Integral and Metric Geometry (Online)

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This workshop was originally planned for November 2020. Due to the Corona pandemic, it had to be postponed to May 2022. It was held as an online workshop.

1   Overview of the Field

Modern geometry consists of numerous, virtually independent fields. A unifying theme of mathematics is the uncovering of deep ties between different, seemingly unrelated facts. The online workshop succeeded to improve our understanding of one such deep link, the Weyl principle, which forms a bridge between the disciplines of integral geometry and valuation theory on one side, and metric geometry and spaces with curvature bounds on the other.

Valuation theory grew out of integral geometry, and studies such geometric quantities as volume, surface area, and their generalizations. Spaces with curvature bounds are geometric objects which are not necessarily smooth, but that retain nevertheless some of the features of smooth spaces. For example, convexity/concavity properties of the distance function can be seen as a non-smooth generalization of the smooth notion of curvature of constant sign.

2   Recent Developments and Open Problems

The Weyl principle predicts strong relations between metric aspects and integral geometry. Some recent developments in this direction that were discussed in the talks are

- Intrinsic volumes and collapse (talks by Alesker, Hofstaetter, Prosanov) [1]

- Weyl principle in a Finsler setting (talks by Ivanov, Wannerer) [3]

- Weyl principle in a pseudo-Riemannian setting (talk by Solanes) [2]

- New geometric inequalities (talks by Hoisington, Kotrbatý, Milman, Scheuer, Wenger) [4, 5]

3   Presentation Highlights

As one of the aims of the conference was to bring together researchers from metric geometry and from integral geometry, three longer survey talks at the interplay between the two areas were organized. In his talk

Curvature measures: yesterday, today, and tomorrow, Joseph Fu (University of Georgia, Athens) gave an overview about historical and current developments related to curvature measures, in particular about the analytic and metric aspects of the Weyl principle. Juan Carlos Alvarez Paiva (University of Lille) gave a talk about **Finsler manifolds from a convex-geometric viewpoint**, in which he explained how important results from convex geometry can be used in the study of Finsler manifolds, and conversely how questions in Finsler geometry lead to new and open conjectures in convex geometry. Dmitri Burago (Penn State University) talked about **Some tools from integral geometry used in different areas of mathematics**. He showed how tools from convex geometry, like Crofton type formulas, can be applied in metric geometry, for instance in the study of Finsler tori and Besicovich' inequality.

Other talks were related to integral currents and isoperimetric inequalities in metric spaces (Sormani, Wenger, Perales); to Alexandrov spaces (Alesker, Prosanov, Petrunin); and to various recent topics in integral geometry and/or metric geometry reaching as far as discrete geometry/analysis (Izmestiev) and classical convex geometry (Werner) yet connected to the theme of the workshop making once again the case for the diversity and breadth of the field.

A particular emphasis was to invite young researcher to present their recent works. Among them were Georg Hofstaetter, Joseph Hoisington, Jan Kotrbatý, Raquel Perales, Roman Prosanov and Julian Scheuer. A higher than usual number of participants were asked to chair talks, matching up with their areas of research, which resulted in an energetic and vibrant exchange afterwards despite having the talks online.

4   Scientific Progress Made and Outcome of the Meeting

It became quite clear during the meeting that the interplay between integral geometry on the one hand side and metric geometry on the other, is even deeper than what we originally expected. In many of the talks we have discovered strong connections between seemingly different topics such as Finsler manifolds, metric spaces with a curvature bound, translation invariant valuations and isoperimetric inequalities. A lot of new questions and conjectures emerged in the lively discussions after the talks. We hope and expect that the workshop contributed in initiating new collaborations and in opening new research directions.

References

[1] Semyon Alesker. Some conjectures on intrinsic volumes of Riemannian manifolds and Alexandrov spaces. *Arnold Math. J.*, 4(1):1–17, 2018.

[2] Andreas Bernig, Dmitry Faifman, and Gil Solanes. Curvature measures on pseudo-Riemannian manifolds. To appear in J. Reine Angewandte Math.

[3] Dmitry Faifman and Thomas Wannerer. The Weyl principle on the Finsler frontier. *Selecta Math. (N.S.)*, 27(2):Paper No. 27, 30, 2021.

[4] Jan Kotrbatý. On Hodge-Riemann relations for translation-invariant valuations. *Adv. Math.*, 390:Paper No. 107914, 28, 2021.

[5] Emanuel Milman and Joe Neeman. The Gaussian double-bubble and multi-bubble conjectures. *Ann. of Math. (2)*, 195(1):89–206, 2022.