in a way favoring the income increase of a group they belong to. The strategy update is stochastic and the magnitude of noise is characterized by a "temperature" parameter using the Fermi-Dirac function that provides smooth transition from 0 to 1 in the strategy adoption probability. Systematic investigations are performed to determine the average frequency of cooperators in the stationary state when varying the payoffs, the size of group and also the number of players who can modify their strategy simultaneously for a fixed noise level. The present dynamical rules support the maintenance of cooperation in two ways. On the one hand, cooperators and defectors can form chessboard like structure providing optimum income for the whole society if the sum of sucker's payoff and temptation to choose defection is sufficiently high. On the other hand, the enforcement of group interest supports cooperation within the region of Prisoner's Dilemma even if only one or two players can choose new strategy within an elementary step.

Speaker: **Christine Taylor** (Program for Evolutionary Dynamics, Harvard University, 1 Brattle Square, Suite 6, Cambridge, MA 02138, USA. E-mail: taylor4@fas.harvard.edu)
Title: *How often should one cooperate?*
Abstract: It is often difficult to determine the exact payoffs in a game, furthermore, payoff values are frequently variable rather than costant. In finite random games, where there are n players, each having finitely many strategies with payoffs that are independent and identical continuous distribtuions, cooperation is is preferable when a Nash equilibrium is not Pareto-optimal. I will give a brief overview on Pareto-inefficiency of pure Nash equilibrium solutions in finite random games, and then explore the relationship between cooperation and self-interest in some two-player multi-strategy games and multi-player two-strategy symmetric games.

Speaker: **Arne Traulsen** (Emmy-Noether-Group, Evolutionary Dynamics, Max-Planck-Institute for Evolutionary Biology, August-Thienemann-Str. 2, 24306 Plön, Germany. E-mail: traulsen@evolbio.mpg.de)
Title: *Some general results for multiplayer games*
Abstract: Evolutionary game dynamics of two players with two strategies has been studied in great detail. These games have been used to model many biologically relevant scenarios, ranging from social dilemmas in mammals to microbial diversity. Some of these games may, in fact, take place between a number of individuals and not just between two. Here we address one-shot games with multiple players. As long as we have only two strategies, we can calculate fixation probabilities and compare them two each other or to neutral selection in the spirit of the 1/3-rule. For games with multiple players and more than two strategies, some statements derived for pairwise interactions no longer hold. For example, in two player games with any number of strategies there can be at most one isolated internal equilibrium. For d players

d with n strategies, there can be at most $(d-1)^{n-1}$ isolated internal equilibria. Multiplayer games show a great dynamical complexity that cannot be captured based on pairwise interactions. The results hold for any game and can easily be applied to specific cases, such as public goods games or multiplayer stag hunts.