

Arithmetic of K3 Surfaces

November 30 – December 5

MEALS

Breakfast* (Buffet): 7:00–9:30 am, Sally Borden Building, Monday–Friday

Lunch* (Buffet): 11:30 am–1:30 pm, Sally Borden Building, Monday–Friday

Dinner* (Buffet): 5:30–7:30 pm, Sally Borden Building, Sunday–Thursday

Coffee Breaks: As per daily schedule, 2nd floor lounge, Corbett Hall

***Please scan your meal card at the host/hostess station in the dining room for each meal.**

MEETING ROOMS

All lectures will be held in Max Bell 159 (Max Bell Building accessible by walkway on 2nd floor of Corbett Hall). LCD projector, overhead projectors and blackboards are available for presentations. Please note that the meeting space designated for BIRS is the lower level of Max Bell, Rooms 155–159. Please respect that all other space has been contracted to other Banff Centre guests, including any Food and Beverage in those areas.

CHECKOUT

Participants are required to checkout of the guest rooms by **12 noon on Friday**. The BIRS facilities (2nd Floor Lounge, Max Bell Meeting Rooms, Reading Room) are available until 3 pm on Friday.

SCHEDULE

Sunday

- 16:00** Check-in begins (Front Desk - Professional Development Centre - open 24 hours)
17:30–19:30 Dinner
20:00 Informal gathering in 2nd floor lounge, Corbett Hall
Beverages and small assortment of snacks available on a cash honour-system.

Monday

- 7:00–8:45** Breakfast
8:45–9:00 Introduction and welcome to BIRS by BIRS Station Manager, Max Bell 159
9:00–10:00 **Olivier Wittenberg**, Brauer groups and rational points of K3 surfaces
10:00–10:20 Coffee Break
10:20–11:05 **Alessandra Sarti**, Automorphism groups of K3 surfaces
11:15–12:00 **Thomas Dedieu**,
Self-rational maps of K3 surfaces, action on rational points and on nodal curves
12:00–13:30 Lunch
13:30–15:45 **Informal discussion**, with:
13:30–14:30 Guided tour of the Banff Centre; meet in the 2nd floor lounge, Corbett Hall
15:00–15:30 Coffee
15:45–16:45 **Matthias Schuett**, K3 surfaces and modular forms
17:00–18:00 **Open problem session**
18:00–19:30 Dinner

Tuesday

- 7:00–9:00** Breakfast
9:00–10:00 **Arnaud Beauville**, Algebraic cycles on K3 and derived equivalences
10:00–10:20 Coffee Break
10:20–11:20 **Tetsuji Shioda**, Mordell-Weil Lattices of Certain Elliptic K3 Surfaces
11:30–12:15 **Martin Bright**,
Computing Brauer-Manin obstructions on diagonal quartic surfaces
12:15–13:30 Lunch
13:30–15:45 **Informal discussion**, with:
15:00–15:30 Coffee
15:45–16:45 **Chad Schoen**, Calabi-Yau threefolds with vanishing third Betti number
17:00–18:00 **Ronald van Luijk**, The analogue of the Batyrev-Manin conjecture for K3 surfaces
18:00–19:30 Dinner

Wednesday

- 7:00–9:00** Breakfast
9:00–10:00 **Arthur Baragar**, Finding automorphisms on K3 surfaces with small Picard number
10:00–10:20 Coffee Break
10:20–11:20 **Joe Silverman**,
Dynamics and canonical heights on K3 surfaces with noncommuting involutions
11:30–13:30 Lunch
Afternoon Free
17:30–19:30 Dinner

Thursday

- 7:00–9:00** Breakfast
9:00–10:00 **Yuri Zarhin**,
Variants of the Tate conjecture with finite coefficients and their applications
10:00–10:20 Coffee Break
10:20–11:20 **Serge Cantat**, Dynamics of automorphisms
11:30–12:15 **Evis Ieronymou**, Transcendental Brauer group and counterexamples to weak approximation for diagonal quartic surfaces
12:15–12:20 **GROUP PHOTO** on the steps in front of Corbett Hall
12:20–13:30 Lunch
13:30–15:30 **Informal discussion**, with:
15:00–15:30 Coffee
15:45–16:45 **Ekaterina Amerik**,
Potential density of rational points on the variety of lines of a cubic fourfold
17:00–18:00 **Noriko Yui**,
On the modularity of certain K3 surfaces with non-symplectic group actions
18:00–19:30 Dinner

Friday

- 7:00–9:00** Breakfast
9:00–10:00 **Michael Stoll**, Searching for rational points on genus 2 Jacobians
10:00–10:20 Coffee Break
10:20–11:20 **Jaap Top**, Arithmetic of a family of K3's with Picard number 19
11:30–13:30 Lunch

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ABSTRACTS

Speaker: **Ekaterina Amerik** (Université de Paris-Sud)

Title: *Potential density of rational points on the variety of lines of a cubic fourfold*

Abstract: This is a joint work with Claire Voisin. Conjecturally, rational points must be potentially dense on any smooth projective variety with trivial canonical class, defined over a number field. This is well-known for abelian varieties, but the simply-connected case is much less understood. For instance, though potential density has been proved by Bogomolov and Tschinkel for certain families of K3 surfaces with special properties, it is unknown whether a “generic” K3 surface is potentially dense over a number field; in particular, there is no example of a potentially dense K3 surface with Picard number 1. Our purpose is to provide such an example in dimension 4. More precisely, we exhibit a family of irreducible holomorphic symplectic fourfolds, such that its “sufficiently general” member defined over a number field has cyclic Picard group and is potentially dense. To find explicit equations for such a variety is still an open problem.

Speaker: **Arthur Baragar** (University of Nevada Las Vegas)

Title: *Finding automorphisms on K3 surfaces with small Picard number.*

Abstract: In this talk we will investigate, via an example, some techniques for finding the group of automorphisms for a K3 surface. Some of the techniques will be general, some will be specific to the example, and some will depend on the small Picard number.

Speaker: **Arnaud Beauville** (Université de Nice)

Title: *Algebraic cycles on K3 and derived equivalences.*

Abstract: The Chow ring of a complex K3 surface is very large, but contains a finite-dimensional subring where most of the action takes place. I will describe this subring, and the recent result of Huybrechts proving that it is stable under derived equivalences.

Speaker: **Martin Bright** (University of Bristol)

Title: *Computing Brauer-Manin obstructions on diagonal quartic surfaces.*

Abstract: I will describe the theoretical process of calculating the algebraic Brauer-Manin obstruction on a surface, and show practically how to do it in the case of a diagonal quartic surface.

Speaker: **Serge Cantat** (Université de Rennes)

Title: *Dynamics of Automorphisms.*

Abstract: Let f be an automorphism of a (projective) K3 surface. I will describe the distribution of periodic points of f and list a few open questions regarding the closure of the set of periodic points.

Speaker: **Thomas Dedieu** (Universität Bayreuth)

Title: *Self-rational maps of K3 surfaces. Action on rational points and on nodal curves.*

Abstract: I will explain how self-rational maps can be used to show that certain special algebraic K3 surfaces have potential density. It is however expected that generic algebraic K3 surfaces do not carry any self-rational map with degree > 1 . By looking at the action of such maps on nodal curves, I will relate this conjecture to another conjecture about the irreducibility of the universal Severi varieties for K3 surfaces.

Speaker: **Evis Ieronymou** (Ecole Polytechnique Fédérale de Lausanne)

Title:

Transcendental Brauer group and counterexamples to weak approximation for diagonal quartic surfaces.

Abstract: Diagonal quartic surfaces are a special subclass of K3 surfaces. Following work of Swinnerton-Dyer and Martin Bright we have some control over the algebraic Brauer group of such surfaces. The role played by elements of the Brauer group which are not algebraic is quite mysterious. In this talk we discuss the construction of such elements, and applications to weak approximation.

Speaker: **Alessandra Sarti** (Université de Poitiers)

Title: *Automorphism Groups of K3 Surfaces.*

Abstract: I will present recent progress in the study of prime order automorphisms of K3 surfaces. An automorphism is called (non-)symplectic if the induced operation on the global nowhere vanishing holomorphic two form is (non-)trivial. After a short survey on the problem, I will describe the topological structure of the fixed locus, the geometry of these K3 surfaces and their moduli spaces.

Speaker: **Chad Schoen** (Duke University)

Title: *Calabi-Yau threefolds with vanishing third Betti number.*

Abstract: Algebraic varieties of the type described in the title do not exist in characteristic 0. In the past 10 years a number of examples have been constructed in positive characteristic. We will describe some of these constructions, several of which involve K3 surfaces and their moduli. We will also briefly touch on some natural questions about these varieties which generalize classical questions about K3 surfaces. This talk, by a non-expert in geometry in positive characteristic, is aimed at non-experts.

Speaker: **Matthias Schuett** (Københavns Universitet)

Title: *K3 surfaces and modular forms.*

Abstract: A classical construction of Shimura associates every Hecke eigenform of weight 2 with rational coefficients to an elliptic curve over \mathbb{Q} . The converse statement that every elliptic curve over \mathbb{Q} is modular, is the Taniyama-Shimura-Weil conjecture, proven by Wiles et al. For higher weight, however, the opposite situation applies: Nowadays we know the modularity for wide classes of varieties, but it is an open problem whether all newforms of fixed weight with rational coefficients can be realised in a single class of varieties. I will present joint work with N. Elkies that provides the first solution to the realisation problem in higher weight: We show that every newform of weight 3 with rational coefficients is associated to a singular K3 surface over \mathbb{Q} .

Speaker: **Tetsuji Shioda** (Rikkyo University)

Title: *Mordell-Weil Lattices of Certain Elliptic K3 Surfaces.*

Abstract: We discuss the rank formula and lattice structure of the Mordell-Weil lattice of certain elliptic surfaces, especially, the elliptic K3 surfaces of Inose-Kuwata, defined by an explicit Weierstrass equation.

Speaker: **Joe Silverman** (Brown University)

Title: *Dynamics and canonical heights on K3 surfaces with noncommuting involutions.*

Abstract: K3 surfaces embedded in $\mathbb{P}^2 \times \mathbb{P}^2$ admit a pair of noncommuting involutions. In this talk I will discuss the orbits of (rational) points under the action of these involutions and construct a pair of canonical height functions that are useful for studying the arithmetic properties of orbits. As time permits, I will describe K3 dynamical analogues of various classical conjectures and theorems, and will discuss analogous results on K3 surfaces embedded in $\mathbb{P}^1 \times \mathbb{P}^1 \times \mathbb{P}^1$ that admit three noncommuting involutions.

Speaker: **Michael Stoll** (Universität Bayreuth)

Title: *Searching for rational points on genus 2 Jacobians.*

Abstract: Let C be a curve of genus 2 over \mathbb{Q} , with Jacobian J . In many situations, we need to know generators of the Mordell-Weil group $J(\mathbb{Q})$ (or at least of a finite index subgroup). As is the case with elliptic curves, these generators can be exponentially large in terms of the coefficients in the curve equation. I will describe a method that allows us to find comparatively large points in $J(\mathbb{Q})$. This method amounts to searching for points on 2-covering spaces X of J , but in practice, we search for points on a certain quotient Y of X that lift to X . This quotient is a Kummer Surface. We can also use a variant that works with 2-covering spaces X of the principal homogeneous space Pic_C^1 of J ; in this case, we replace the quotient Y by a \mathbb{P}^1 -bundle over Y that has a nice explicit description. This amounts to a partial 4-descent on J and has allowed me to find some points of logarithmic height close to 100.

Speaker: **Jaap Top** (Universiteit Groningen)

Title: *Arithmetic of a family of K3's with Picard number 19.*

Abstract: This will be an expository talk. The aim is to present some of the techniques (Nikulin involutions, Shioda-Inose structures, recognizing a genus 2 curve from the Kummer surface of its jacobian, correspondences) which Bert van Geemen and I used in a joint paper (2006), to establish a relation between two families of K3 surfaces.

Speaker: **Ronald van Luijk** (Universiteit Leiden)

Title: *The analogue of the Batyrev-Manin conjecture for K3 surfaces.*

Abstract: The Batyrev-Manin conjecture predicts for any Fano variety the asymptotic growth of the number of rational points of bounded height on the variety as a function of the height bound. This growth is strongly related to the geometry of the variety, in particular to its Picard number. We will see some heuristics and experiments that suggest an analogue of the conjecture for K3 surfaces.

Speaker: **Olivier Wittenberg** (CNRS - École Normale Supérieure)

Title: *Brauer groups and rational points of K3 surfaces.*

Abstract: This will be an expository talk. Rational points of K3 surfaces over number fields do not always satisfy weak approximation or even the Hasse principle. Understanding which K3 surfaces enjoy these properties and which do not is one of the basic problems in the study of the arithmetic of K3 surfaces. I will discuss examples and known results in this direction, most of which involve Brauer groups, and mention a few open questions as well.

Speaker: **Noriko Yui** (Queen's University)

Title: *On the modularity of certain K3 surfaces with non-symplectic group actions.*

Abstract: This is a joint work with Ron Livné (Jerusalem) and Matthias Schütt (Copenhagen). We consider complex K3 surfaces with a non-symplectic group acting trivially on the algebraic cycles. Vorontsov and Kondo classified those K3 surfaces with transcendental lattice of minimal rank. The purpose of this talk is to study the Galois representations associated to these K3 surfaces. The rank of transcendental lattices is even and varies from 2 to 20, excluding 8 and 14. We show that these K3 surfaces are dominated by Fermat surfaces and hence they are all of CM type. We establish the modularity of the Galois representations associated to the transcendental parts of these K3 surfaces. Time permitting, we briefly discuss mirror symmetry for these K3 surfaces.

Speaker: **Yuri Zarhin** (Penn State University)

Title: *Variants of the Tate conjecture with finite coefficients and their applications.*

Abstract: We'll discuss interactions between various conjectures of Tate that deal with the algebraicity of Galois-invariant cohomology classes and homomorphisms of abelian varieties and their Tate modules over finitely generated fields. We also consider an analogue of Tate's conjecture on homomorphisms that deals with points of sufficiently large order (instead of Tate modules). Applications to finiteness results for Brauer groups will be given (joint work with Alexei Skorobogatov).