Banff International Research Station

for Mathematical Innovation and Discovery

Recent Developments in Numerical Methods for Nonlinear Hyperbolic Partial Differential Equations and their Applications
9/1/2008–9/5/2008

MEALS

*Breakfast (Buffet): 7:00–9:30 am, Sally Borden Building, Monday–Friday
*Lunch (Buffet): 11:30 am–1:30 pm, Sally Borden Building, Monday–Friday
*Dinner (Buffet): 5:30–7:30 pm, Sally Borden Building, Sunday–Thursday
Coffee Breaks: As per daily schedule, 2nd floor lounge, Corbett Hall
***Please remember to scan your meal card at the host/hostess station in the dining room for each meal.**

MEETING ROOMS

All lectures will be held in Max Bell 159 (Max Bell Building accessible by walkway on 2nd floor of Corbett Hall). LCD projector, overhead projectors and blackboards are available for presentations. *Please note that the meeting space designated for BIRS is the lower level of Max Bell, Rooms 155–159. Please respect that all other space has been contracted to other Banff Centre guests, including any Food and Beverage in those areas.*

SCHEDULE

Sunday	
16:00	Check-in begins (Front Desk - Professional Development Centre - open 24 hours)
	Lecture rooms available after 16:00
17:30–19:30	Buffet Dinner, Sally Borden Building
20:00	Informal gathering in 2nd floor lounge, Corbett Hall
	Beverages and small assortment of snacks available on a cash honour-system.
Monday	
7:00–8:45	Breakfast
8:45–9:00	Introduction and Welcome to BIRS by BIRS Station Manager, Max Bell 159
9:00–9:45	Randy LeVeque
9:45–10:00	Break
10:00–10:45	Remi Abgrall
10:45–11:15	Coffee Break, 2nd floor lounge, Corbett Hall
11:15–12:00	Chi-Wang Shu
12:00–14:30	Lunch
14:30–14:45	Group Photo; meet on the front steps of Corbett Hall
14:45–15:30	Jian-Guo Liu
15:30–16:00	Coffee Break, 2nd floor lounge, Corbett Hall.
16:00–16:45	Chiu-Yen Kao
16:45–17:00	Break
17:00–17:30	Informal discussion
17:30–19:30	Dinner

Tuesday

7:00–8:45	Breakfast
8:45–9:30	Ian Mitchell
9:30–9:45	Break
9:45–10:30	Hongkai Zhao
10:30–11:00	Coffee Break, 2nd floor lounge, Corbett Hall.
11:00–11:45	Alexander Kurganov
11:45–13:45	Lunch
13:00–13:40	Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall
13:45–14:30	Yongtao Zhang
14:30–14:45	Break
14:45–15:30	Alexander Vladimirsky
15:30–16:00	Coffee Break, 2nd floor lounge, Corbett Hall.
16:00–16:45	Tao Tang
16:45–17:00	Break
17:00–17:30	Informal discussion
17:30–19:30	Dinner

Wednesday

7:00–8:45	Breakfast
8:45–9:30	Shi Jin
9:30–9:45	Break
9:45–10:30	Jianliang Qian
10:30–11:00	Coffee Break, 2nd floor lounge, Corbett Hall.
11:00–11:45	Pino Martin
11:45–13:45	Lunch
Afternoon	Excursion

Thursday

7:00–8:45	Breakfast
8:45–9:30	Pierre Degond
9:30–9:45	Break
9:45–10:30	Roberto Ferretti
10:30–11:00	Coffee Break, 2nd floor lounge, Corbett Hall.
11:00–11:45	Yingjie Liu
11:45–13:45	Lunch
13:45–14:30	Irene Gamba
14:30–14:45	Break
14:45–15:30	Marshall Slemrod
15:30–16:00	Coffee Break, 2nd floor lounge, Corbett Hall.
16:00–16:45	Susana Serna
16:45–17:00	Break
17:00–17:30	Informal discussion
17:30–19:30	Dinner

Friday

7:00–9:00	Breakfast
9:00	Informal discussion
11:30–13:30	Lunch

Checkout by 12 noon.

** 5-day workshops are welcome to use the BIRS facilities (2nd Floor Lounge, Max Bell Meeting Rooms, Reading Room) until 3 pm on Friday, although participants are still required to checkout of the guest rooms by 12 noon. **

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Recent Developments in Numerical Methods for Nonlinear Hyperbolic Partial Differential Equations and their Applications 9/1/2008–9/5/2008

ABSTRACTS
(in alphabetic order by speaker surname)

Speaker: **Remi Abgrall** (Bordeaux)
Title: *Construction and Evaluation of High Order Residual Distribution Schemes for Compressible Flows*
Abstract: We are interested in the numerical approximation of steady hyperbolic problems

$$\text{div } F(u) = 0, \qquad u = g \text{ on the inflow boundary} \tag{1}$$

which are defined on an open set $\Omega \subset R^d$, $d = 2, 3$ with weak Dirichlet boundary conditions defined on the inflow boundary[1]. In (1), the vector of unknown u belongs to R^p, and the flux F is $F = (f_1, \dots, f_d)$. We present a systematic construction which is genuinely high order and has a very compact stencil. Applications in compressible flows will be given.
Author : R. Abgrall, A Larat, M Ricchiuto

Speaker: **Pierre Degond** (Universit Paul Sabatier)
Title: *Fluid models for swarming problems*
Abstract: We will discuss two models of individual and collective behaviour among animal populations. The first one is a continuum model derived from a widely used discrete particle model due to T. Vicsek et al (Phys. Rev. Lett. 1995). The second one is a new model proposed on the basis of biological experiments on fish behaviour: the persistent turning walker.

Roberto Ferretti Stability of high-order large time-step schemes and relationship between Semi-Lagrangian and Lagrange-Galerkin techniques Abstract: Two major implementation, Semi-Lagrangian and Lagrange-Galerkin, of large time-step schemes are compared. The first has a cleaner and more general stability analysis, but cannot in practice be implemented in its exact version. The second allows for a simpler implementation, but its theoretical analysis is still incomplete. We prove that SL schemes can be regarded as LG schemes, provided a suitable Galerkin basis is defined. This also allows to show stability of SL schemes in a wider range of situations.

Speaker: **Irene Gamba** (University of Texas, Austin)
Title: *Spectral-Lagrangian solvers for non-linear Boltzmann type equations* Abstract: We present a deterministic spectral solver for the non-linear Boltzmann Transport Equation (energy conservative and non-conservative) for rather general collision kernels. The computation of the non-linear Boltzmann Collision integral and the lack of appropriate conservation properties due to spectral methods has been taken care by framing the conservation properties in the form of a constrained minimization problem which is solved easily using a Lagrange multiplier method. We benchmark our code with several examples of models for Maxwell type of interactions, (elastic or inelastic) for which explicit solution formulas are known. The numerical moments are compared with exact moments formulas and the numerical non-equilibrium

[1] $\vec{n}$ is the inward normal.

probability distributions functions are compared to the general asymptotic results. In the case of space inhomogeneous boundary value problems, the numerical method captures the discoutinuous behahior of the probability distribution functions solution of the BTE with diffusive boundary conditions and sudden changes in boundary temperature, as predicted by Y. Sone for solutions of the BTE and computed by Aoki, et al '91, using alternative models. This work is in collaboration with my ICES student Harsha Tharkabhushanam.

Speaker: **Shi Jin** (University of Wisconsin, Madison)
Title: *Eulerian Gaussian beam methods for the Schrodinger equation in the semiclassical regime*

Speaker: **Chiu-Yen Kao** (Ohio State University)
Title: *Central Schemes for a new class of entropy solutions of the Buckley-Leverett equation*
Abstract: Buckley-Leverett (BL) equation arises in two-phase flow problem in porous media. It models the oil recovery by water-drive in one-dimension. Here we propose a central scheme for an extension of the BL equation which includes the dynamic effects in the pressure difference between the two phases and results in a third order mixed derivatives term in the modified BL equation. The numerical scheme is able to capture the admissible shocks which is the so-called nonclassical shock due to the violation of the Oleinik entropy condition.

Speaker: **Alexander Kurganov** (Tulane University)
Title: *Interface Tracking Method: Applications to Compressible Multifluids and to Flows in Domains with Moving Boundaries*
Abstract: I will first present an interface tracking method for compressible multifluids, which are assumed to be immiscible. The major problem in designing a robust numerical problem for such systems is appearance of "mixed" cells, in which the fluids are artificially mixed due to the numerical averaging of the conservative quantities. The key idea of our method is to completely avoid using the data from the "mixed" cells and replacing them with the data obtained via the interpolation between the reliable data from the neighboring single-fluid cells. The interpolation is performed in the phase space so that the obtained values are physically meaningful.

I will then show how this method can be extended to compressible fluids in domains with moving boundaries. The fluid domain is placed in a rectangular computational domain of a fixed size, which is divided into Cartesian cells. At every time moment, there are three types of cells: internal, boundary (which play the role of the "mixed" cells), and external ones. The numerical solution is evolved in internal cells only, and the data required to compute the fluxes at the boundaries are computed using a solid wall ghost-cell extrapolation and an interpolation in the phase space.

Speaker: **Randall J. LeVeque** (University of Washington)
Title: *Quadrilateral Grids and Finite Volume Methods on the Sphere*
Abstract: I will describe some recent work with Donna Calhoun and Christiane Helzel on the development of finite volume methods for solving PDEs on the sphere and in other nonrectangular domain. The goal is to use logically rectangular grids that have nearly uniform cell sizes (unlike latitude-longitude or polar coordinates, for example). The grid mappings we use are nonsmooth and nonorthogonal, but second order accuracy can still be achieved.

Speaker: **Jian-Guo Liu** (University of Maryland)
Title: *All speed asymptotic preserving schemes for fluid and plasma*
Abstract: I will present a class of asymptotic preserving schemes for fluid and plasma which works for all Mach numbers. For small Mach numbers, usual difficulties in time stepping stiffness and numerical dissipations are completely removed by an elegant decomposition in the scheme construction. And the incompressible flow behavior is accurately captured. While for Mach numbers on the order of 1, the effectiveness of the usual shock capture scheme remains.

Speaker: **Yingjie Liu** (Georgia Tech)
Title: *Hierarchical Reconstruction for Discontinuous Galerkin Method on Triangular Meshes with a WENO-type Linear Reconstruction*
Abstract: In this work we further study the hierachical reconstruction (HR) (Liu, Shu, Tadmor and Zhang, SINUM '07) for discontinuous Galerkin method (DG) on triangular meshes. A WENO-type linear reconstruction is developed on each hierachical stage, in which the weights are essentially independent of the mesh shape and thus are relatively simple. This linear reconstruction is better at reducing the abrupt shift of stencils (which may cause degeneration of accuracy) for unstructured meshes and has better performance in resolving contacts. Several new phenomena and techniques are found during the research. For example, the linear DG method tends to have smaller and smaller CFL number as the order goes higher and higher. However, when the nonlinear HR is applied to each time step of the DG method, we find the CFL number can be almost as large as in the first order case. We also find that applying HR multiple times for higher order methods, e.g., 4th order or higher, can generate better resolution. This is somehow against our intuition that multiple times of reconstructions could lead to over-smearing.
(by Zhiliang Xu, Yingjie Liu and Chi-Wang Shu)

Speaker: **Pino Martin** (Princeton University)
Title: *Numerical challenges for direct and large-eddy simulations of highly compressible turbulence*
Abstract: The detailed simulation of compressible turbulent flows requires solving the conservation of mass, momentum and energy equations. For direct numerical simulations (DNS) and large eddy simulations (LES), a wide range of possible turbulent length scales and time scales must be resolved by the numerical method. Thus, DNS and LES require accurate representation of time-dependent propagation of high wavenumber (or high frequency), small amplitude waves. In addition, compressible turbulent flows are characterized by shockwaves that result in a sudden change of the fluid properties. Strong fluctuations can lead to the formation of transient shocklets, and boundary conditions and flow geometry might result in stronger, more permanent shocks. Therefore, methods for compressible. turbulent flows require robust shock capturing, as well as minimal numerical dissipation and dispersion errors. We have worked on refashioning weighted essentially non-oscillatory methods for the simulation of compressible turbulence. This work has enabled efficient and accurate DNS of turbulent flows that are relevant to reusable launch vehicles and scramjet engines.

In our current work, we are using converged DNS data of canonical flows, such as highly compressible isotropic turbulence interacting with shock waves, and DNS data of shock wave and boundary layer interactions to assess and develop a robust and efficient LES methodology for this type of flows. The formulation of LES requires the use of filtering operations. We are exploring a class of shock-confining filters (SCF) to avoid filtering across shocks. We find that linear filtering consistently causes data to exhibit anomalies immediately downstream of the main shock and that these anomalies are avoided by the application of SCF.

In this talk, I will demonstrate the reduction of numerical dissipation given by using linearly and non-linearly optimized WENO methods, and how the adaptation mechanism of WENO prevents a further reduction of the numerical dissipation. In addition, I will illustrate the need for nonlinear filtering procedures to enable robust LES of compressible turbulence. Finally, I will describe the DNS database of shock containing turbulent flows that is available for download to the community.

Speaker: **Ian Mitchell** (University of British Columbia)
Title: *The Ellipsoidal Particle Level Set*
Abstract: In the particle level set (PLS) method [Enright et al, JCP 2002] Eulerian level sets were combined with unstructured Lagrangian marker particles to produce a surface evolution algorithm with some of the best features of both: simple approximations of geometric quantities from the implicit surface function and accurate volume preservation in convective flows from the particles, all without any need for complex data structures to manage connectivity. However, despite the success of PLS many challenges remain; for example, PLS does not properly delete particles when the characteristics along which they are travelling

merge, even though pure level set methods correctly adapt to this situation. In this talk I outline how we can use ellipsoidal particles to overcome some limitations of the basic PLS. The results are demonstrated on reachability problems and the algorithms are implemented in the Toolbox of Level Set Methods.

Speaker: **Jianliang Qian** (Michigan State University)
Title: *Eulerian Gaussian Beams for Semi-Classical Solutions of Schrödinger Equations*
Abstract: We propose Gaussian-beam based Eulerian methods to compute semi-classical solutions of the Schrödinger equation. Traditional Gaussian beam type methods for the Schrödinger equation were based on the Lagrangian ray tracing. We develop a new Eulerian framework which uses global Cartesian coordinates, level-set based implicit representation and Liouville equations. The resulting method gives uniformly distributed phases and amplitudes in phase space simultaneously. To obtain semi-classical solutions to the Schrödinger equation with different initial wave functions, we only need to slightly modify the summation formula. This yields a very efficient method for computing semi-classical solutions to the Schrödinger equation. For instance, the proposed algorithm requires only $O(sNn^2)$ operations to compute s different solutions with s different initial wave functions under the influence of the same potential, where $N = O(1/\hbar)$, $\hbar$ is the Planck constant, and $n \ll N$ is the number of computed beams. Numerical experiments indicate that this Eulerian Gaussian beam approach yields accurate semi-classical solutions even at caustics.

Speaker: **Susana Serna** (UCLA)
Title: *A characteristic-based nonconvex entropy-fix upwind scheme for the ideal magnetohydrodynamic equations*
Abstract: We present an analysis of the wave structure of the ideal magnetohydrodynamic (MHD) equations. We propose an appropriate spectral decomposition in local wavefields allowing to detect nonconvex singularities. We design a characteristic-based upwind scheme that resolves the wave dynamics by local characteristic fields using the proposed complete system of eigenvectors. The new scheme is able to detect local regions containing nonconvex singularities and to handle an entropy correction by prescribing a local viscosity that ensures convergence to the entropy solution. A high order accurate version of the scheme is tested through a set of one and two dimensional MHD problems.

Speaker: **Chi-Wang Shu** (Brown University)
Title: *An overview on high order schemes for hyperbolic problems*
Abstract: In this talk we will give an overview of algorithm development and application, on high order schemes for hyperbolic problems. We will mainly discuss the finite difference weighted essentially non-oscillatory (WENO) schemes, finite volume WENO schemes, and discontinuous Galerkin (DG) finite element methods. A comparison of their relevant advantages and disadvantages will be given. We will also discuss a few recent developments including well balanced high order schemes, a high order conservative Lagrangian type scheme for the compressible Euler equations, and superconvergence and time evolution of discontinuous Galerkin finite element solutions for hyperbolic equations.

Speaker: **Marshall Slemrod** (University of Wisconsin)
Title: *Conservation Laws: Transonic Flow and Differential Geometry*
Abstract: In this talk I will show how the equations for steady 2 D gas dynamics and the equations for embedding a two dimensional Riemannian manifold into three dimensional Euclidean space. Furthermore I will show how the Murat- Tartar method of compensated compactness plays a crucial role in proving existence of weak solutions to both problems. the method all suggest a numerical algorithm for resolvng transonic flow around an airfoil.

Speaker: **Tao Tang** (The Hong Kong Baptist University)
Title: *Moving grid methods and Multi-mesh methods*
Abstract: In this talk, we present an adaptive moving grid strategy for partial differential equations in two- and three-space dimensions. The algorithm automatically adjusts the size of the finite elements

in the physical domain to resolve the relevant scales in multiscale physical systems while minimizing computational costs. Some subtle issues in the moving mesh scheme, in particular the solution interpolation from the old mesh to the new mesh and the choice of monitor functions, will be addressed. Since the mesh redistribution procedure normally requires to solve large size matrix equations (arising from discretizing the Euler-Lagrange equations or a minimization problem), we will describe a procedure to decouple the matrix equation to a much simpler block-tridiagonal type which can be solved by multi-grid methods efficiently.

To demonstrate the performance of the proposed moving mesh strategy, the algorithm is implemented in moving finite element computations for multiphase flows and dendritic growth simulations. Numerical results on two- and three-dimensional simulations will be presented.

In dendritic growth simulations, the governing equations used are the phase field equations, where the regularity behaviors of the relevant dependent variables. The behaviors of the thermal field function and the phase field function can be very different. To enhance the computational efficiency, we approximate these variables on different h-adaptive meshes. The coupled terms in the system are calculated based on the multi-mesh h-adaptive method. It is illustrated numerically that the multi-mesh technique is useful in solving phase field equations and can save storage and the CPU time significantly.

Speaker: **Alexander Vladimirsky** (Cornell University)
Title: *Causal numerical methods for hyperbolic problems*
Abstract: The direction of "information flow" is well-defined (even if not a priori known) for hyperbolic PDEs. Lagrangian-framework methods exploit this by efficiently computing solutions along characteristics, but suffer from two other deficiencies: highly non-uniform resolution and crossing of characteristics. Eulerian-framework discretizations have the advantage of easily controlled resolution and high accuracy, but the resulting discretized equations are often nonlinear and coupled, resulting in expensive iterative schemes. The idea behind "causal methods" is to use Lagrangian information to choose better Eulerian discretizations, so that the resulting system can be decoupled and solved efficiently. We will illustrate this principle by describing causal methods for viscosity solutions of static Hamilton-Jacobi PDEs arising in optimal control (including randomly-terminated and multi-criterion-optimized processes) and causal methods for multivalued solutions of general first-order PDEs.

Speaker: **Yong-Tao Zhang** (University of Notre Dame)
Title: *Uniformly accurate discontinuous Galerkin fast sweeping methods*
Abstract: In this talk, I will present our recent work on developing uniformly accurate discontinuous Galerkin fast sweeping methods for solving Eikonal equations. In order to achieve both high order accuracy and fast convergence rate (linear computational complexity), the central question is how to enforce the causality property of Eikonal equations in the compact discontinuous Galerkin (DG) local solver. We design the causality indicators which guide the information flow directions for the DG local solver. The initial values of causality indicators are provided by the first order fast sweeping method, and corrected by the second order DG local solver during the iterations. We observe both a uniform second order accuracy in L^∞ norm and fast convergence speed (linear computational complexity) in the numerical examples.

Speaker: **Hongkai Zhao** (UC Irvine)
Title: *Contration property of the fast sweeping method* Abstract: I will discuss a few nice contraction properties of upwind scheme combined with Gauss-Seidel iteration for stationary Hamilton-Jacobi equations. These properties are essential for the fast convergence of the fast sweeping method. I will also talk about a few interesting implications on numerical error for both the solution and its gradient based on these contraction properties.